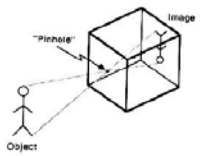
Pinhole Camera Model

- Aka perspective projection model



- For each scene point only 1 ray can enter camera
- Pinhole = center of projection through which all light passes
- Pinhole too big ⇒ blurring
  Pinhole too small ⇒ diffraction-based blurring
- Long exposure time needed

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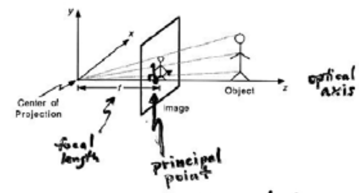
Perspective Projection



- Image plane orthogonal to z axis (called optical axis)
- Camera frame origin at center of projection
- 3D scene point $P=(X,Y,Z)^T$ projects to image point $p'=(x',y',z')^T$ where $z'=f$ (focal length)

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Before the Discovery of Perspective

di Bartolo, "The Nativity of the Virgin" (c. 1400)

di Giovanni Fei, "The Presentation of the Virgin" (c. 1400)

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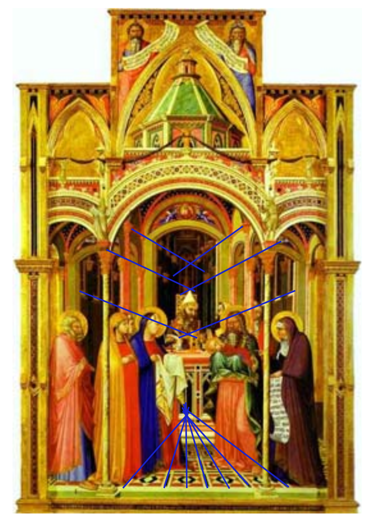

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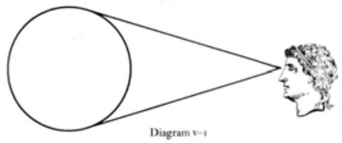
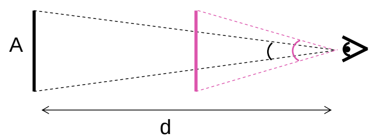
Ambrogio Lorenzetti (1342) *The presentation in the temple.* Panel, Uffizi, Florence

Natural Perspective



Euclid's Optics (300 BC)

- Visual ray: from point on object to eye
- Visual cone: from contour of object to eye
- Euclid's Law: diminution in visual angle with distance



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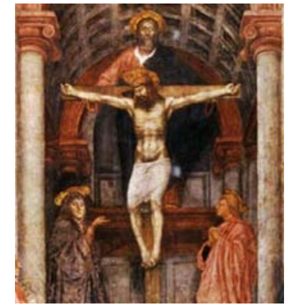
Italian Renaissance



- Linear perspective
  - Illusionistic 3D space
  - Sculptural body
  - Natural pose, individual expression
  - Humanized suffering

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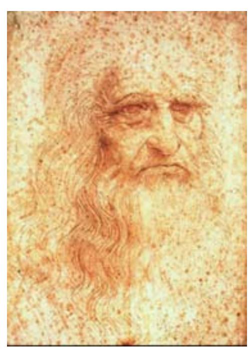
"Perspective is nothing else than the seeing of an object through a sheet of glass, on the surface of which may be marked all the things that are behind the glass."

-- Leonardo



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Alberti's Window

Hieronomous Rodeem (1531) *Johan II of Bavaria.* Woodcut.

"First of all, on the surface on which I am going to paint, I draw a rectangle of whatever size I want, which I regard as an open window, through which the subject to be painted is seen."

-- Alberti (1435-6)

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Alberti's Veil

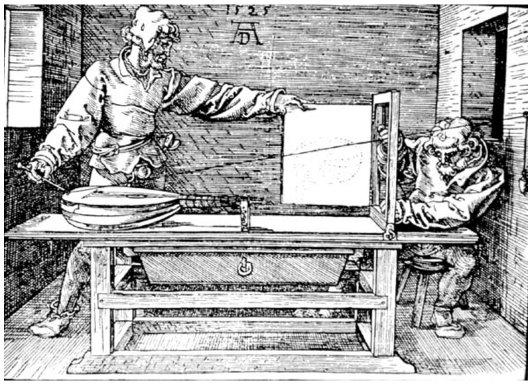
• Grid system

Albrect Dürer (c. 1525) *Draughtsman drawing a reclining nude.* Woodcut.

Reconstructed view through

Point-Plotting Method

• Use strings to embody Euclid's visual rays



Albrect Dürer (c. 1525) *Two draughtsmen plotting points for the drawing of a lute in foreshortening.* Woodcut.

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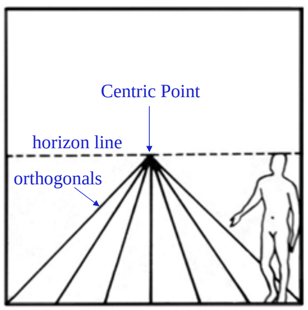
Alberti's Method (1435): "Construzione Legittima"



1. Draw "open window", with a human figure 3 braccia high
2. Mark baseline in units of 1 braccio
3. Draw <u>Centric Point</u> at eye level (determines severity of convergence)
4. Draw <u>orthogonals</u>
5. Draw horizon line

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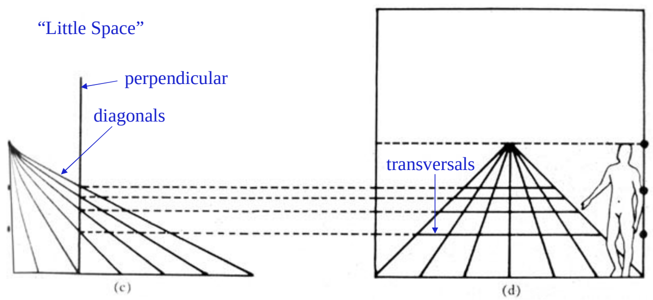

6. Draw "little space", with a point at the height of the Centric Point (like elevation view, with eye point)
7. Draw baseline with units of 1 braccio (like ground plane)
8. Draw a vertical line (like picture plane)
9. Draw <u>diagonals</u> (like visual rays)
10. Draw <u>transversals</u> at intersections

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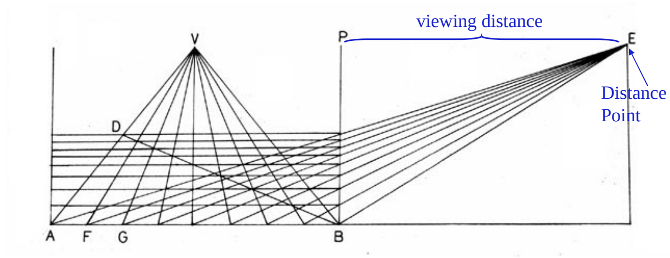
Modified Alberti Method

- Slide the “little space” over so the right side of the rectangle becomes the picture plane
- DB is a “check line” for verifying correctness



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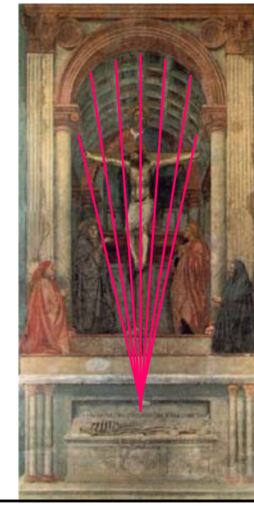
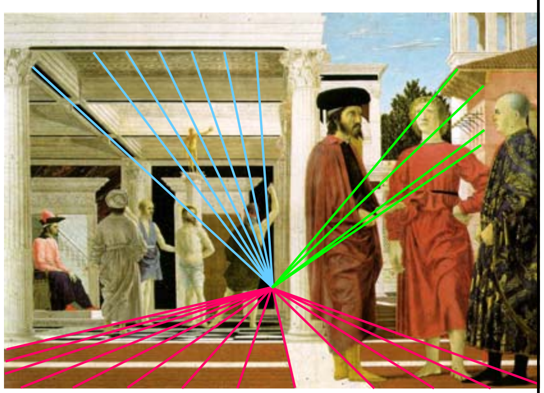
Masaccio’s “Trinity” (c. 1425-8)

- The oldest existing example of linear perspective in Western art
- Use of “snapped” rope lines in plaster
- Vanishing point **below** orthogonals implies looking **up** at vaulted ceiling



Piero della Francesca, “Flagellation of Christ” (c. 1455)

- Carefully planned
- Strong sense of space
- Low eye level



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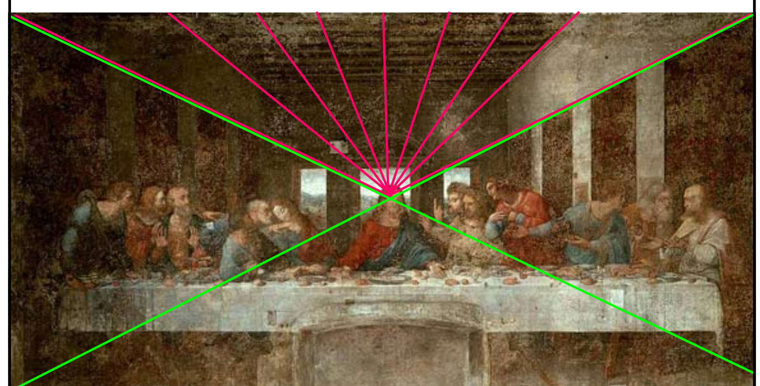
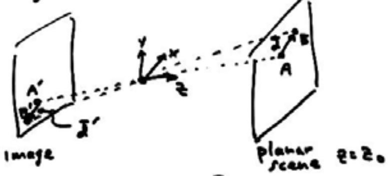
Leonardo da Vinci, “Last Supper” (c. 1497)

- Use of perspective to direct viewer’s eye
- Strong perspective lines to corners of image



Properties of Perspective Projection

- Object size changes as it translates along z axis (scale effect)



Magnification $|m| = \frac{|\bar{d}'|}{|\bar{d}|} = \left|\frac{f}{z_0}\right|$

$\Rightarrow$ distance b/w points <u>not</u> preserved

- As f gets smaller, more world points project onto finite image plane $\Rightarrow$ more <u>wide angle</u> image
- As f gets larger, more <u>telescopic</u>
- Lines in 3D project to lines in 2D

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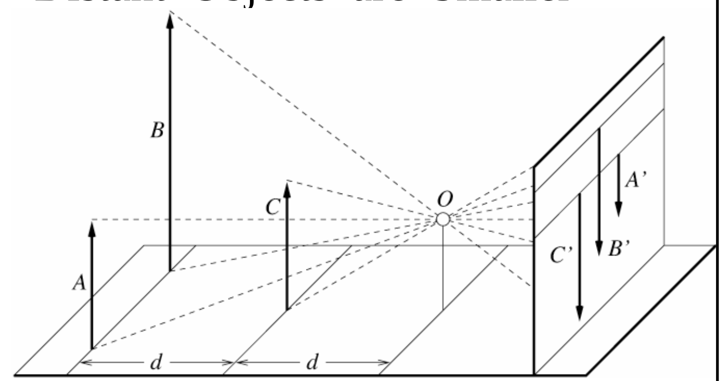
Distant Objects are Smaller



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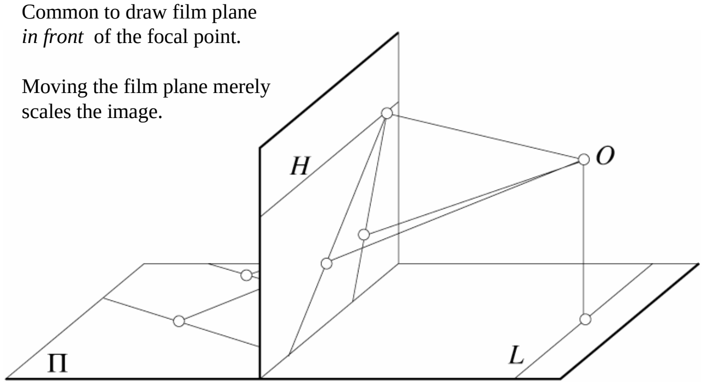
Parallel Lines Meet

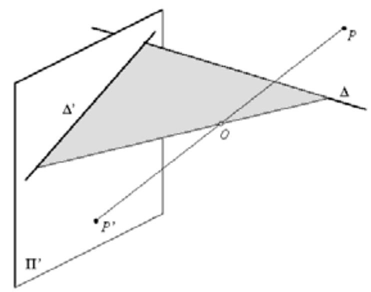
Common to draw film plane *in front* of the focal point.

Moving the film plane merely scales the image.



Geometric Properties of Projection

- Points go to points
- Lines go to lines
- Planes go to whole image
- Polygons go to polygons
- Degenerate cases
  - line through focal point to point
  - plane through focal point to line



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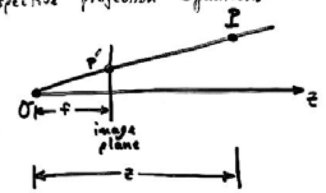
Perspective Projection (cont.)

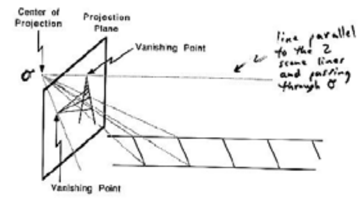
- Perspective projection equations

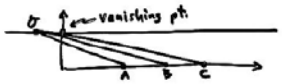


$$\frac{P'}{f} = \frac{P}{z} \quad \Rightarrow$$

$$\begin{cases} x' = \frac{fx}{z} \\ y' = \frac{fy}{z} \\ z' = f \end{cases}$$

- Vanishing point = point in image beyond which projection of straight line cannot extend





- Focus of Expansion (FOE)
  When camera translates, trajectories of image points appear to move towards or away from a fixed point called FOE which is common vanishing pt. because all pts moving along straight lines relative to camera

Vanishing Points

- each set of parallel lines (= direction) meets at a different point
  - The *vanishing point* for this direction
- Sets of parallel lines on the same plane lead to *collinear* vanishing points
  - The line is called the *horizon* for that plane
- Good ways to spot faked images
  - scale and perspective don't work
  - vanishing points behave badly
  - supermarket tabloids are a great source

Using Homogeneous Coordinates

- Translation by (a, b, c)

$$\begin{cases} x' = x - a \\ y' = y - b \\ z' = z - c \end{cases}$$

or

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$P' = \mathcal{T} P$$

- Scale Change by (S_x, S_y, S_z)

$$\begin{cases} x' = x S_x \\ y' = y S_y \\ z' = z S_z \end{cases}$$

or

$P' = \mathcal{S} P$ where

$$\mathcal{S} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Rotation about coordinate axes (counterclockwise looking towards origin) by [redacted] θ

Ex. About z-axis:

$$\begin{cases} x' = x\cos\theta - y\sin\theta \\ y' = x\sin\theta + y\cos\theta \\ z' = z \end{cases}$$

$$\Rightarrow R_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Also,

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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- Any transformation involving translation, scale or rotation can be written as

$$P' = MP$$

where M constructed by composing transformation matrices

Ex.

$$\begin{bmatrix} 1 & 0 & a \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \alpha\cos\theta & \beta\sin\theta & \alpha(a\cos\theta - b\sin\theta) \\ -\alpha\sin\theta & \beta\cos\theta & \beta(a\sin\theta + b\cos\theta) \\ 0 & 0 & 1 \end{bmatrix}$$

- Translations are commutative, rotations are not
- General transformation matrix of form:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & T_x \\ a_{21} & a_{22} & a_{23} & T_y \\ a_{31} & a_{32} & a_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Affine Transformation

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Projection using Homogeneous Coords

- Perspective Projection

$$x' = \frac{fx}{z}, \quad y' = \frac{fy}{z}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{fx}{z} \\ \frac{fy}{z} \\ f \\ 1 \end{bmatrix} \Leftrightarrow \underbrace{\begin{bmatrix} x_h \\ y_h \\ z_h \\ z/f \end{bmatrix}}_{\text{image pt}} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 0 \end{bmatrix}}_{T_{perspective}} \underbrace{\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}}_{\text{scene pt}}$$

- Orthographic Projection

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & f \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{T_{orthographic}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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Camera Matrix

- Turn previous expression into HC's
  - HC's for 3D point are (X,Y,Z,T)
  - HC's for point in image are (U,V,W)

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/f & 0 \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

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Projection Matrix for Orthographic Projection

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$

- *Note*: Since image plane at $z=f$, perspective projection equation can be written as:

$$\begin{bmatrix} x_h \\ y_h \\ w \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

and $\therefore \begin{cases} x' = x_h / w \\ y' = y_h / w \end{cases}$

$\Rightarrow$ Camera = linear projective transform from 3D projective space to 2D projective plane

- 3×4 matrix called *camera perspective projection matrix*

Camera Parameters

- Issue
  - camera may not be at the origin, looking down the *z*-axis
    - extrinsic parameters
  - one unit in camera coordinates may not be the same as one unit in world coordinates
    - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

$$\begin{pmatrix} U \\ V \\ W \end{pmatrix} = \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{intrinsic parameters} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \text{Transformation} \\ \text{representing} \\ \text{extrinsic parameters} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ T \end{pmatrix}$$