

1. Statistical estimation

- The estimate of population mean μ is sample mean \bar{y}
- The estimate of population standard deviation σ is sample standard deviation s .

2. The standard error of the mean is

$$SE_{\bar{y}} = \frac{s}{\sqrt{n}}$$

which is a measure of the reliability or precision of \bar{y} as an estimate of μ : the smaller the SE, the more precise the estimate.

Consider: What is the distinction between standard error and standard deviation?

3. Confidence interval

- The construction of confidence interval:
If the sample size is n , sample mean is \bar{y} , and the standard error is $SE_{\bar{y}}$, then the $(1 - \alpha)\%$ confidence interval for μ is constructed as follows:

$$\bar{y} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

where the critical value $t_{\frac{\alpha}{2}}$ is determined from Student's t distribution with $df = n - 1$. For instance, if $\alpha = 10$, then the 90% confidence interval is $\bar{y} \pm t_{0.5} SE_{\bar{y}}$.

- The interpretation of a confidence interval:
Suppose the 95% confidence interval of μ is (a, b) , which of the following statement is true?
 - $\Pr\{a < \mu < b\} = 95\%$
 - We are 95% confidence that the population mean μ is between a and b .
 - $\Pr\{a < \bar{y} < b\} = 95\%$

- We are 95% confidence that the sample mean \bar{y} is between a and b .
- If we take 100 samples from the population and construct 100 95% confidence intervals. Then there will be 95 confidence intervals containing μ .
- $\Pr\{\text{the next sample will give us a confidence interval that contains } \mu\}=0.95$

4. Planning a study to estimate μ

To get a desired standard error, the sample size should be:

$$n \geq \left(\frac{\text{Guessed SD}}{\text{Desired SE}} \right)^2$$

Exercise:

Y follows a normal distribution with mean 20 and standard deviation 2.

Take a sample from the population and get these data:

19.10672 20.49547 19.20281 16.81740 19.18170 19.44320 18.34311 19.51481
19.22503 25.53221 20.24905 18.80119 21.46908 `sx`

- get sample mean \bar{Y} and sample standard deviation s
- the sample error
- $\Pr\{19 < \bar{Y} < 22\}$
- the 90% confidence interval for μ
- if we want the standard error to be less than 0.1, how large should the sample be?