STAT371 DISCUSSION 4 September 29, 2002

TA: Ruiyan Luo
Office: 4268 CSSC
Office hours: TW 2:30–3:30pm
Phone number: 262-8182
E-mail: rluo@stat.wisc.edu

1. Bionomial distribution

- Four conditions for a binomial random varialbe:
  - Binary outcomes: there are two possible outcomes for each trial (success and failure)
  - Independent rials: the outcomes of the trials are independent of each other
  - n is fixed: the number of trials, n, is fixed in advance
  - Same value of p: the probability of a success on a single trial is the same for all trials
- The binomial distribution formula
  For a binomial random variable, the probability that the n trials result in j successes (and $n-j$ failures) is given by the following formula:
  $$Pr\{j successes\} =_n C_j p^j (1-p)^{n-j}$$
  where ${}_nC_j = \frac{n!}{j!(n-j)!}$ and $x! = x(x-1)(x-2)...(2)(1)$, $0! = 1$.
  ${}_nC_j$ has some properties:
  $${}_nC_0 =_n C_n = 1$$
  $${}_nC_j =_n C_{n-j}$$
- Properties of binomial distribution:
  - expectation (or mean) of X is np.
  - variance is $np(1-p)$; standard deviation is $\sqrt{np(1-p)}$
  - the binomial distribution is symmetric if and only if $p = 0.5$

2. Normal distribution

- If Y follows a normal distribution with mean μ and standard deviation σ, then it is common to write $Y\tilde{}N(\mu, \sigma)$. Its density function is

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{y-\mu}{\sigma})^2}$$

- By standardization formula

$$Z = \frac{Y - \mu}{\sigma}$$

random variable Z has density function

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

, which is called standard normal distribution with mean 0 and standard deviation 1.

- Pr{Z is between a and b}= area under the standard normal curve between a and b. The tabel in the book gives the area under the normal curve below a spedified value of z.
Pr$\{Z \leq z\}$=area to the left of z (given in table)
Pr$\{Z \geq z\}$=area to the right of z=1-area to the left of z
Pr$\{a \leq Z \leq b\}$=area to the left of b − area to the left of a

- Given a probability α, from the normal table we can get Z_α such that $Pr\{Z \leq Z_\alpha\} = 1 - \alpha$, then $Y_\alpha = Z_\alpha\sigma + \mu$ satisfying that $Pr\{Y \leq Y_\alpha\} = 1 - \alpha$.