ZMATHT22 Made by Tommy Zhao on June 4, 2010 for the Whitney Young Math Team

Version: 2.2, Size: 12018 b, Compatible with at least Ti-83 Plus, Ti-83 Plus Silver, Ti-84, and Ti-84 Silver

WARNING!: This program uses modes of a+bi, degree, grid off, plots off, and zoom standard. The input for angles should always be in degrees mode. This program also uses L_1, L_2, and variables A to U and does not delete them so that (1) you can exit the program faster, and (2) you can look at the answer if you know which variable (usually A) or list (usually L_1) they were stored. In menus where there are less than 5 options, there will be more than 1 CANCEL (Back) because this program is easy to control since 6 is always cancel (back) (except in the main menu) and 7 is always exit. For best results, use the latest version.

The latest version of this program will be posted online for download upon request.

MATH PROGRAM OPERATIONS

ALGEBRA

Prime Factorization – given a number, it will find the prime factorizations of that number. For example, given 90090, it will spit out {2,3,3,5,7,11,13}. This program uses L_1, and L_2.

Factors if N – given a number, it will find all the factors of that number and tell you the number of factors. For example, given 48, it will tell you that there's 10 number of factors, which are {1,2,3,4,6,8,12,16,24,48}

Simplify Radicals – given a radical, it will simplify the radical. For example, given 5*sqrt(6525), it will spit out 75*sqrt(29).

Multiply polynomial – given values, it will multiply 2 roots, 3 roots, 4 roots, 3 binomials or 2 trinomials. For example, given (1x+2)(3x+4)(5x+6)(7x+8), it will spit out 105x^4+596X^3+1244X^2+1136X+384.

Quadratic Formula and Factoring – given a trinomial, it will solve it, find the discriminate, and factor (if possible) the trinomial as well. For example, given -8X^2-18X-4, it will spit out X=-2,-.25 and factors are (4x+1),(x+2), and -2.

ANALYTICAL GEOMETRY

Shoelace Theorem – given points, it will find the area between the points.

Midpoint, Line, and Distance – given 2 points, it will find the midpoint, the formula of the line that connects the 2 points, and the distance between the 2 points. For example, given points (1,2),(3,4), it will spit out midpoint is (2,3), line of the 2 points is y=1X+1, and distance is 2.828427125 or 2*sqrt(2)

Midpoint, Plane, Area, Volume of Parallelepiped – given 3 3D points, it will find the midpoint, the formula of the plane that the 3 points lie on, the area between the 3 points, and the volume of the parallelepiped if those points were, instead, vectors. For example, given points (1,3,2),(3,-1,6), and (5,2,0) it will spit out midpoint is

(3,1.3333333,2.6666667), the plane is 6x+10y+7z-50=1, the area is 13.60147051 or sqrt(185), and the volume is 10 or sqrt(100). This can also do the “Bunny Loop” method by inputting 0s for all 3 zs.

Point Refraction (Reflection) – given a point and something to refract (reflect) over (another point, line, x or y-axis, or rotated axes) it will find the image (new object). If it refracts (reflects) over a line, it will also find the distance so it can be used to find the distance between 2 parallel lines given that you know a point on one of the line and the formula for the other line. For example, from ITCM 2005 J-S 8 person #16, given point (7,-1) to refract over line Y=X+4, it will say that the new image is (-5,11) and the distance between the point and the line is 8.485281374 or 6*sqrt(2). The radical distance will only be given if the number in the radical is an integer. And if the distance isn’t in nice radical form, then the answer is stored in variable B for you to use a radical estimator program if you have one.

Axes Rotation – given AX^2+BXY+CY^2+DX+EY+F=0, it will identify the conic section for you using the discriminate, tell you what angle to rotate the axes about to eliminate the xy-term, and give you the rotated axis formula where the xy-term is 0.

Trigonometry Triangles – given SSS, SSA, SAS, SAA, or ASA in degrees, it will find the missing side(s), angel(s), and area of the triangle It will also deal with the infamous SSA case by showing 2 solutions (if 2 triangles exists) as well as 0 solutions (if no triangles exists). For example, given SSA of 10, 11, and 20 degrees (#15 of 2009-2010 CCML Precalc accelerated contest #4), it will spit out that the 1st triangle have angles of 22 and 138 degrees, a side of 20, and an area of 37 and the 2nd triangle have angles of 2 and 158 degrees, a side of 1, and an area of 2. The triangle picture will always be the same, but it’s there to tell you the relative angels and sides and how they are related to one another. Side a will always be the shortest while side c will always be the longest.

Precalculus

Synthetic Division – given a polynomial and a divisor, it will do synthetic division and find the roots if the remainder is 0. For example, given 1X^3+2X^2+3X+-6 to be divided by X-1, u get 1X^2+3X+6 and the remainder is 0 so the roots are 1, -1.5-1.9364i, and -1.5+1.9364i. The roots will only be given if the remainder is 0.

Divide Polynomial – given a trinomial (Or polynomial of a lower degree) over another, it will find the intercepts and asymptotes. WARNING: The results will be wrong if there are holes in the graphs so make sure one cancels out the common binomials manually before plugging the equation into this program.

Binomial Theorem – given info of (aX+bY)^n, it will execute the binomial theorem if n is no greater than 19. This uses list 1.

Vectors – given vector(s), it will find the component vectors, unit vector, magnitude and direction of the vector, the angle between 2 vectors, the sum of the vectors, the projection of the 1st vectors onto the 2nd vector, the dot product, and the cross product of the 1st vector cross the 2nd vector. Given vectors <1,2,3>,<4,5,6>, it will spit out that there is 12.933154 degree between the 2 vectors, the magnitude of the sum is 12.4499, the direction above is x-axis of the sum is 54.462322 degrees, the direction above the y-axis of the sum is 52.125016 degrees, the projection of the 1st vector onto the 2nd vector is the vector <1.6623377,2.779221,2.4935065>, the dot product is 32, and the cross product is <-3,6,-3>.

Complex Numbers – given info, it will find the standard form, polar (trig) form, DeMoivre’s power, or DeMoivre’s roots in standard form or polar form in degrees and radian mode.

Statistics of L_1 – given list 1, it will find the arithmetic mean, geometric mean, median, standard deviation, and variance. Because the binomial theorem, prime factorization, trig triangles, and a lot others also uses L_1 if you want to find the statistics of list 1, you should run this program before you run the others, unless you want the statistics of the numbers that the other programs have placed in L_1. This gives you an error (which I don’t know how to prevent at the moment) if you do not have a list 1.

REVISIONS

Version 2.2 June 4, 2010

- Fixed the line between 2 points so that the variables are correct.
- Renamed refraction to reflection.
- Fixed factoring if -1 is a factor.
- Improper exiting is improved so that axes won’t be turned off, unless you exit out on a screen that uses text.

Version 2.1 May 27, 2010

- Fixed simplify radical error if A=0.
- The answers are now displayed in fraction form if possible.
- Added in shoelace theorem and removed clock angels. Thanks to Shunping Xie.

Version 2.0 May 25, 2010

- Fixed statistics so that it turns back on the axes and function because doing the calculations in case of overflow.
- Canceled more errors, such as, the binomial’s n^{th} power has to be an integer for the binomial theorem to work.
- Reused the “INVALID INPUT” display trigger to save memory.
- Added in volume of parallelepiped, given 3 vectors.
- Conjoined refraction between the x-axis and the y-axis.
- Tweaked the program so that it’s more condensed and organized to help future edits as well as other programmers who are interested in looking through this program.
- Fixed the drawing of the triangle when zoom isn’t standard. Thanks to Shunping Xie.
- Fixed complex roots in polar form so that it shows invalid for inputting a negative magnitude.
- Still more aesthetic changes.

Version 1.9 May 24, 2010

- Still more aesthetic changes. Renamed a few program. Such as changing “Refraction” to “Pnt Refraction”, “Mdpt Dist Line” to “Mdpt Line Dist”, or “Mpt Area Plane” to “Mpt Plane Area”.
- Added in finding the image (refracted) on rotated axes.
- Added in showing what angel to rotate the axes to eliminate the xy-term and solving for the rotated axes equation.
- Fixed the SSA trig triangle so that there will be no triangle if it is trying to do arcsin of something that is greater than 1. (For some reason, it didn’t used to give me an error; instead, it outputs none of the number of the input.)
- Added in showing the square of radicals for most radicals.
- Improved complex roots so that it now does it in degrees and radians (in terms of pi).

Version 1.8 May 19, 2010

- Added in solving for the discriminate when given a trinomial to solver/factor.
- Placed statistics to the top of the program, so that if it isn’t protected and there is no L_1 and you click goto, you will not have to standby for a minute as the calculator scrolls through this entire program.
- Put Stats under Precalculus menu.
- Improved the dividing polynomial program so that it uses draw and have a lot more conditional operations to cancel out the errors.
- Deleted dividing polynomials since it took too much memory as programming it had too many conditional operations. (And yes, this change does cancel out the previous change stated above.) People should know to first remove holes if any, the x intercepts are the zeros of the numerator, the y intercept is the constant of the numerator over the constant of the denominator, vertical asymptotes are the zeros of the denominator, the horizontal asymptotes are the limits as x approaches infinity, and the slant asymptotes are the numerator divided by the numerator without the remainder.
- Revised the main menu option 6 so that it NO LONGER says 2009-2010 Whitney Young Math Team.

Version 1.7 May 17, 2010

- Still more aesthetic changes.
- Revised trinomial factoring.
- Revised vectors.
- Revised refraction from a point to a line to show distance in the form sqrt(.
- Program name change.
- Revised conic identification so it tells you what angel to rotate to eliminate the xy-term.
- Revised prime factorization for speed.
- Revised DeMorive’s Roots for standard form so it uses draw (an aesthetic change).

Version 1.6May 5, 2010

- Protect.
- Fix a lot of error cases where you divide by 0.

Version 1.5 May 5, 2010

- Added back geometry clock since I had room in geometry menu and it doesn’t take much RAM.

- A lot more other aesthetic changes.
- Upgraded the equation of a plane given 3 points so that it gives the area between the 3 points and the midpoint of the three points.

Version 1.4 May 4, 2010

- Improved Integral factoring program.
- Greatly improved trig triangles.
- HUGE ASTEHTIC CHANGE by reformatting it to use lower case (except for warnings and menu titles). Also, the format of the program itself is now .8x instead of .83.
- Added in equation of a plane given 3 points.
- Removed Geometry clock because those question doesn't pop up often.

Version 1.3May 3, 2010

- Added Integer factors.

Version 1.2 May 1, 2010

- Semi-fixed list 1 stats problem. Thanks to Mark Koziel.
- Modified DeMoivre's powers so it's more useful.

Version 1.1 April 29, 2010

- Fixed trig triangle formulas.
- Fixed slope formula between 2 points.
- Upgraded the binomial theorem so that it does (**AX**+**BY**)^N instead of (**XA**+**YB**)^N.
- Upgraded the conic identification program so that it shows no conic sections if there are no conic sections.
- Upgraded various other programs to shows "INVALID" rather than giving you an error.
- I reuse a few triggers for more than one program, namely the radical simplifier, to save RAM.
- The quadratic formula is done and shown in case you need it in (X±sqrt(Y))/Z form.
- Fixed an error that gives you the wrong sign from the synthetic division roots.
- Dividing polynomial has been revised so it more likely to solve the intercepts. However, beware of holes in the graph.
- Vectors can now be done in 2D for those who want to use it for precalculus rather than multivariable calculus or Physics C. It's actually still the exact same program but the k components are set to 0 so you don't have to enter them. Thanks to (because of) students in Precalculus, who doesn't work with 3D vectors.
- The complex numbers now works.
- Now operational under Mirage OS. Thanks to Shunping Xie.

NOTE: I am sorry if there are any errors (there shouldn't be much because I have easily spent over 200 hours programming and 3 calculator battery changes) on this program that ate my spring break and free and non-free time and I will try to get them fixed when I hear about them. And yes, I do spend a lot of time on math projects. If you like this one, my geometry backyard project with Ms. Gitles and the trigonometry problem project with Ms. Gustavson should also WOW you, although both of them weren't 100% complete when I turned them in. This calculator program math project that I'm doing specifically for math team, as you can see, is also incomplete. I am also working on a physics formula program at the moment, but I don't think I can finish in time of my graduation, so yea… Good luck and have fun. Program brought to you by Duracell (for battery-ing loads of my program testing), Ti calculator, Ti Graph Link and Ti Connect, Starcraft (for its programming experience), Ms. Gitles (for the area, surface area, and volume section of the geometry, which is no longer in the program because of memory and because there is a lot other apps and programs for area, and volume), Ms. Chhabra (for teaching Precalculus), Professor Epp (and yes, she is the author of the discrete math with applications book) (for teaching Discrete Mathematics, a prerequisite of computer science), Professor Groth (for teaching Precalculus and teaching me how to use solver on the calculator, and also suggesting to fully utilize the calculator), Professor Butterworth (for teaching Calculus I and Calculus II), Professor Catoiu (for teaching Calculus III and Multivariable Calculus I (for the plane equation program)), MIT Open Courseware (for teaching Multivariable Calculus), James Stewart (for authoring my Precalculus, Calculus, and Multivariable Calculus books, where I reviewed many equations to write this program), Ms. Gustavson (for teaching almost everything from the beginning of algebra 2 to the end of Precalculus, and who also showed many proofs, such as where the quadratic formula came from, and who told us not to be the one using the calculator, but the one to program/understand it) (and yes, I have had 3 Precalculus educators, 8 math classes in the span of 14 months, and (although not relevant) 7 science classes in the span of 2 years), and the 2009-2010 Whitney Young Math Team-ers and coaches, Ms. Au and Mr. Moran, for teaching me so many different ways to solve math problems. This program is for you, math teamers (especially for the upcoming senior math teamers).

Program sub-authors: Although I did not copy letter by letter for any of the sub-programs in this current program, a few of them are based off other people's programs. Wanting to give credit when it's due, prime factorization and simplifying radicals came from the formula team, factors of N is based off of John Joe Friedman's Test program, the shoelace theorem is based off Shunping Xie's AAAMATH, the trig triangle picture are based off a program by C Baker, and the binomial theorem is based off a program by Tony S.

Any errors, comments, or questions can be sent to tzhao451@yahoo.com

Scheduled Physics Program Release Date: July 11, 2010 (just a random date to set as the goal, but it's also 1 month after my graduation), but don't count on it. It's going to being a formula program that covers 3 years of physics, but projects like these usually takes longer than I expect them to. The program is now like 5% done and a preview of this program is now available.

Newtonian Mechanics

Kinematics & Dynamics

Kinematics

Position, Velocity, and Acceleration

position=x=fnInt(v,t)= fnInt(fnInt(a,t),t) in m

velocity or speed=v= nDeriv(x,t)=fnInt(a,t) in m/s

velocity is vector, speed isn't

acceleration=a=nDeriv(v,t) =nDeriv(nDeriv(x,t),t) in m/s^2

Linear Motion Equations

$x=x_0+.5(v+v_0)t$

$x=x_0+v_0t+.5at^2$

$x=x_0+vt-.5at^2$

$v=v_0+at$

$v^2=v_0^2+2a(x-x_0)$

$a=(v-v_0)/t$

number of g's=a/g in g's

Falling Objects

$y=y_0+.5(v+v_0)t$

$y=y_0+v_0t-.5gt^2$

$y=y_0+vt+.5gt^2$

$v=v_0-gt$

$v^2=v_0^2-2g(y-y_0)$

$g=(v_0-v)/t$

g=acceleration of gravity=9.8 m/s^2=32.2 ft/s^2

Projectiles

vertical velocity = v_y

horizontal velocity = v_x

$v_x=v_{x0}$

$x=x_0+v_{x0}t=x_0+v_xt$

Same Height Projectiles

range=$v_0^2\sin(2\theta)/g$ in m

max height=$v_0^2\sin^2\theta/(2g)$ in m

flight time=$2v_{0y}/g$ in s

Vectors and Vector Products

Vectors

$a_x=a\cos\theta$, $a_y=a\sin\theta$

$a=\sqrt{(a_x^2+a_y^2)}$

$\tan\theta=a_y/a_x$

Dot Product

a dot b=$ab\cos\theta=a_ib_j+a_ib_j+a_ib_j$

Cross Product

a cross b=$ab\sin\theta$=determinate in the direction determined by the RHR=$<a_jb_k-a_kb_j, a_kb_i-a_ib_k, a_ib_j-a_jb_i>$

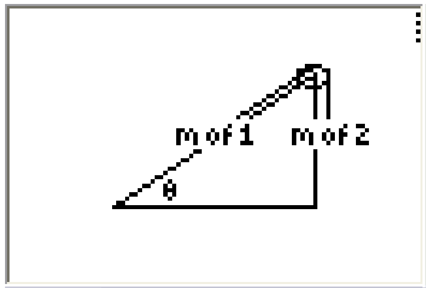
Dynamics

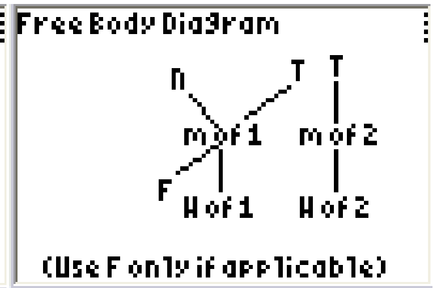
Mechanical Force

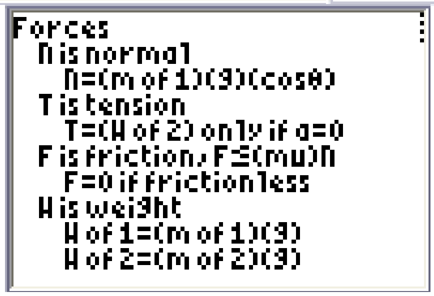
$F=ma$=nDeriv(p,t)=$(p-p_0)/(t-t_0)=J/(t-t_0)=W/((r-r_0)\cos\theta)=P/(v\cos\theta)=\tau/(r\sin\theta)=-kx=-Gm_1m_2/r^2$ in N

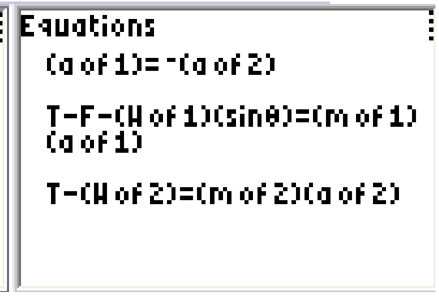
Negative means force opposes displacement or force is attractive (as opposed to repulsive).

Ramp Forces (Example)









$a_1=-a_2=g(m_2-m_1\sin\theta-+\mu m_1\cos\theta)/(m_1+m_2)$

$a_2=g(m_1\sin\theta-m_2+-\mu m_1\cos\theta)/(m_1+m_2)$

$T=m_2(a_2+g)=(m_1m_2g)(1+\sin\theta+-\mu\cos\theta)/(m_1+m_2)$

Use 1st operation of the +- or -+ sign if m_1 accelerates right.

Redo friction

Use F to oppose net force on m_1 if applicable.

If direction is based on friction (that is, after trying both + and – and the acceleration does not remain constant positive or constant negative), then no net force.

Friction

Static friction on non-moving objects

Kinetic friction on moving objects

$F_{fric}\leq\mu N$

$F_{fric}\leq F_{opposing}$

Max $F_{fric}=\mu_s N$

$\mu=\tan\theta_{\text{block slide on ramp w/ const speed}}$

Vertical Circles

Add forces toward center, subtract forces away center and set $F_{net}=ma_c= mv^2/r$

Planes turn by banking the wings at an angle of $\tan^{-1}(v^2/(rg))$.

Curves should be banked at an angle of $\tan^{-1}(v^2/(rg))$ so cars will not skid.

Derived from $N\cos\theta=mg$ and $N\sin\theta=mv^2/r$

Critical Velocity (Derivation)

$F_{\text{Tension or Normal}}+mg=F_c$

$F_{\text{Tension or Normal}}+mg=ma_c$

$\lim(F_{\text{Tension or Normal}}+mg ,F_{\text{Tension or Normal}},0)= mv^2/r$

$mg=mv^2/r$

$v_c=\sqrt{(rg)}$

Drag Force

$D=-bv$ or $D=-bv^2$

b is the drag constant

The negative means drag force opposes velocity.

If $D=-bv$, then b is in kg/s and terminal velocity=mg/b

If D=-bv2, then b is in kg/m and terminal velocity=$\sqrt{(mg/b)}$

Drag Derivation on Motion Equations (Derivation)

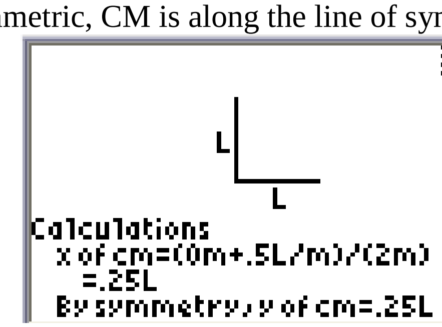
$F_{net}=ma$

$mg-bv=mdv/dt$

dt=mdv/(mg-bv)
fnInt(1,t,0,t)= fnInt(m/(mg-bv),v,0,v)   let u=mg-bv, du=-bdv
t=(-m/b)fnInt(1/u,u)
t=(-m/b)(ln(u)-ln(u_0))
-bt/m=ln(mg-bv)-ln(mg)
-bt/m=ln((mg-bv)/(mg))
e^(-bt/m)=(mg-bv)/(mg)
mg(e^(-bt/m))=mg-bv
bv=mg(1-e^(-bt/m))
v(t)=v_t(1-e^(-bt/m))

Work, Energy, Power
- Work
  - Work
    - W=Fdcosθ=fnInt(F,r) in J
  - Work-Energy Theorem
    - $KE-KE_0+U-U_0=W$
  - Law Conservation of Energy
    - $KE-KE_0+U-U_0=0$
  - Friction and Energy
    - $KE-KE_0+U-U_0=W_{friction}$
  - Work and Power
    - W=Pt
- Energy
  - Kinetic Energy (Derivation)
    - K=fnInt(F,x)=fnInt(ma,x)=(m)fnInt(nDeriv(v,t),x)=(m)fnInt(nDeriv(x,t),v)=(m)fnInt(v,v)=.5mv^2=.5p^2/m in J
  - Gravitational Potential Energy
    - U=mgh=-G$m_1 m_2$/r in J
  - Spring Potential Energy
    - U=.5kx^2 in J
  - Rotational Energy
    - R=.5Iω^2=.5I$(v/r)^2$ in J
- Power
  - P=nDeriv(W,t)=W/t=Fd/t in W or J/s
  - If v is constant, P=Fvcosθ

Systems of Particles, Linear Momentum
- Center of Mass
  - r_{cm}=sum(seq(mr))/M=fnInt(r,m)/M
    - m is the tiny piece of mass
    - r is the distance along axis of calculation from the reference point to center of the tiny piece of mass
    - M is the total mass
  - If symmetric, CM is along the line of symmetry



- Momentum and Impulse

Momentum

$p=mv$ in Ns or kgm/s

Impulse

$J=fnInt(F,t)=F(t-t_0)=p-p_0=m(v-v_0)$ in Ns or kgm/s

Elastic Collisions: Bounce Off

Momentum is Conserved

$m_A(v_{A0}-v_A)=m_B(v_B-v_{B0})$

KE is Conserved

$m_A(v_{A0}{}^2-v_A{}^2)=m_B(v_B{}^2-v_{B0}{}^2)$

$v_A=[(m_A-m_B)v_{A0}+2m_Bv_B]/(m_A+m_B)$

Inelastic Collisions: Stick

Momentum is Conserved

$m_Av_{A0}+m_Bv_{B0}=(m_A+m_B)v$

$v=(m_Av_{A0}+m_Bv_{B0})/(m_A+m_B)$

KE is NOT conserved

$KE-KE_0=K_A-K_{A0}+K_B-K_{B0}$

2D Collisions

Same Equations

Work on x and y components separately.

Non-head-on elastic collisions of the same mass come apart at 90 degrees.

Circular Motion and Rotation

Uniform Circular Motion

period=$T=1/f$ in s

frequency=$f=1/T$ in Hz

$v_c=2\pi r/T=2\pi rf=\omega r$ in m/s

$a_c=v^2/r=4\pi^2r/T^2=4\pi^2rf^2=2\pi v/T=2\pi vf=\omega^2r$ in m/s^2

$F_c=ma_c$

Angular Momentum and its Conservation

Angular Momentum

$L=I\omega=r$ cross p in kgm^2/s

If object B added onto apparatus

$L=L_0$

$\omega=I_A\omega_0/(I_A+I_B)$

Rotational Kinetic Energy

$KE_{rot}=.5I\omega^2$ in J

Rotational Work

$W=\tau\theta=Fr\theta=Fs$ in J

Power

$P=W/t=\tau\theta/t$ in W

If ω is constant, $P=\tau\omega$

Inertia

$I=?mr^2=fnInt(r^2,m)=sum(seq(mr^2))=fnInt(r^2\rho,V)$ in kgm^2

Resist assuming all the mass is concentrated at the center of mass.

Parallel Axis Theorem

Use the parallel axis theorem when a pendulum rotates about a pivot point that is parallel to its rotation about the center of mass.

$I=I_{CM}+mr^2$

Perpendicular Axis Theorem

Use the perpendicular axis theorem when there are moments of inertia of two perpendicular axes through the same point in the plane of the object.

$I=I_x+I_y$

Uniform Thin Rod, Axis Perpendicular to Length (Example)

$dm/M=dx/L$ so $dm=(M/L)dx$

$I=\text{fnInt}(x^2,m)=(M/L)\text{fnInt}(x^2,x,-h,L-h)$

$=[(M/L)(x^3/3)]_{-h}^{L-h}=(1/3)M(L^2-3Lh+3h^2)$

Torque and Rotational Statics

$\tau=r$ cross $F=rF\sin\theta=I\alpha=\text{nDeriv}(L,t)$ in mN

Static Equilibrium

$F_{up}=F_{down}$

$T_{cw}=\tau_{ccw}$

You are allowed to move forces in the direction they point for easier calculations.

Rotational Kinematics

angular displacement$=\theta-\theta_0=(s-s_0)/r$ in rad

angular speed$=\omega=(\theta-\theta_0)/t=(s-s_0)/(rt)=v/r=2\pi f=2\pi/T$ in rad/s

angular acceleration$=\alpha=(\omega-\omega_0)/t=a_T/r$ in rad/s^2

centripetal acceleration$=a_c=v^2/r=4\pi^2r/T^2=4\pi^2rf^2=2\pi v/T=2\pi vf=\omega^2r$ in m/s^2

tangential acceleration$=a_T=(v-v_0)/T=\alpha r$ in m/s^2

acceleration$=a=\sqrt{(a_c^2+a_T^2)}$ in m/s^2

$\theta=\theta_0+.5(\omega+\omega_0)t$

$\theta=\theta_0+\omega_0t+.5\alpha t^2$

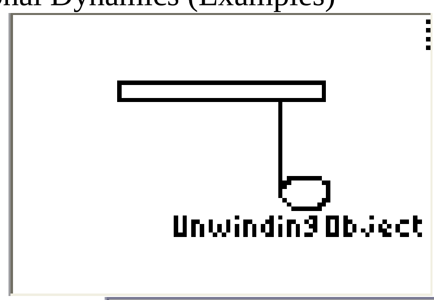
$\theta=\theta_0+\omega t-.5\alpha t^2$

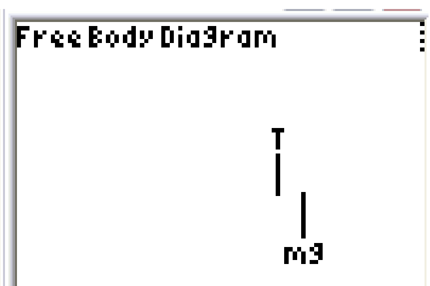
$\omega=\omega_0+\alpha t$

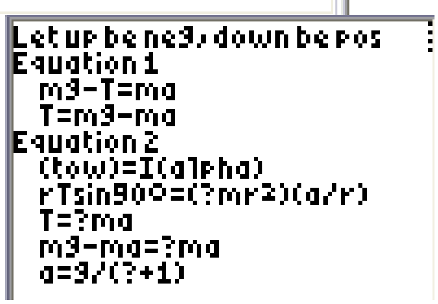
$\omega^2=\omega_0^2+2\alpha(\theta-\theta_0)$

$\alpha=(\omega-\omega_0)/t$

Rotational Dynamics (Examples)







How would you set this up if you want to keep up as positive?

Big Wheel (Picture Not Yet Created)

Set Up Equations

$mg-T=Ma$

(tow)=I(alpha)

$Tr=.5MR^2(a/r)$

$T=mg-Ma=mg-(mg)/(.5R^2/r^2+1)$

$a=(mg-T)/M=(mg)/(.5MR^2/r^2+M)$

Oscillation and Gravitation

Simple Harmonic Motion

Spring

$T=2\pi\sqrt{(m/k)}=2\pi/\omega$ in s

$f=\sqrt{(k/m)}/(2\pi)$ in Hz
$\omega=2\pi/T=2\pi f=\sqrt{(k/m)}$ in rad/s
$k=\omega^2 m$ in N/m

Pendulum
$T=2\pi\sqrt{(L/g)}=2\pi/\omega$ in s
$f=(1/(2\pi))\sqrt{(g/L)}$ in Hz
$\omega=2\pi/T=2\pi f=\sqrt{(g/L)}$ in rad/s
$h=L(1-\cos\theta)$ in m

Physical Pendulum: Mass NOT at End
$T=2\pi\sqrt{(I/(mgd))}$
Mass should cancel
d is dist from center of mass to pivot point

SHM and Energy
$x(t)=A\cos(\omega t+\varphi)$
$v(t)=-A\omega\sin(\omega t+\varphi)$
$a(t)=-A\omega^2\cos(\omega t+\varphi)$
amplitude=A
angular frequency=$\omega=2\pi/T=2\pi f$
phase constant=φ=initial condition
$\varphi=0$ if $x=A$
$U=.5kx^2=.5kA^2\cos^2(\omega t)=.5\omega^2 mA^2\cos^2(\omega t)$ in J
$KE=.5mv^2=.5m\omega^2A^2\sin^2(\omega t)=.5kA^2\sin^2(\omega t)$ in J
$E=KE+U=.5kA^2=.5m\omega^2A^2$ in J

Newton's Law of Gravity
$F=-Gm_1m_2/r^2$ in N
$U=-Gm_1m_2/r$ in J
$G=6.67*10^{-11}\ Nm^2/kg^2$

To get Object to Height h (Derivation)
$KE-KE_0+U-U_0=0$
$0-.5mv_0^2-GMm/(R_0+h)+GMm/R_0=0$
$v_0=\sqrt{(2GM(1/R_0-1/(R_0+h)))}$

To get Object into Orbit (Derivation)
$F_c=F_g$
$mv^2/R=GMm/R^2$
$v=\sqrt{(GM/R)}$

Escape Velocity (Derivation)
$KE-KE_0+U-U_0=0$
$0-.5mv_0^2-GMm/(R_0+h)+GMm/R_0=0$
$v_{esc}=\lim(\sqrt{(2GM(1/R_0-1/(R_0+h)))},R_0+h,\infty)$
$v_{esc}=\sqrt{(2GM/R_0)}$

There is no gravity inside a hollow spherical shell. Only account gravity for the mass is beneath (closer toward the center than) the object.

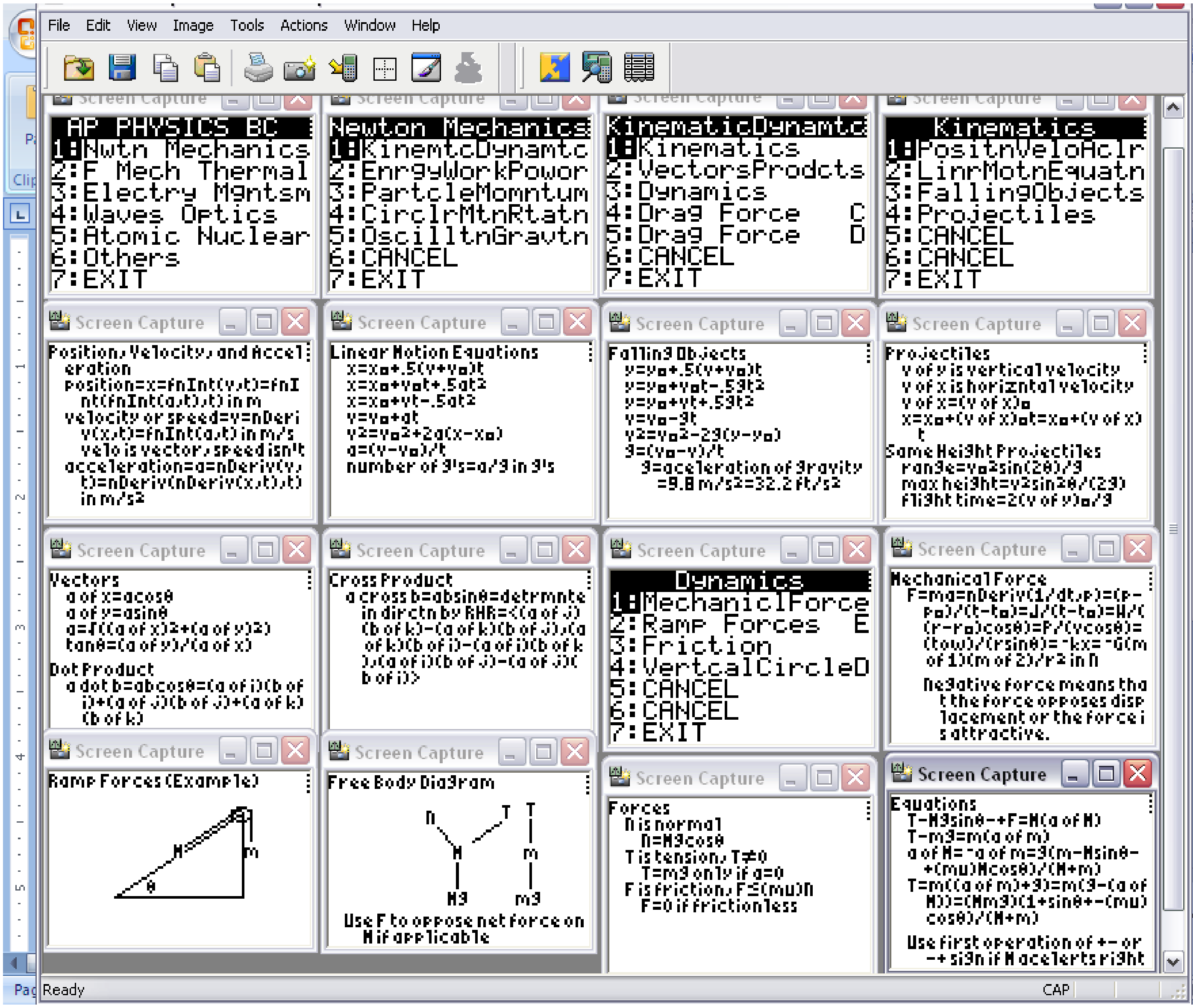
Orbits of Planets and Satellites
Kepler's Laws
$T_A^2/R_A^3=T_B^2/R_B^3$
Angular momentum is conserved as planets orbit the sun.

Any questions, concerns, errors, or comments may be emailed to tzhao451@yahoo.com
Your name will be added in the credits and change log for any corrections or suggestions used unless you ask for your name to be removed.

(Will be) Created (or Finished) by Kai Da Tommy Zhao (before school starts, hopefully)

Physics formula calculator programs are neither disapproved nor endorsed by AP Physics BC teachers McIntosh or Gallo. McIntosh said he does not mind you putting equation into your calculator because it may help you memorize them better. Gallo said as long as you can do it on the exams, then you can do it on his free-response tests. You are allowed to use as many calculators as you like in the AP Physics exam and you are allowed to take them both in 1 year (although it's not recommended by teachers because of the work load, it's recommended by me since you can sleep in class or do other classes' work or get a deeper understanding of Physics if the lesson overlaps). Also, since there is a time conflict with the AP Physics BC exam, you will have an extra 10 days to study for the exam(s) you want to reschedule. Good luck and have fun in physics.

With Best Wishes,
Kai Da Tommy Zhao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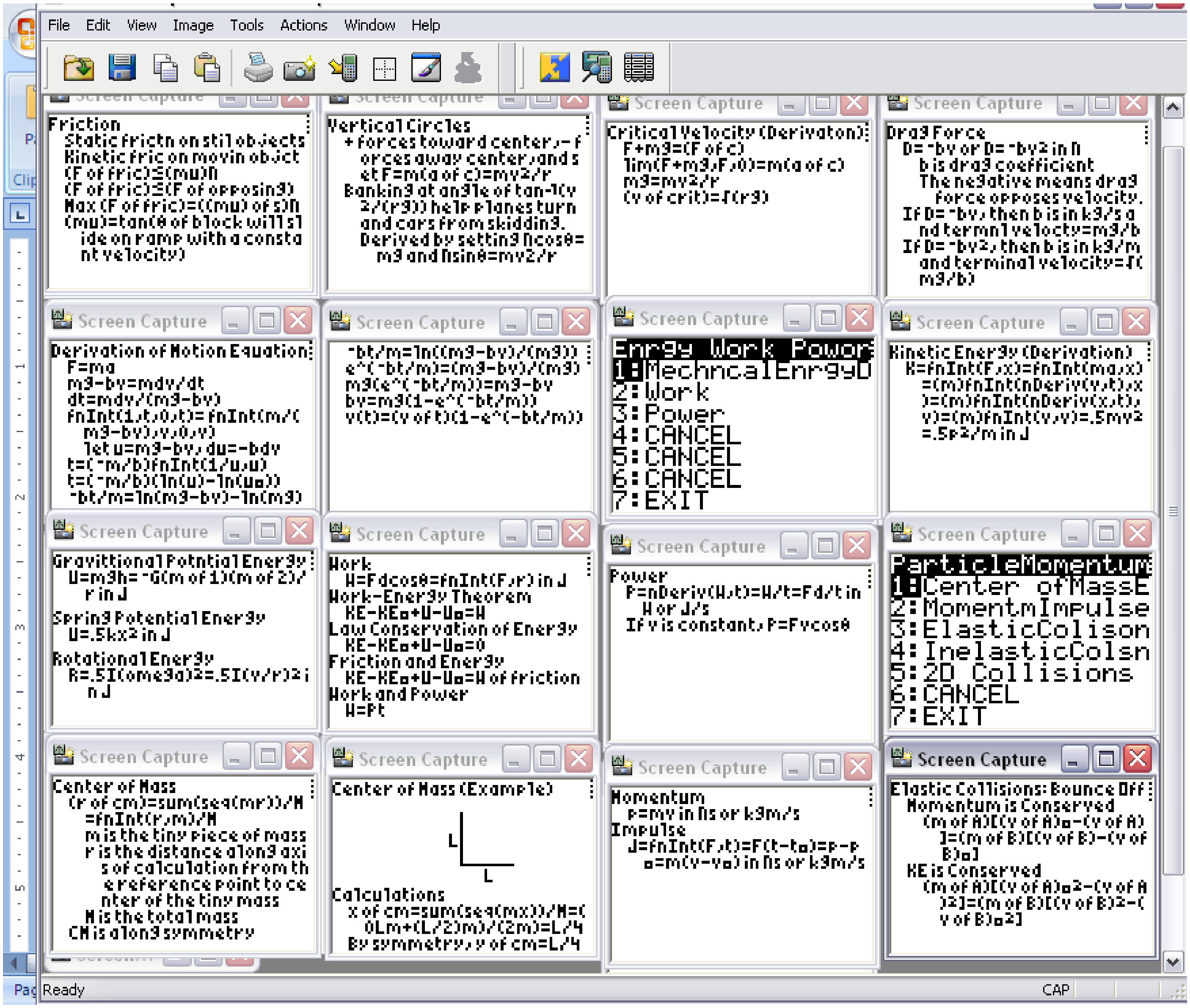
AP PHYSICS BC
1:Nwtn Mechanics
2:F Mech Thermal
3:Electry Mgntsm
4:Waves Optics
5:Atomic Nuclear
6:Others
7:EXIT
Newton Mechanics
1:KinemtcDynamtc
2:EnrgyWorkPower
3:PartcleMomntum
4:CirclrMtnRtatn
5:OscilltnGravtn
6:CANCEL
7:EXIT
KinematicDynamtc
1:Kinematics
2:VectorsProdcts
3:Dynamics
4:Drag Force C
5:Drag Force D
6:CANCEL
7:EXIT
Kinematics
1:PositnVeloAclr
2:LinrMotnEquatn
3:FallingObjects
4:Projectiles
5:CANCEL
6:CANCEL
7:EXIT
Position, Velocity, and Acceleration
position=x=fnInt(v,t)=fnInt(fnInt(a,t),t) in m
velocity or speed=v=nDeriv(x,t)=fnInt(a,t) in m/s
velo is vector, speed isn't
acceleration=a=nDeriv(v,t)=nDeriv(nDeriv(x,t),t) in m/s²
Linear Motion Equations
x=x₀+.5(v+v₀)t
x=x₀+v₀t+.5at²
x=x₀+vt-.5at²
v=v₀+at
v²=v₀²+2a(x-x₀)
a=(v-v₀)/t
number of g's=a/g in g's
Falling Objects
y=y₀+.5(v+v₀)t
y=y₀+v₀t-.5gt²
y=y₀+vt+.5gt²
v=v₀-gt
v²=v₀²-2g(y-y₀)
g=(v₀-v)/t
g=acceleration of gravity =9.8 m/s²=32.2 ft/s²
Projectiles
v of y is vertical velocity
v of x is horizntal velocity
v of x=(v of x)₀
x=x₀+(v of x)₀t=x₀+(v of x)t
Same Height Projectiles
range=v₀²sin(2θ)/g
max height=v²sin²θ/(2g)
flight time=2(v of y)₀/g
Vectors
a of x=acosθ
a of y=asinθ
a=√((a of x)²+(a of y)²)
tanθ=(a of y)/(a of x)
Dot Product
a dot b=abcosθ=(a of i)(b of i)+(a of j)(b of j)+(a of k)(b of k)
Cross Product
a cross b=absinθ=detrmnte in dirctn by RHR=<(a of j)(b of k)-(a of k)(b of j),(a of k)(b of i)-(a of i)(b of k),(a of i)(b of j)-(a of j)(b of i)>
Dynamics
1:MechanicalForce
2:Ramp Forces E
3:Friction
4:VertcalCircleD
5:CANCEL
6:CANCEL
7:EXIT
Mechanical Force
F=ma=nDeriv(1/dt,p)=(p-p₀)/(t-t₀)=J/(t-t₀)=W/((r-r₀)cosθ)=P/(vcosθ)=(tow)/(rsinθ)=⁻kx=⁻G(m of 1)(m of 2)/r² in N
Negative force means that the force opposes displacement or the force is attractive.
Ramp Forces (Example)
θ
M
m
Free Body Diagram
N
T
T
M
m
Mg
mg
Use F to oppose net force on M if applicable
Forces
N is normal
N=Mgcosθ
T is tension, T≠0
T=mg only if a=0
F is friction, F≤(mu)N
F=0 if frictionless
Equations
T-Mgsinθ-+F=M(a of M)
T-mg=m(a of m)
a of M=⁻a of m=g(m-Msinθ-+(mu)Mcosθ)/(M+m)
T=m((a of m)+g)=m(g-(a of M))=(Mmg)(1+sinθ+-(mu)cosθ)/(M+m)
Use first operation of +- or -+ sign if M accelerts right
Ready
CAP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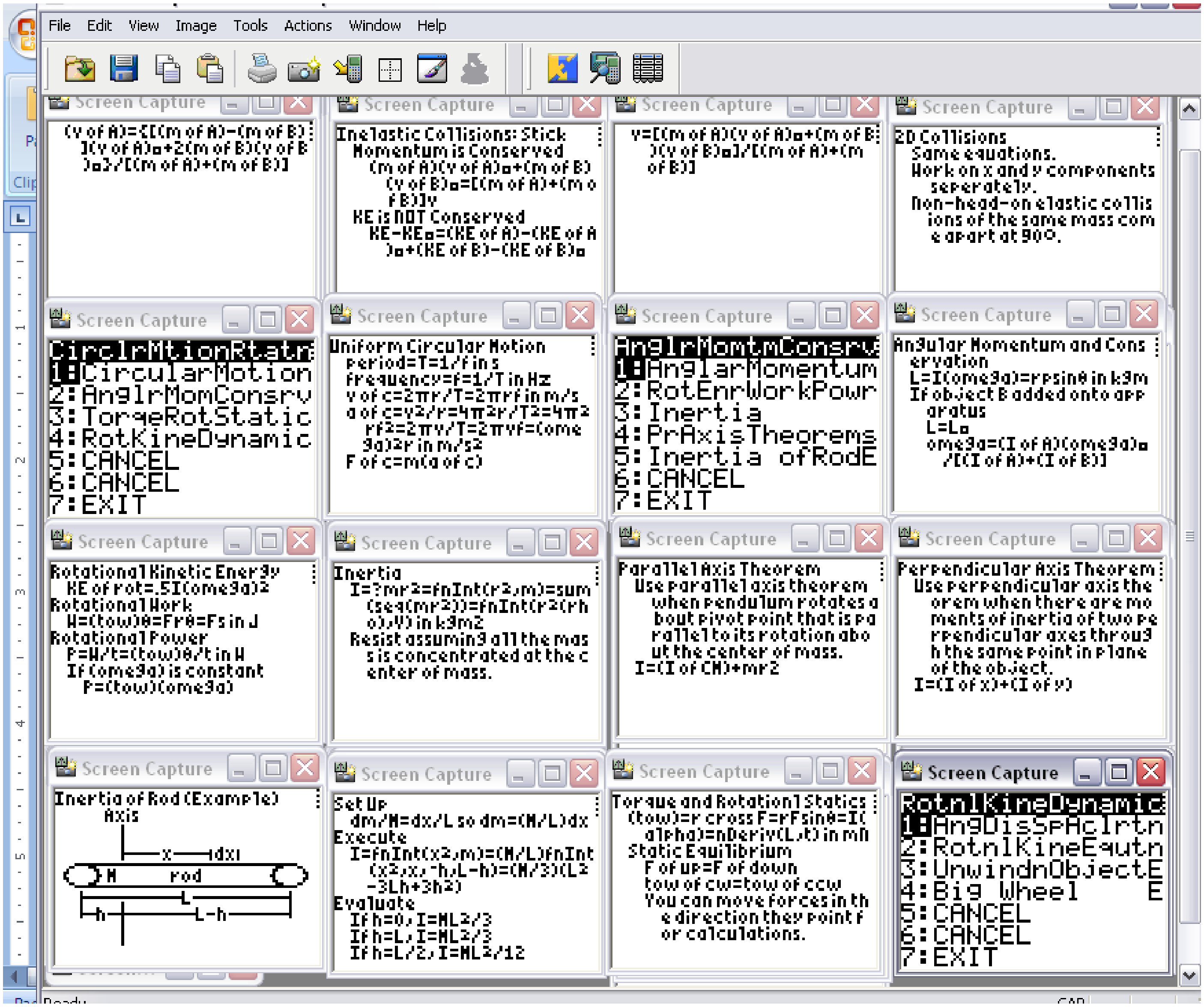
Friction
Static frictn on stil objects
Kinetic fric on movin object
(F of fric)≤(mu)N
(F of fric)≤(F of opposing)
Max (F of fric)=((mu) of s)N
(mu)=tan(θ of block will slide on ramp with a constant velocity)
Vertical Circles
+forces toward center, -forces away center, and set F=m(a of c)=mv2/r
Banking at angle of tan-1(v2/(rg)) help planes turn and cars from skidding.
Derived by setting Ncosθ=mg and Nsinθ=mv2/r
Critical Velocity (Derivaton):
F+mg=(F of c)
lim(F+mg,F,0)=m(a of c)
mg=mv2/r
(v of crit)=√(rg)
Drag Force
D=-bv or D=-bv² in N
b is drag coefficient
The negative means drag force opposes velocity.
If D=-bv, then b is in kg/s and termnl veloctiy=mg/b
If D=-bv², then b is in kg/m and terminal velocity=√(mg/b)
Derivation of Motion Equation:
F=ma
mg-bv=mdv/dt
dt=mdv/(mg-bv)
fnInt(1,t,0,t)=fnInt(m/(mg-bv),v,0,v)
let u=mg-bv, du=-bdv
t=(-m/b)fnInt(1/u,u)
t=(-m/b)(ln(u)-ln(uo))
-bt/m=ln(mg-bv)-ln(mg)
-bt/m=ln((mg-bv)/(mg))
e^(-bt/m)=(mg-bv)/(mg)
mg(e^(-bt/m))=mg-bv
bv=mg(1-e^(-bt/m))
v(t)=(v of t)(1-e^(-bt/m))
Enrgy Work Power
1:MechncalEnrgyD
2:Work
3:Power
4:CANCEL
5:CANCEL
6:CANCEL
7:EXIT
Kinetic Energy (Derivation)
K=fnInt(F,x)=fnInt(ma,x)=(m)fnInt(nDeriv(v,t),x)=(m)fnInt(nDeriv(x,t),v)=(m)fnInt(v,v)=.5mv²=.5p²/m in J
Gravittional Potntial Energy
U=mgh=-G(m of 1)(m of 2)/r in J
Spring Potential Energy
U=.5kx² in J
Rotational Energy
R=.5I(omega)²=.5I(v/r)² in J
Work
W=Fdcosθ=fnInt(F,r) in J
Work-Energy Theorem
KE-KEo+U-Uo=W
Law Conservation of Energy
KE-KEo+U-Uo=0
Friction and Energy
KE-KEo+U-Uo=W of friction
Work and Power
W=Pt
Power
P=nDeriv(W,t)=W/t=Fd/t in W or J/s
If v is constant, P=Fvcosθ
ParticleMomentum
1:Center of MassE
2:MomentmImpulse
3:ElasticColison
4:InelasticColsn
5:2D Collisions
6:CANCEL
7:EXIT
Center of Mass
(r of cm)=sum(seq(mr))/M=fnInt(r,m)/M
m is the tiny piece of mass
r is the distance along axis of calculation from the reference point to center of the tiny mass
M is the total mass
CM is along symmetry
Center of Mass (Example)
L
L
Calculations
x of cm=sum(seq(mx))/M=(0Lm+(L/2)m)/(2m)=L/4
By symmetry, y of cm=L/4
Momentum
p=mv in Ns or kgm/s
Impulse
J=fnInt(F,t)=F(t-to)=p-po=m(v-vo) in Ns or kgm/s
Elastic Collisions: Bounce Off
Momentum is Conserved
(m of A)[(v of A)o-(v of A)]=(m of B)[(v of B)-(v of B)o]
KE is Conserved
(m of A)[(v of A)o²-(v of A)²]=(m of B)[(v of B)²-(v of B)o²]
Ready
CAP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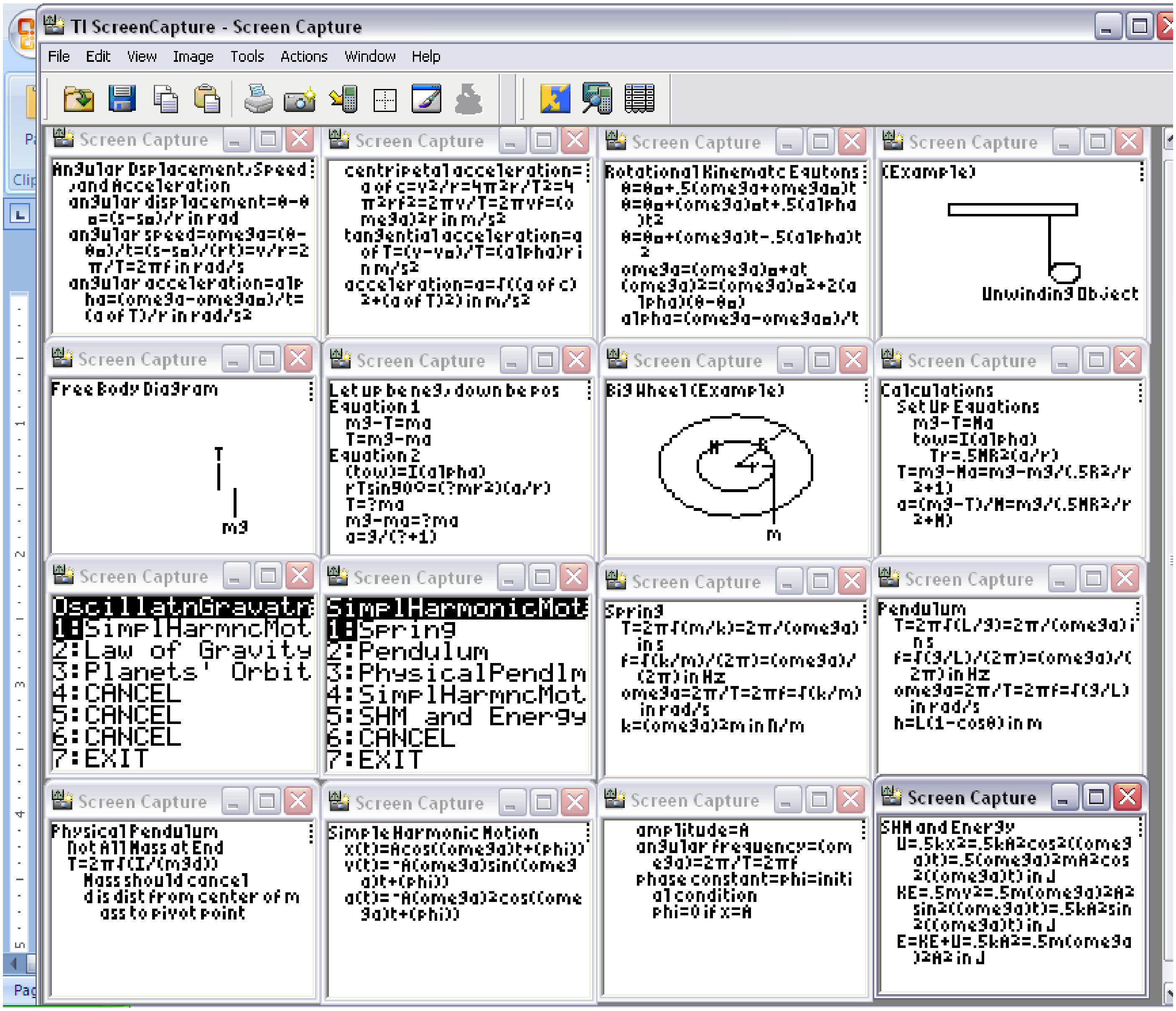
(v of A)=[[(m of A)-(m of B)](v of A)o+2(m of B)(v of B)o]/[(m of A)+(m of B)]
Screen Capture
Inelastic Collisions: Stick
Momentum is Conserved
(m of A)(v of A)o+(m of B)(v of B)o=[(m of A)+(m of B)]v
KE is NOT Conserved
KE-KEo=(KE of A)-(KE of A)o+(KE of B)-(KE of B)o
Screen Capture
v=[(m of A)(v of A)o+(m of B)(v of B)o]/[(m of A)+(m of B)]
Screen Capture
2D Collisions
Same equations.
Work on x and y components seperately.
Non-head-on elastic collisions of the same mass come apart at 90°.
Screen Capture
CirclrMtionRtatn
1:CircularMotion
2:AnglrMomConsrv
3:TorqeRotStatic
4:RotKineDynamic
5:CANCEL
6:CANCEL
7:EXIT
Screen Capture
Uniform Circular Motion
period=T=1/f in s
frequency=f=1/T in Hz
v of c=2πr/T=2πrf in m/s
a of c=v²/r=4π²r/T²=4π²rf²=2πv/T=2πvf=(omega)²r in m/s²
F of c=m(a of c)
Screen Capture
AnglrMomtmConsrv
1:AnglarMomentum
2:RotEnrWorkPowr
3:Inertia
4:PrAxisTheorems
5:Inertia ofRodE
6:CANCEL
7:EXIT
Screen Capture
Angular Momentum and Conservation
L=I(omega)=rpsinθ in kgm
If object B added onto apparatus
L=Lo
omega=(I of A)(omega)o/[(I of A)+(I of B)]
Screen Capture
Rotational Kinetic Energy
KE of rot=.5I(omega)²
Rotational Work
W=(tow)θ=Frθ=Fs in J
Rotational Power
P=W/t=(tow)θ/t in W
If (omega) is constant
P=(tow)(omega)
Screen Capture
Inertia
I=Σmr²=fnInt(r²,m)=sum(seq(mr²))=fnInt(r²(rho),V) in kgm2
Resist assuming all the mass is concentrated at the center of mass.
Screen Capture
Parallel Axis Theorem
Use parallel axis theorem when pendulum rotates about pivot point that is parallel to its rotation about the center of mass.
I=(I of CM)+mr2
Screen Capture
Perpendicular Axis Theorem
Use perpendicular axis theorem when there are moments of inertia of two perpendicular axes through the same point in plane of the object.
I=(I of x)+(I of y)
Screen Capture
Inertia of Rod (Example)
Axis
x
dx
M
rod
L
h
L-h
Screen Capture
Set Up
dm/M=dx/L so dm=(M/L)dx
Execute
I=fnInt(x²,m)=(M/L)fnInt(x²,x,-h,L-h)=(M/3)(L²-3Lh+3h²)
Evaluate
If h=0, I=ML²/3
If h=L, I=ML²/3
If h=L/2, I=ML²/12
Screen Capture
Torque and Rotational Statics
(tow)=r cross F=rFsinθ=I(alpha)=nDeriv(L,t) in mN
Static Equilibrium
F of up=F of down
tow of cw=tow of ccw
You can move forces in the direction they point for calculations.
Screen Capture
RotnlKineDynamic
1:AngDisSpAclrtn
2:RotnlKineEqutn
3:UnwindnObjectE
4:Big Wheel E
5:CANCEL
6:CANCEL
7:EXIT
Ready
CAP


TI ScreenCapture - Screen Capture

AnGular DsPlacement,Speed
,and Acceleration
angular displacement=θ-θo=(s-so)/r in rad
angular speed=omega=(θ-θo)/t=(s-so)/(rt)=v/r=2π/T=2πf in rad/s
angular acceleration=alpha=(omega-omegao)/t=(a of T)/r in rad/s²

centripetal acceleration=a of c=v²/r=4π²r/T²=4π²rf²=2πv/T=2πvf=(omega)²r in m/s²
tangential acceleration=a of T=(v-vo)/T=(alpha)r in m/s²
acceleration=a=√((a of c)²+(a of T)²) in m/s²

Rotational Kinematc Eqatons
θ=θo+.5(omega+omegao)t
θ=θo+(omegao)t+.5(alpha)t²
θ=θo+(omega)t-.5(alpha)t²
omega=(omega)o+at
(omega)²=(omega)o²+2(alpha)(θ-θo)
alpha=(omega-omegao)/t

(Example)
Unwinding Object

Free Body Diagram
T
mg

Let up be neg, down be pos
Equation 1
mg-T=ma
T=mg-ma
Equation 2
(tow)=I(alpha)
rTsin90°=(?mr²)(a/r)
T=?ma
mg-ma=?ma
a=g/(?+1)

Big Wheel (Example)
M R r
m

Calculations
Set Up Equations
mg-T=Ma
tow=I(alpha)
Tr=.5MR²(a/r)
T=mg-Ma=mg-mg/(.5R²/r²+1)
a=(mg-T)/M=mg/(.5MR²/r²+M)

OscillatnGravatn
1:SimplHarmncMot
2:Law of Gravity
3:Planets' Orbit
4:CANCEL
5:CANCEL
6:CANCEL
7:EXIT

SimplHarmonicMot
1:Spring
2:Pendulum
3:PhysicalPendlm
4:SimplHarmncMot
5:SHM and Energy
6:CANCEL
7:EXIT

Spring
T=2π√(m/k)=2π/(omega) in s
f=√(k/m)/(2π)=(omega)/(2π) in Hz
omega=2π/T=2πf=√(k/m) in rad/s
k=(omega)²m in N/m

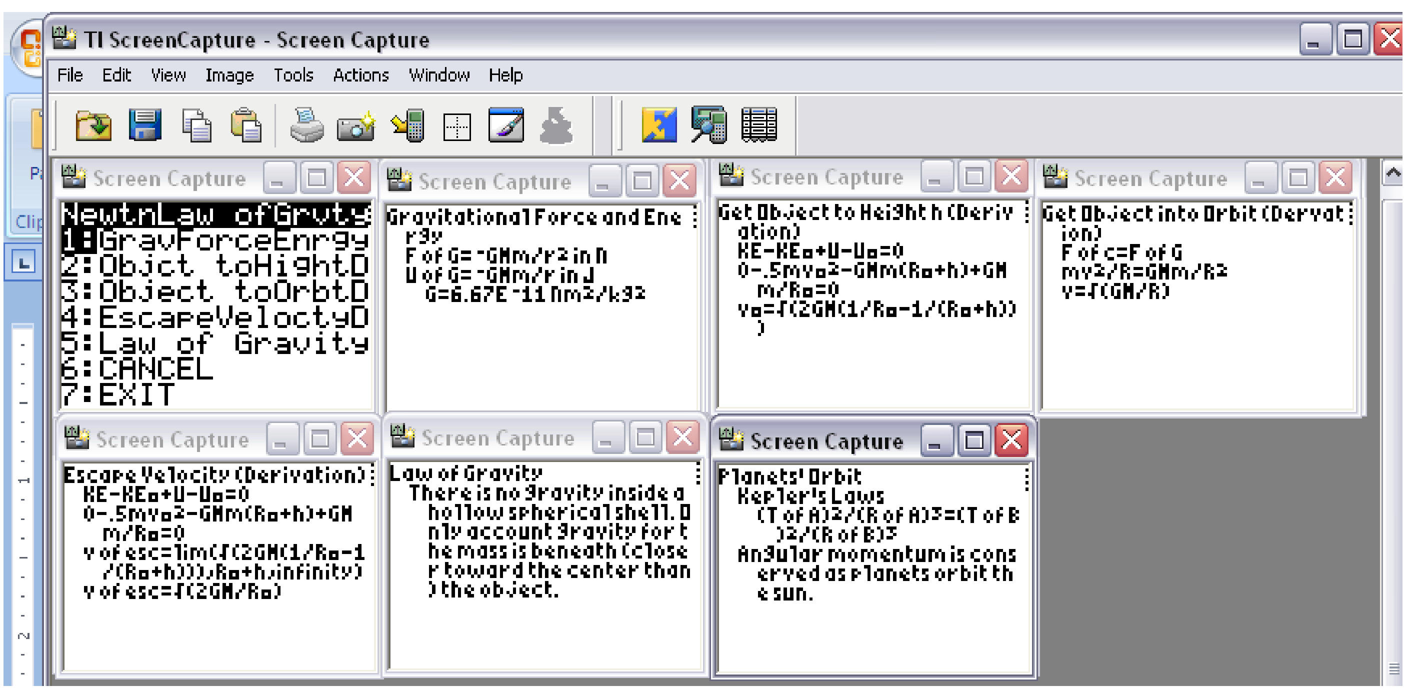
Pendulum
T=2π√(L/g)=2π/(omega) in s
f=√(g/L)/(2π)=(omega)/(2π) in Hz
omega=2π/T=2πf=√(g/L) in rad/s
h=L(1-cosθ) in m

Physical Pendulum
Not All Mass at End
T=2π√(I/(mgd))
Mass should cancel
d is dist from center of mass to pivot point

Simple Harmonic Motion
x(t)=Acos((omega)t+(phi))
v(t)=-A(omega)sin((omega)t+(phi))
a(t)=-A(omega)²cos((omega)t+(phi))

amplitude=A
angular frequency=(omega)=2π/T=2πf
phase constant=phi=initial condition
phi=0 if x=A

SHM and Energy
U=.5kx²=.5kA²cos²((omega)t)=.5(omega)²mA²cos²((omega)t) in J
KE=.5mv²=.5m(omega)²A²sin²((omega)t)=.5kA²sin²((omega)t) in J
E=KE+U=.5kA²=.5m(omega)²A² in J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NewtnLaw ofGrvty
1:GravForceEnrgy
2:Objct toHightD
3:Object toOrbtD
4:EscapeVeloctyD
5:Law of Gravity
6:CANCEL
7:EXIT

Gravitational Force and Energy
F of G=-GMm/r² in N
U of G=-GMm/r in J
G=6.67E-11 Nm²/kg²

Get Object to Height h (Derivation)
KE-KEo+U-Uo=0
0-.5mvo²-GMm(Ro+h)+GMm/Ro=0
vo=√(2GM(1/Ro-1/(Ro+h)))

Get Object into Orbit (Dervation)
F of c=F of G
mv²/R=GMm/R²
v=√(GM/R)

Escape Velocity (Derivation)
KE-KEo+U-Uo=0
0-.5mvo²-GMm(Ro+h)+GMm/Ro=0
v of esc=lim(√(2GM(1/Ro-1/(Ro+h))),Ro+h,infinity)
v of esc=√(2GM/Ro)

Law of Gravity
There is no gravity inside a hollow spherical shell. Only account gravity for the mass is beneath (closer toward the center than) the object.

Planets' Orbit
Kepler's Laws
(T of A)²/(R of A)³=(T of B)²/(R of B)³
Angular momentum is conserved as planets orbit the sun.