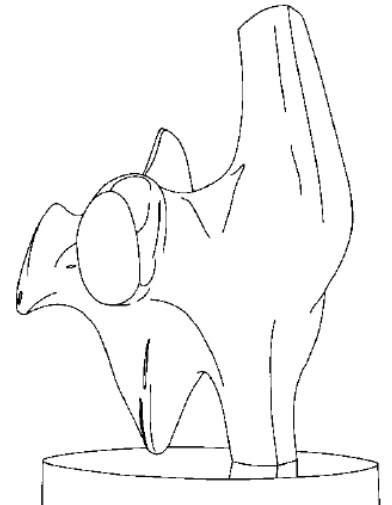
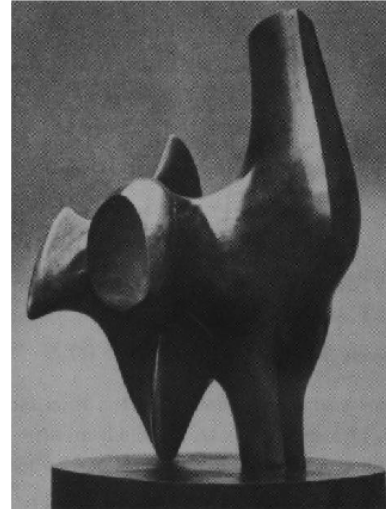
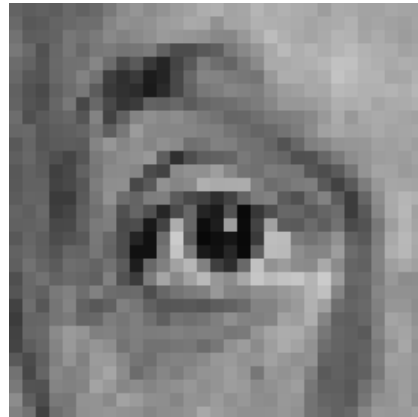


Last Lecture



Edge Detection



Filtering



Pyramid

High Dynamic Range Image Reconstruction from Hand-held Cameras

Pei-Ying Lu Tz-Huan Huang Meng-Sung Wu
Yi-Ting Cheng Yung-Yu Chuang

National Taiwan University

The world is of high dynamic range

scene

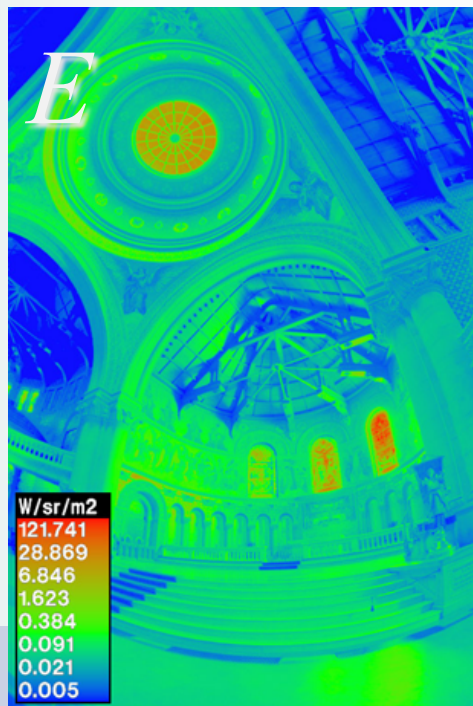
short exposure

long exposure



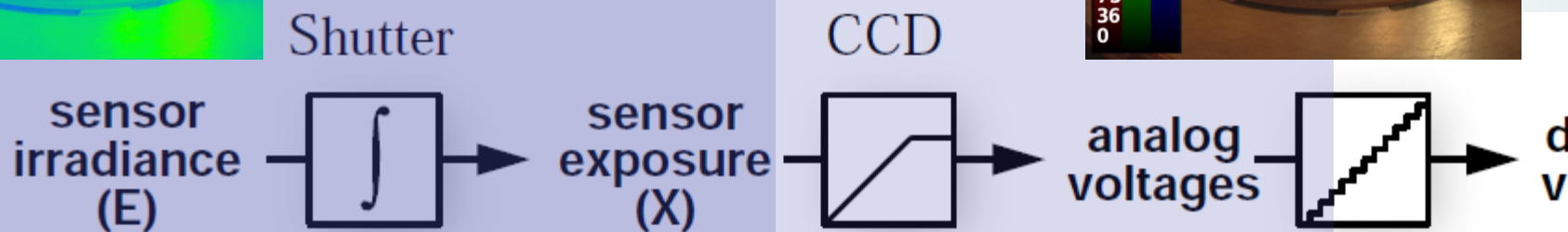
2009
MIAMI

Camera pipeline

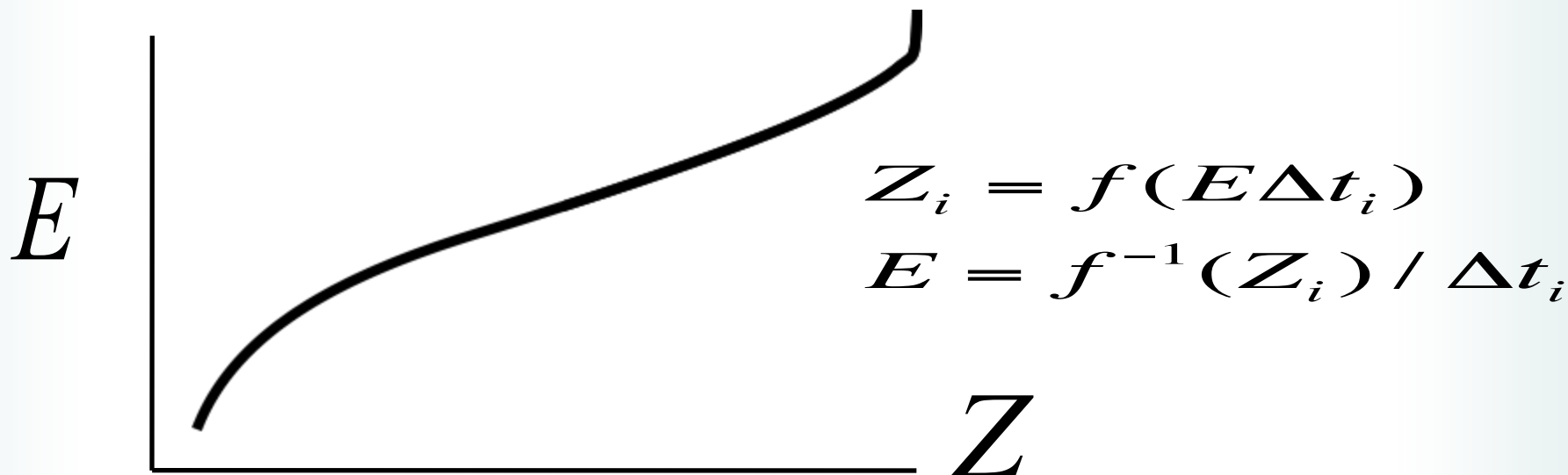


$$X = \int_{t=0}^{\Delta t} E dt = E \Delta t$$

$$Z = f(X) = f(E \Delta t)$$



HDR image reconstruction



HDR image reconstruction

- Recovering High Dynamic Range Radiance Maps from Photographs, SIGGRAPH 1997.
- Radiometric Self Calibration, CVPR 2001.
- Estimation-theoretic approach to dynamic range enhancement using multiple exposures, JEI 2003.
- All assume static cameras and thus require tripods.

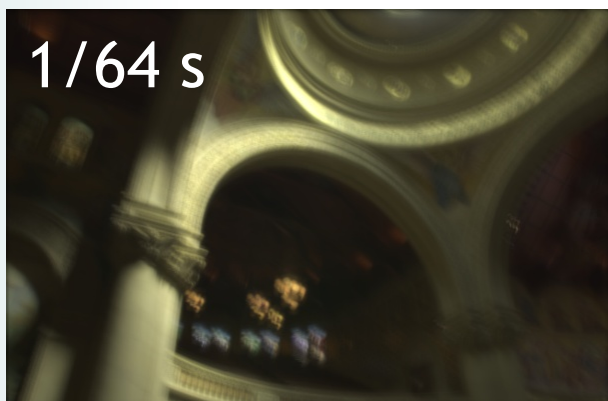
Images from hand-held camera

- Challenge #1: image mis-alignment



Images from hand-held camera

- Challenge #1: image mis-alignment
- Challenge #2: image blur



A naïve approach

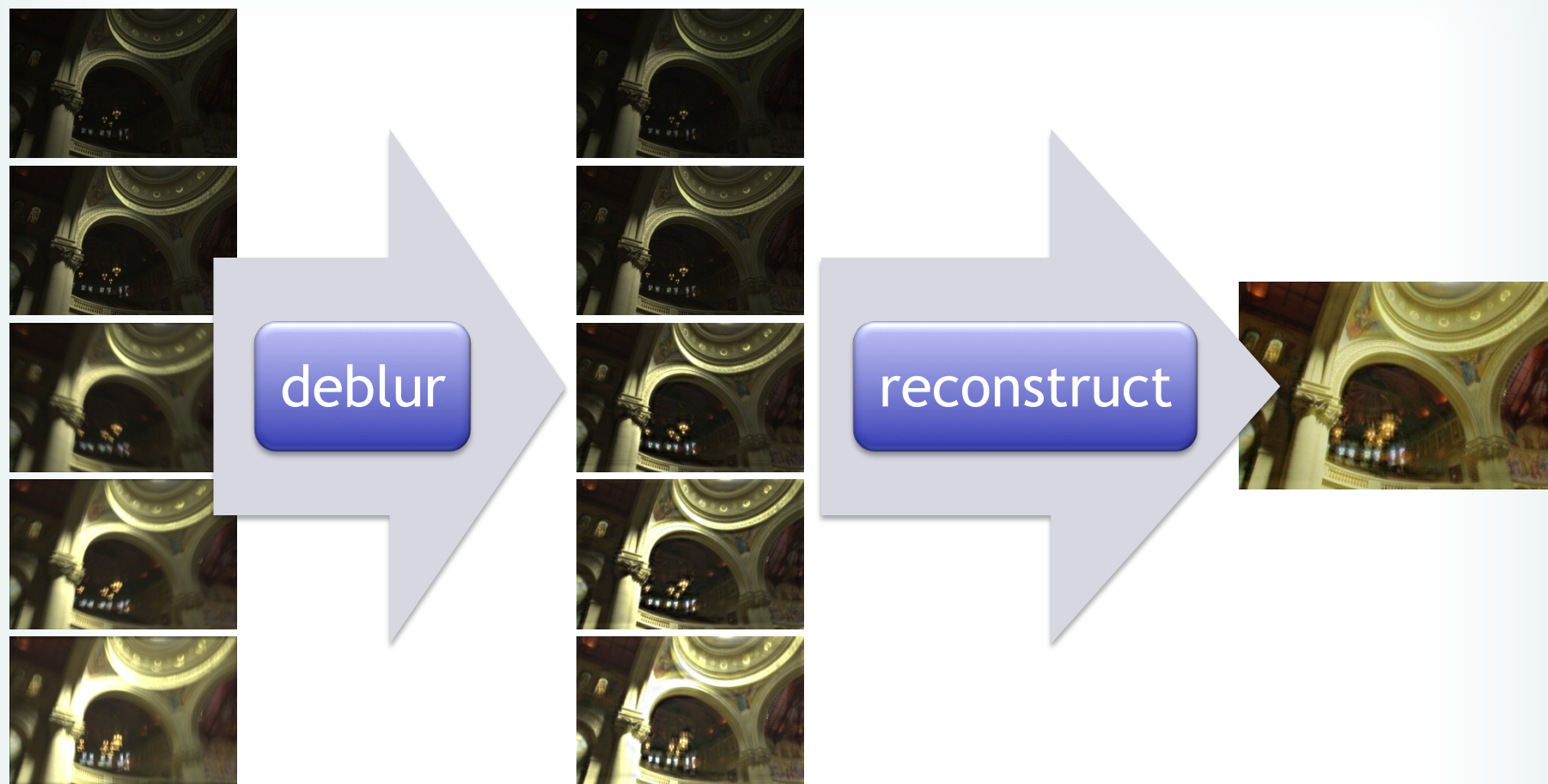


Image blurring process

convolution



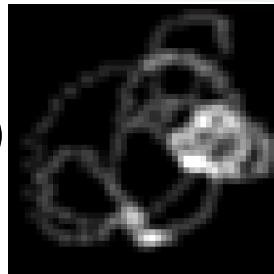
blur image

=



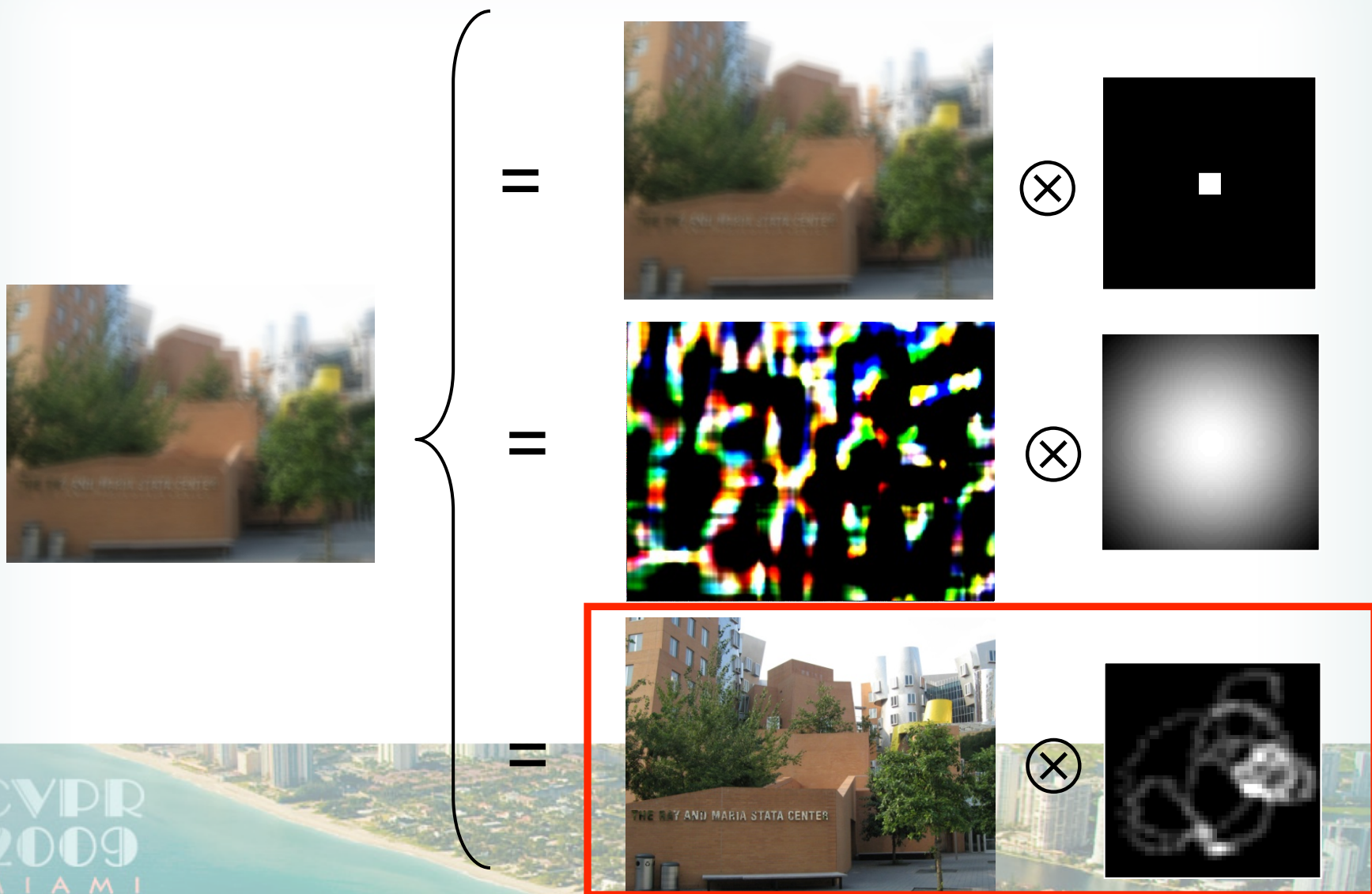
sharp image

⊗



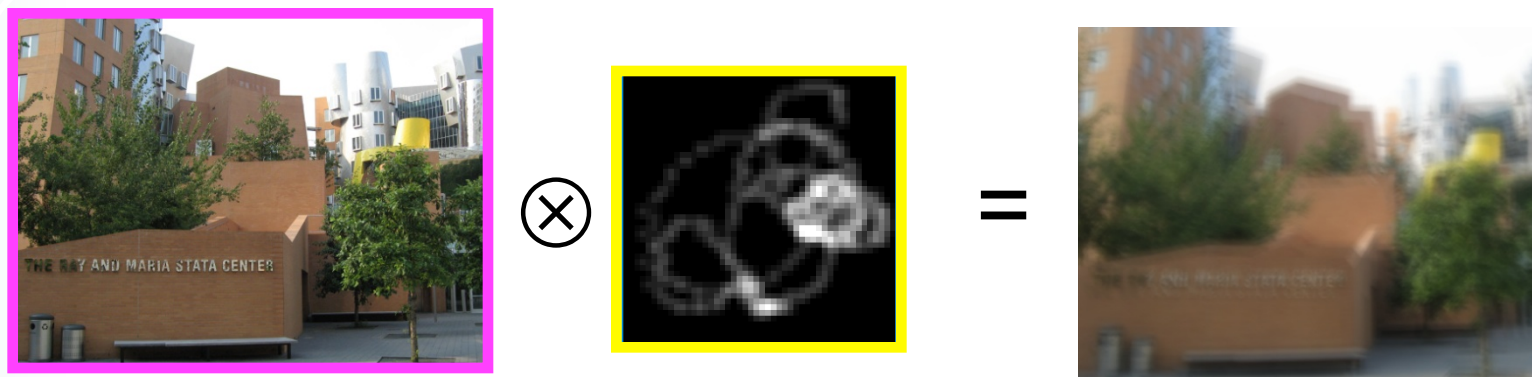
blur
kernel

An under-determined problem



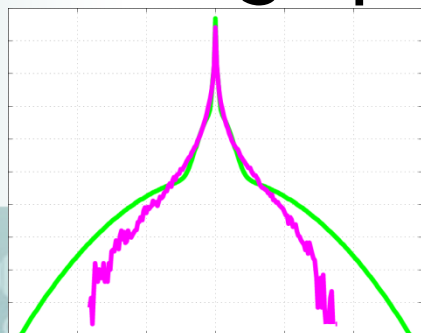
Adding priors

1. Reconstruction constraint:



Fergus et. al. SIGGRAPH 2006

2. Image prior:



distribution
of gradients

3. Blur prior:



positive
& sparse

Adding observation (a noisy image)



blurred image



noisy image



deblurred image

A naïve approach

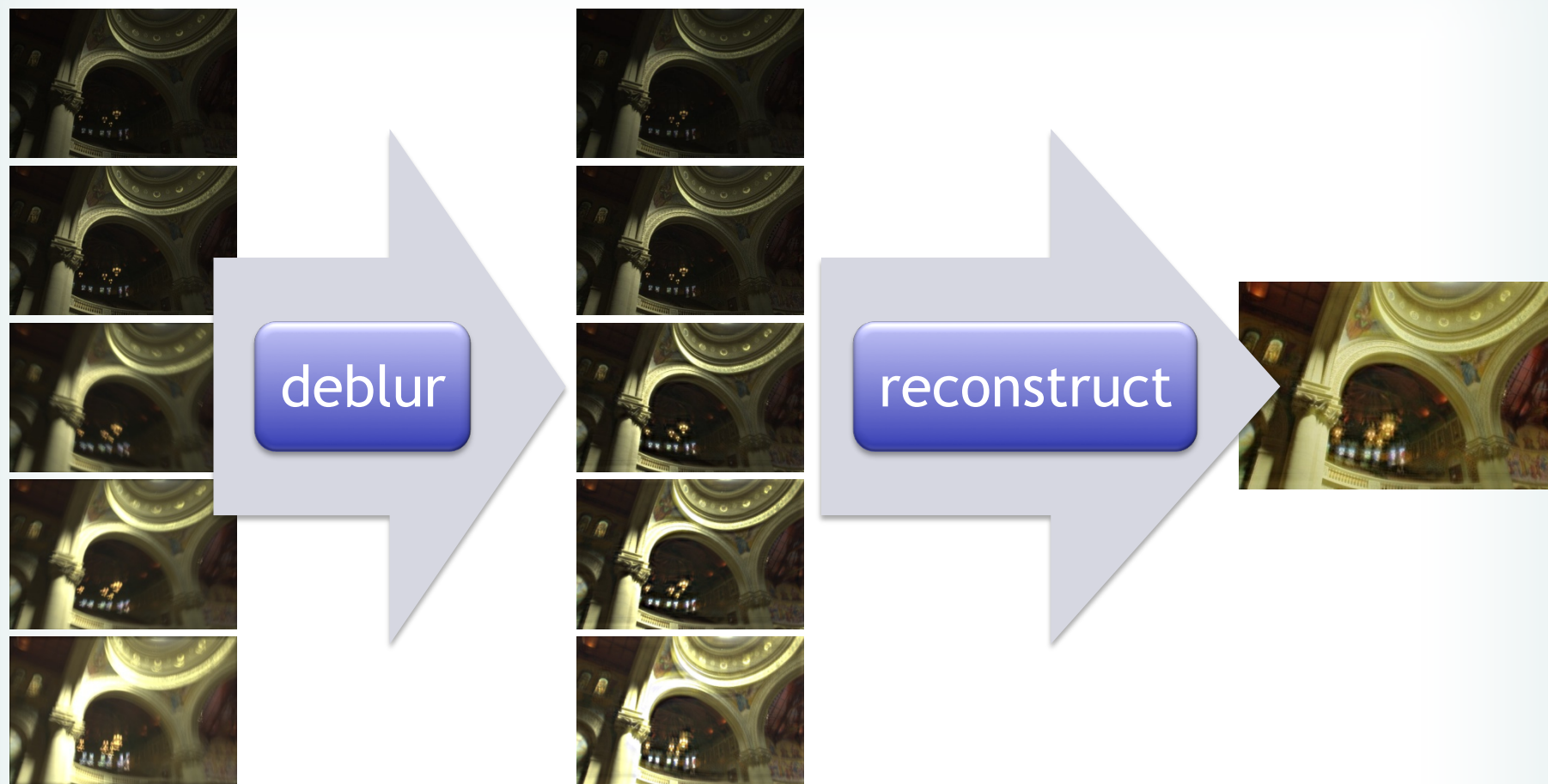


Image formation model

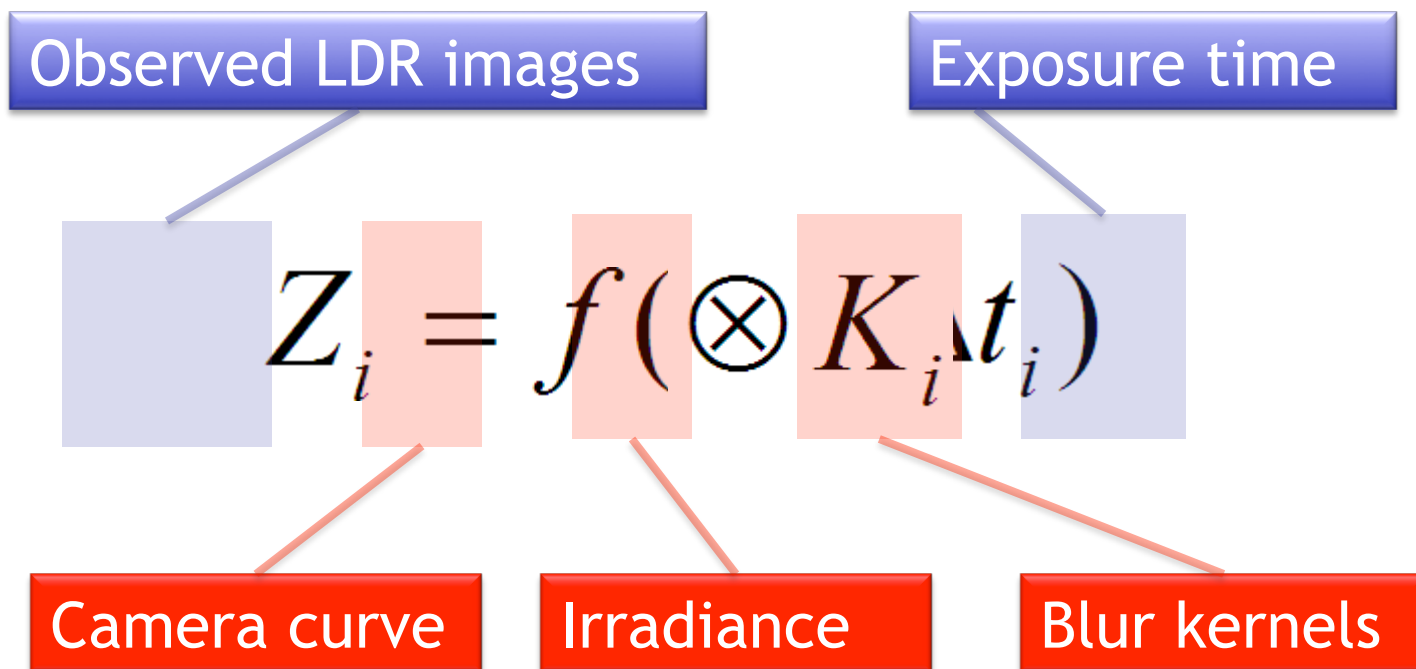
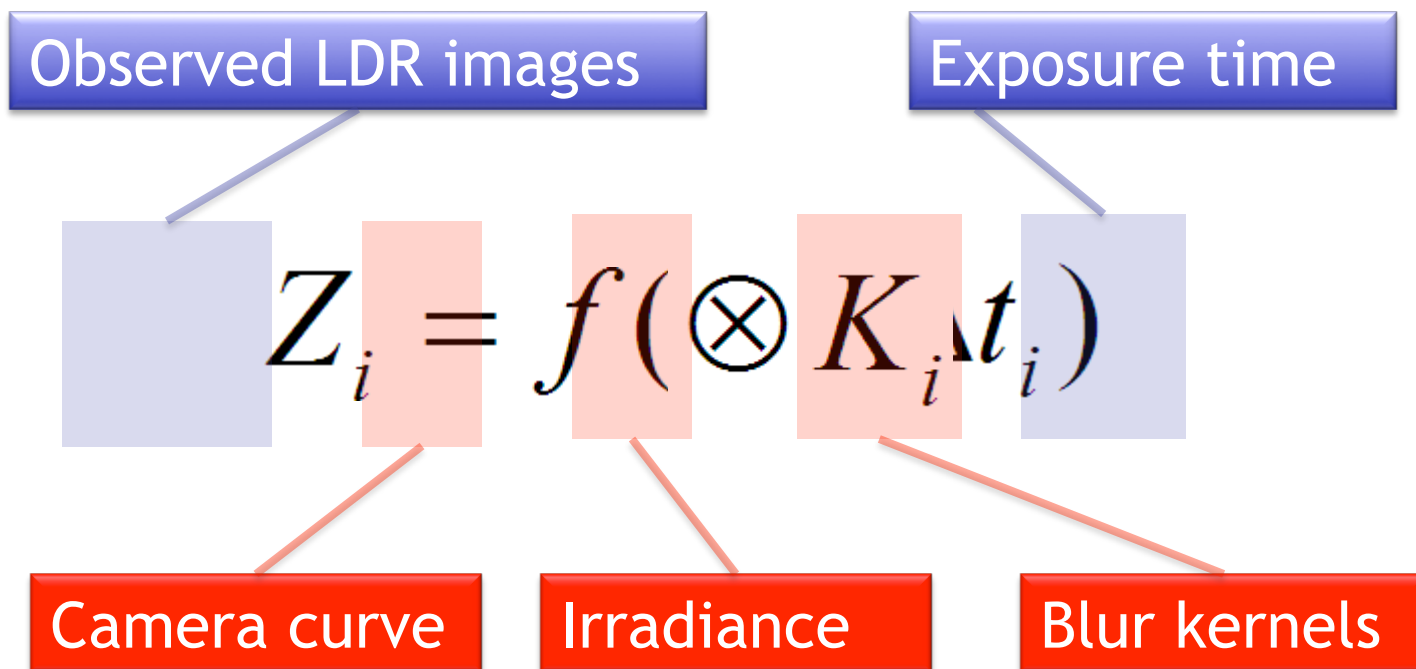


Image formation model



Main Idea

$$Z_{ij} = f((E \otimes K_j)_i \Delta t_j)$$

$$f^{-1}(Z_{ij}) / \Delta t_j = (E \otimes K_j)_i$$

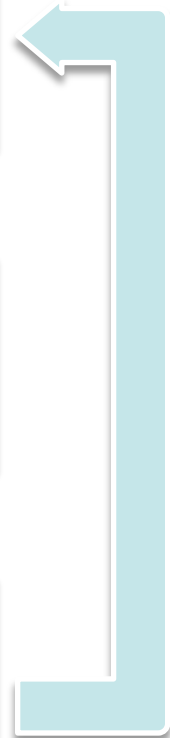
$$\arg \min_{E, K_i, f} \sum_{i=1}^N \sum_{j=1}^P \left\| (E \otimes K_j)_i - f^{-1}(Z_{ij}) / \Delta t_j \right\|^2$$

Iterative solution

Fix E, f to solve K_j

Fix K_j, f , to solve E

Fix E, K_j to solve f



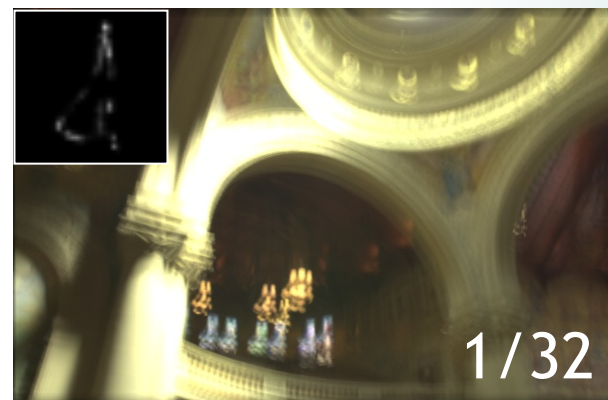
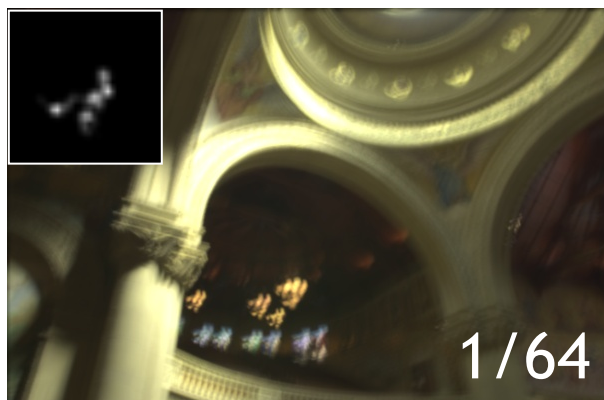
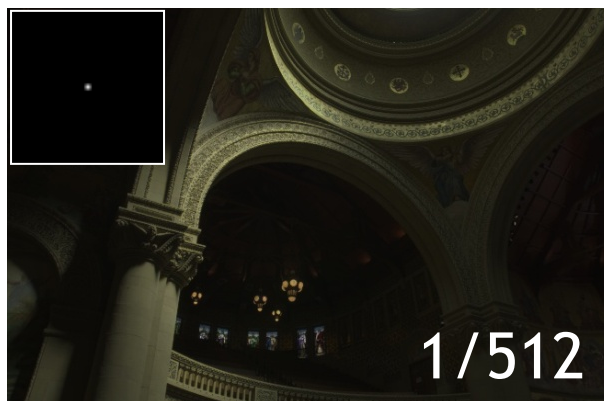
Optimization of f

$$g = f^{-1}$$

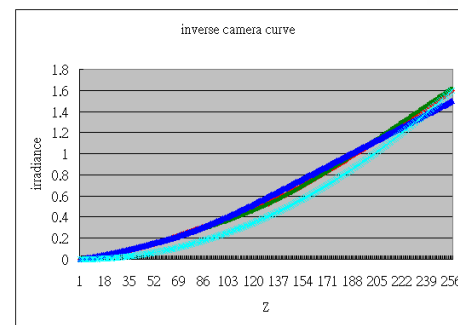
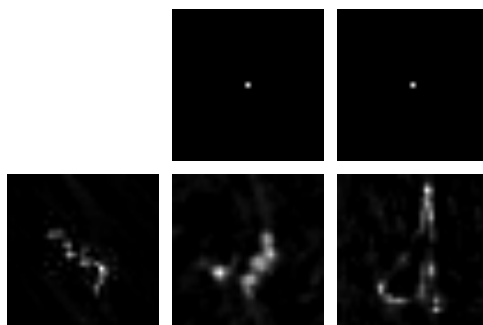
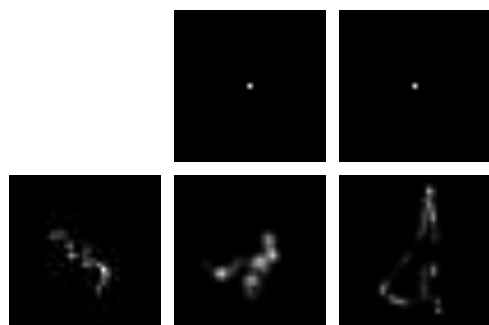
$$\arg \min_g \sum_{i=1}^N \sum_{j=1}^P \left\| (E \otimes K_j)_i - g(Z_{ij}) / \Delta t_j \right\|^2 + \lambda \sum_{z=Z_{\min}+1}^{Z_{\max}-1} g''(z)^2$$

$$w(z) = \begin{cases} z - Z_{\min}, & \text{for } z \leq \frac{1}{2}(Z_{\min} + Z_{\max}) \\ Z_{\max} - z, & \text{for } z > \frac{1}{2}(Z_{\min} + Z_{\max}) \end{cases}$$

A synthetic example



Our results



Groundtruth

Our result

Camera curve

Comparisons

Groundtruth



Our result



Fergus et al.



Yuan et al.



Groundtruth



CVPR
2009
MIAMI

Our result



CVPR
2009
MIAMI

Yuan et al.



CVPR
2009
MIAMI

Fergus et al.



CVPR
2009
MIAMI

Real example I



CVPR
2009
MIAMI

Comparisons



Close-up comparisons



Initial guess



Our result



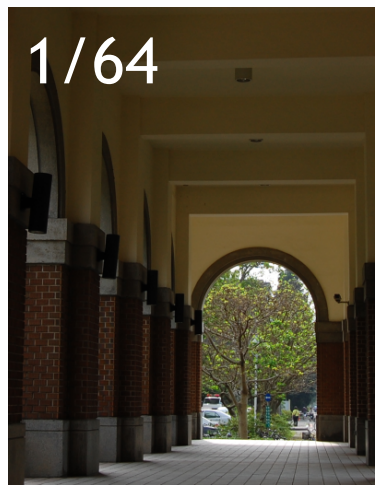
Yuan et al.



Fergus et al.



Real example II



Comparisons

Initial guess



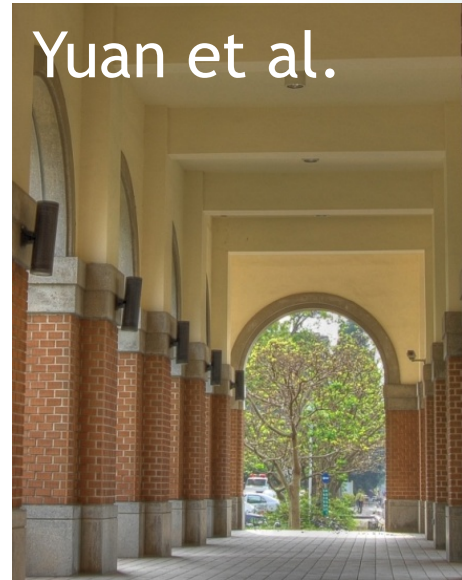
Our result



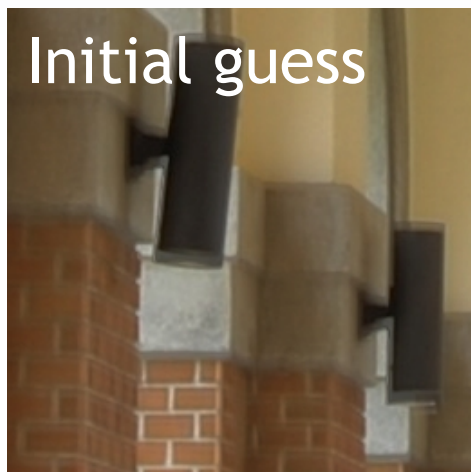
Fergus et al.



Yuan et al.



Close-up comparisons



Initial guess



Our result



Yuan et al.



Fergus et al.



Conclusions

- A technique for reconstructing a non-blurred HDR image from a set of differently exposed and blurred images taken with a hand-held camera.
- A unified formulation for recovering the irradiance image, blur kernels and the camera response curve.

Future Work

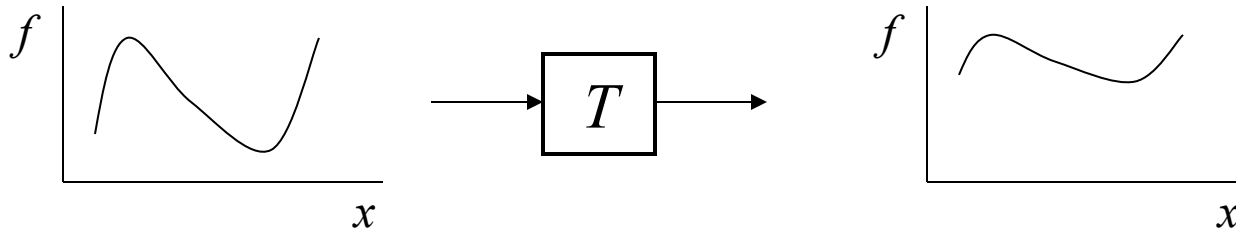
- Adding priors for kernels, HDR images and camera curves.
- Handling the shift-variant cases.

Next

Image Transformation

Image Warping

- image filtering: change **range** of image
 - $g(x) = T(f(x))$



- image warping: change **domain** of image
 - $g(x) = f(T(x))$

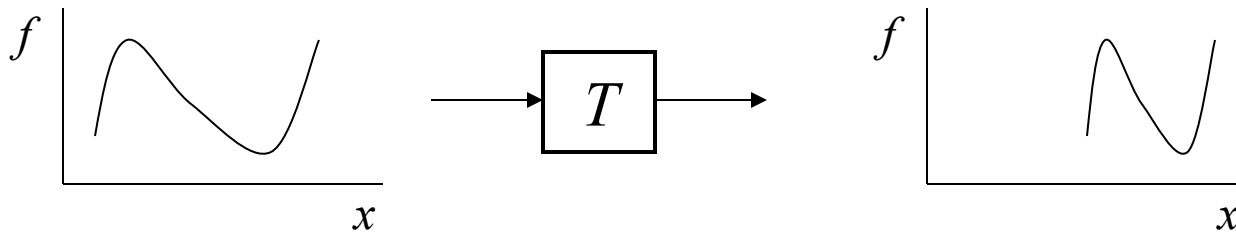
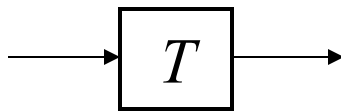
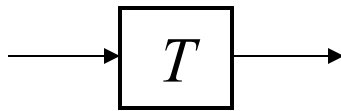


Image Warping

- image filtering: change **range** of image
 - $g(x) = T(f(x))$



- image warping: change **domain** of image
 - $g(x) = f(T(x))$



Parametric (global) warping

- Examples of parametric warps:



translation



rotation



aspect



affine



perspective

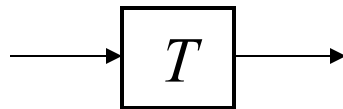


cylindrical

Parametric (global) warping



$$\mathbf{p} = (x, y)$$



$$\mathbf{p}' = (x', y')$$

- Transformation T is a coordinate-changing machine:
- $\mathbf{p}' = T(\mathbf{p})$
- What does it mean that T is global?
 - Is the same for any point \mathbf{p}
 - can be described by just a few numbers (parameters)

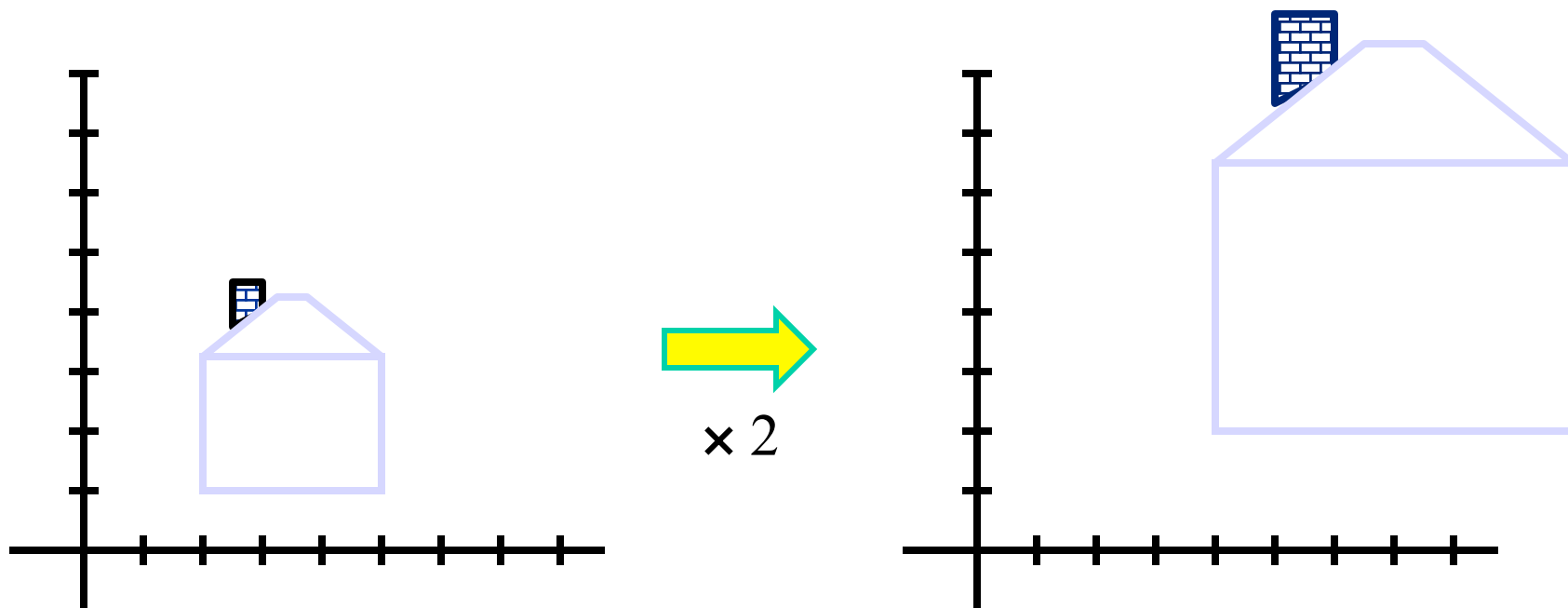
- Let's represent T as a matrix:

- $$\mathbf{p}' = \mathbf{M}\mathbf{p}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \end{bmatrix}$$

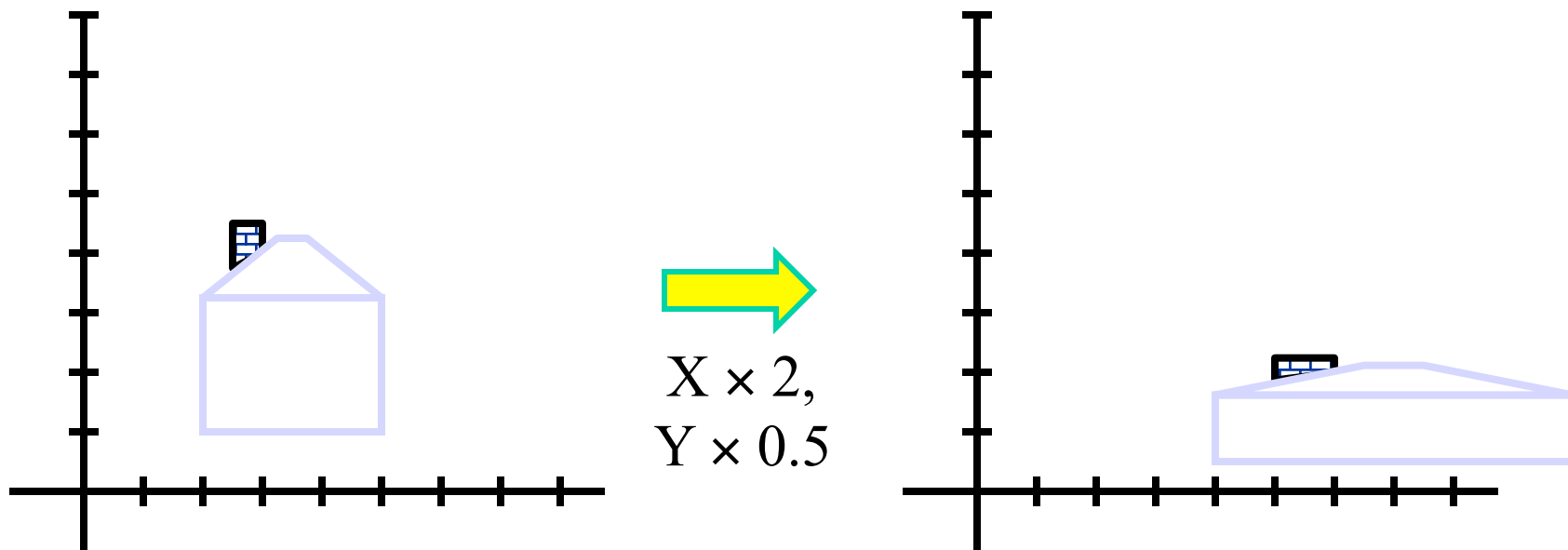
Scaling

- *Scaling* a coordinate means multiplying each of its components by a scalar
- *Uniform scaling* means this scalar is the same for all components:



Scaling

- *Non-uniform scaling*: different scalars per component:



Scaling

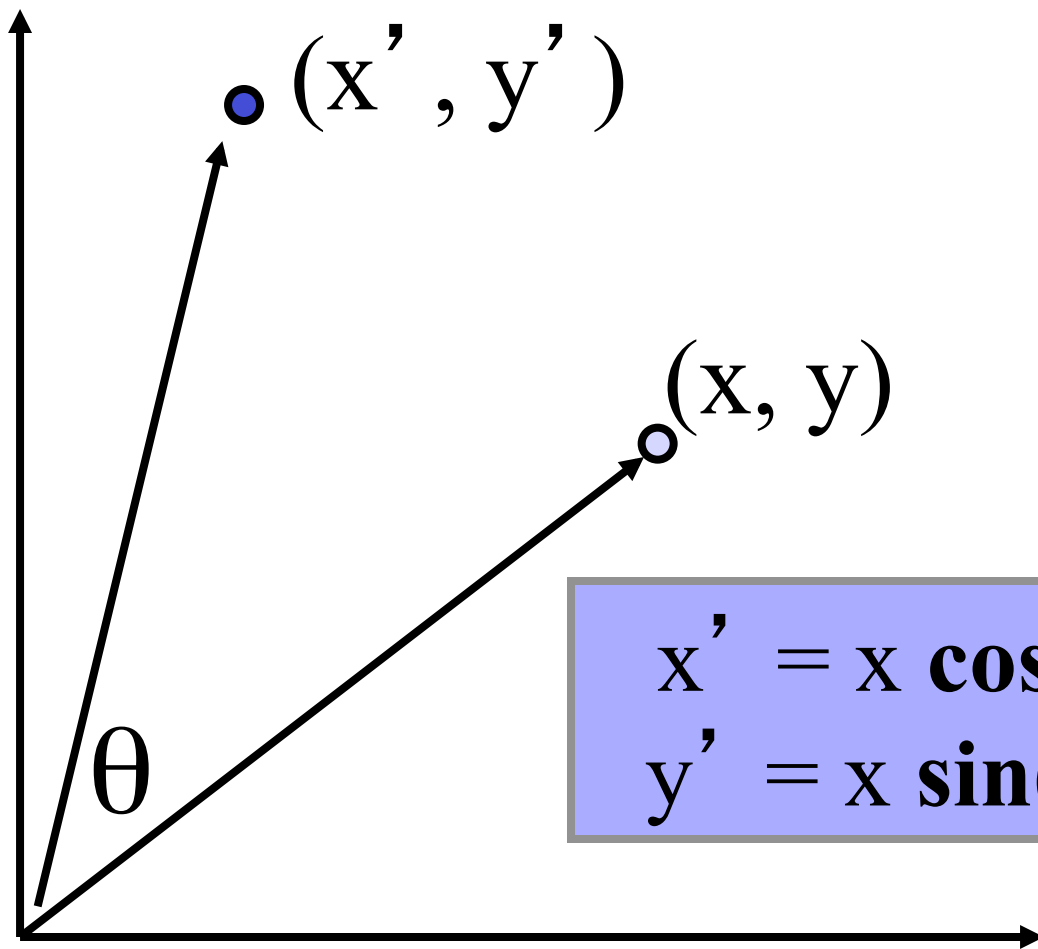
- Scaling operation: $x' = ax$
 $y' = by$

- Or, in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}}_{\text{scaling matrix } S} \begin{bmatrix} x \\ y \end{bmatrix}$$

What's inverse of S?

2-D Rotation



$$\begin{aligned}x' &= x \cos(\theta) - y \sin(\theta) \\ y' &= x \sin(\theta) + y \cos(\theta)\end{aligned}$$

2-D Rotation

- This is easy to capture in matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \underbrace{\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}}_{\mathbf{R}} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Even though $\sin(\theta)$ and $\cos(\theta)$ are nonlinear functions of θ ,
 - **x' is a linear combination of x and y**
 - **y' is a linear combination of x and y**
- What is the inverse transformation?
 - Rotation by $-\theta$
 - For rotation matrices

$$\mathbf{R}^{-1} = \mathbf{R}^T$$

2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Identity?

$$\begin{aligned}x' &= x \\ y' &= y\end{aligned}\quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Scale around (0,0)?

$$\begin{aligned}\mathbf{x}' &= s_x * \mathbf{x} \\ \mathbf{y}' &= s_y * \mathbf{y}\end{aligned}\quad \begin{bmatrix} \mathbf{x}' \\ \mathbf{y}' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \end{bmatrix}$$

2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Rotate around (0,0)?

$$\begin{aligned}x' &= \cos \Theta * x - \sin \Theta * y \\y' &= \sin \Theta * x + \cos \Theta * y\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?

$$\begin{aligned}x' &= x + sh_x * y \\y' &= sh_y * x + y\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Mirror about Y axis?

$$\begin{aligned}x' &= -x \\ y' &= y\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$\begin{aligned}x' &= -x \\ y' &= -y\end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Matrices

- What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$x' = x + t_x \quad \text{NO!}$$

$$y' = y + t_y$$

Only linear 2D transformations
can be represented with a 2x2 matrix

All 2D Linear Transformations

- Linear transformations are combinations of ...

- Scale,
- Rotation,
- Shear, and
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Properties of linear transformations:

- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous Coordinates

- **Q: How can we represent translation as a 3x3 matrix?**

$$x' = x + t_x$$

$$y' = y + t_y$$

Homogeneous Coordinates

- ***Homogeneous coordinates***

- represent coordinates in 2 dimensions with a 3-vector

$$\begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous Coordinates

- **Q: How can we represent translation as a 3x3 matrix?**

$$x' = x + t_x$$

$$y' = y + t_y$$

- **A: Using the rightmost column:**

$$\textbf{Translation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

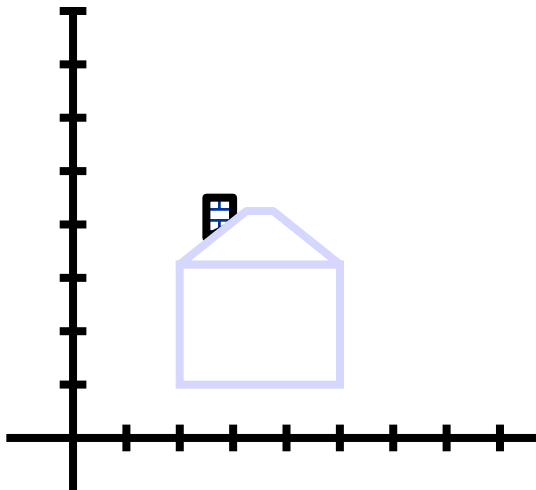
Translation

- Example of translation

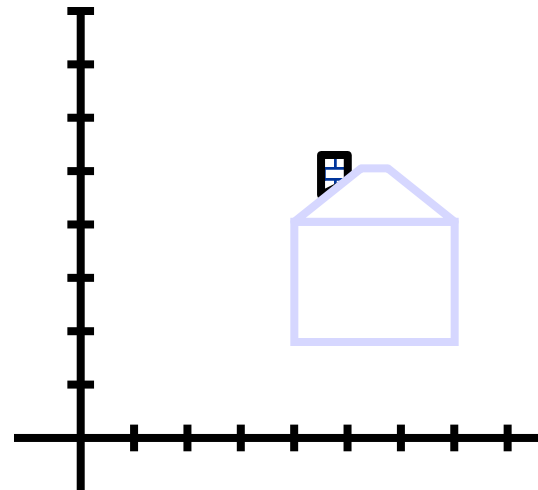
Homogeneous Coordinates



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

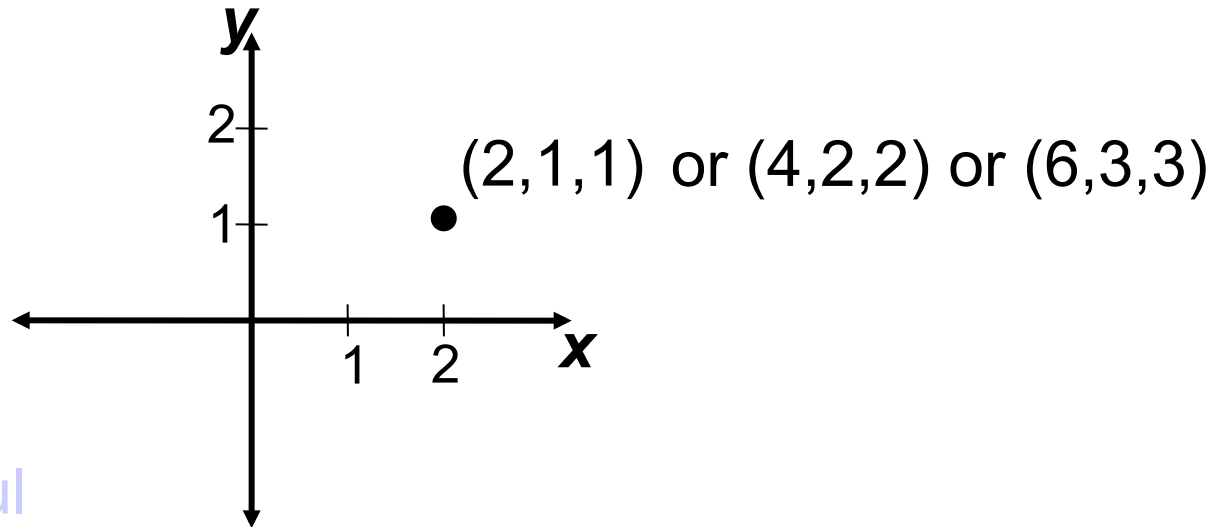


$$\begin{aligned} t_x &= 2 \\ t_y &= 1 \end{aligned}$$



Homogeneous Coordinates

- Add a 3rd coordinate to every 2D point
 - (x, y, w) represents a point at location $(x/w, y/w)$
 - $(x, y, 0)$ represents a point at infinity
 - $(0, 0, 0)$ is not allowed



Convenient
coordinate system to
represent many useful
transformations

Basic 2D Transformations

- Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} \mathbf{x}' \\ \mathbf{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} \mathbf{x}' \\ \mathbf{y}' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ 1 \end{bmatrix}$$

Shear

Affine Transformations

- Affine transformations are combinations of ...
 - Linear transformations, and
 - Translations
- Properties of affine transformations:
 - Origin does not necessarily map to origin
 - Lines map to lines
 - Parallel lines remain parallel
 - Ratios are preserved
 - Closed under composition

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

Projective Transformations

- Projective transformations ...
$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$
 - Affine transformations, and
 - Projective warps
- Properties of projective transformations:
 - Origin does not necessarily map to origin
 - Lines map to lines
 - Parallel lines do not necessarily remain parallel
 - Ratios are not preserved
 - Closed under composition

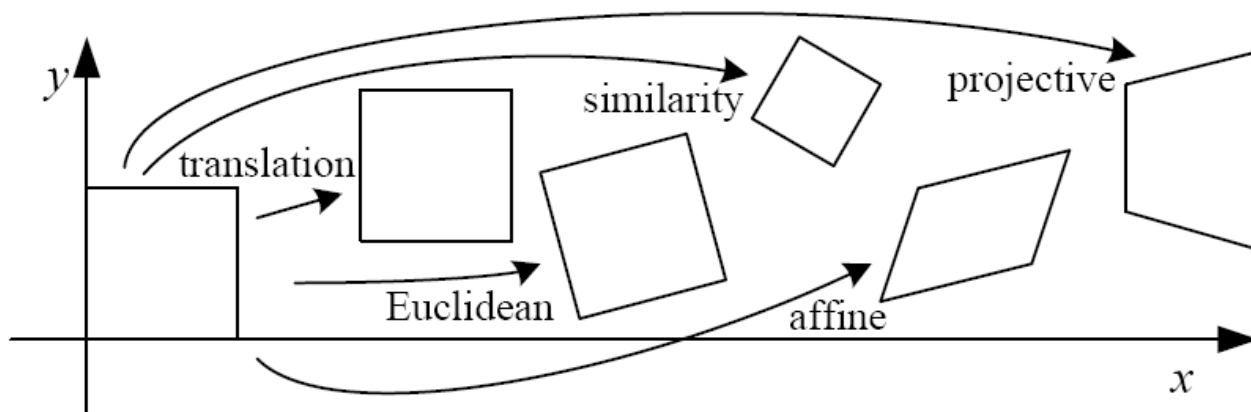
Matrix Composition

- Transformations can be combined by matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$\mathbf{p}' = \mathbf{T}(t_x, t_y) \mathbf{R}(\Theta) \mathbf{S}(s_x, s_y) \mathbf{p}$

2D image transformations

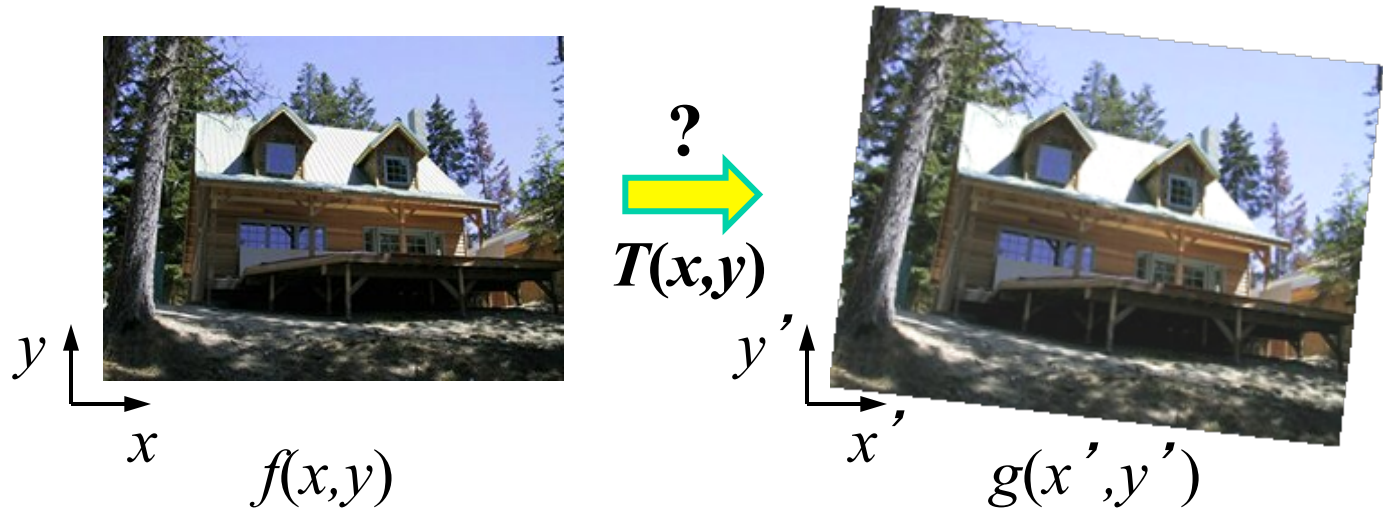


Name	Matrix	# D.O.F.	Preserves:	Icon
translation	$\begin{bmatrix} \mathbf{I} & \mathbf{t} \end{bmatrix}_{2 \times 3}$			
rigid (Euclidean)	$\begin{bmatrix} \mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$			
similarity	$\begin{bmatrix} s\mathbf{R} & \mathbf{t} \end{bmatrix}_{2 \times 3}$			
affine	$\begin{bmatrix} \mathbf{A} \end{bmatrix}_{2 \times 3}$			
projective	$\begin{bmatrix} \tilde{\mathbf{H}} \end{bmatrix}_{3 \times 3}$			

These transformations are a nested set of groups

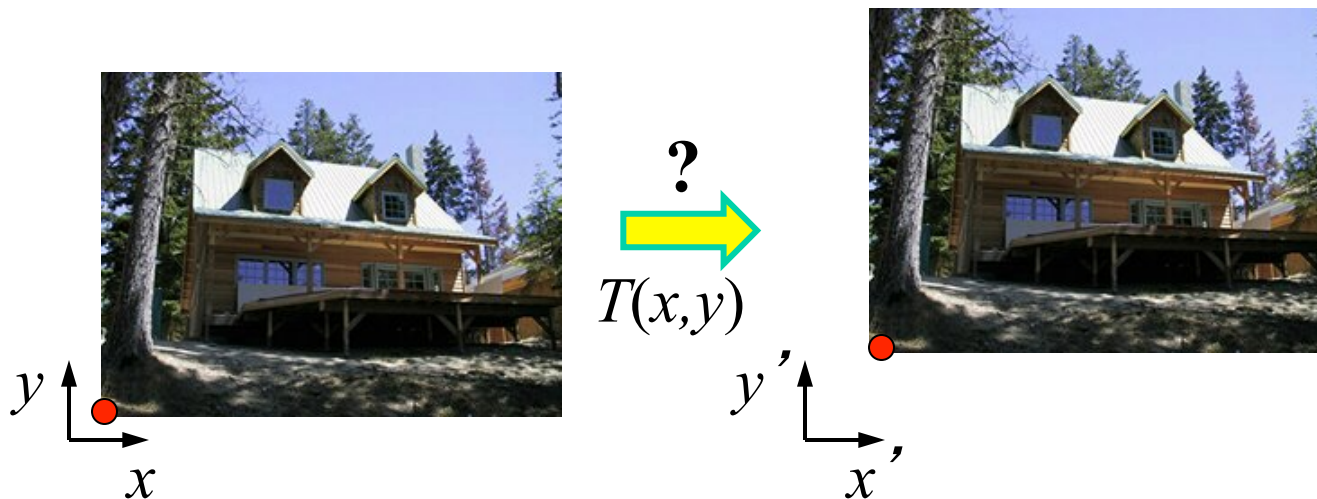
- Closed under composition and inverse is a member

Recovering Transformations



- What if we know f and g and want to recover the transform T ?
 - Using correspondences
 - How many do we need?

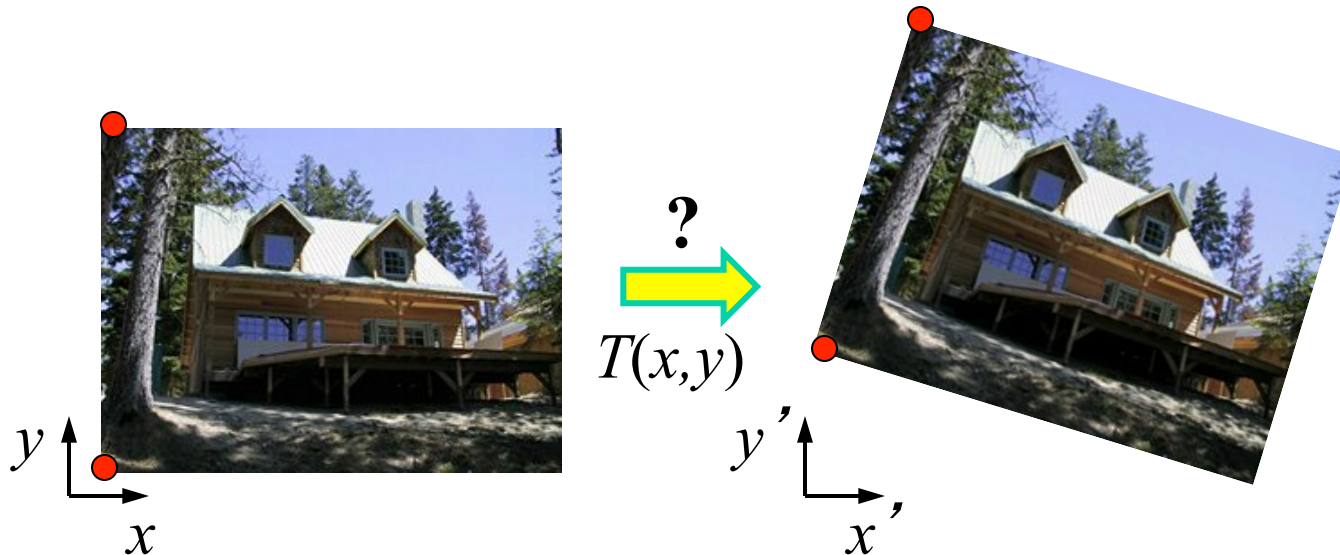
Translation: # correspondences?



- How many correspondences needed for translation?
- How many Degrees of Freedom?
- What is the transformation matrix?

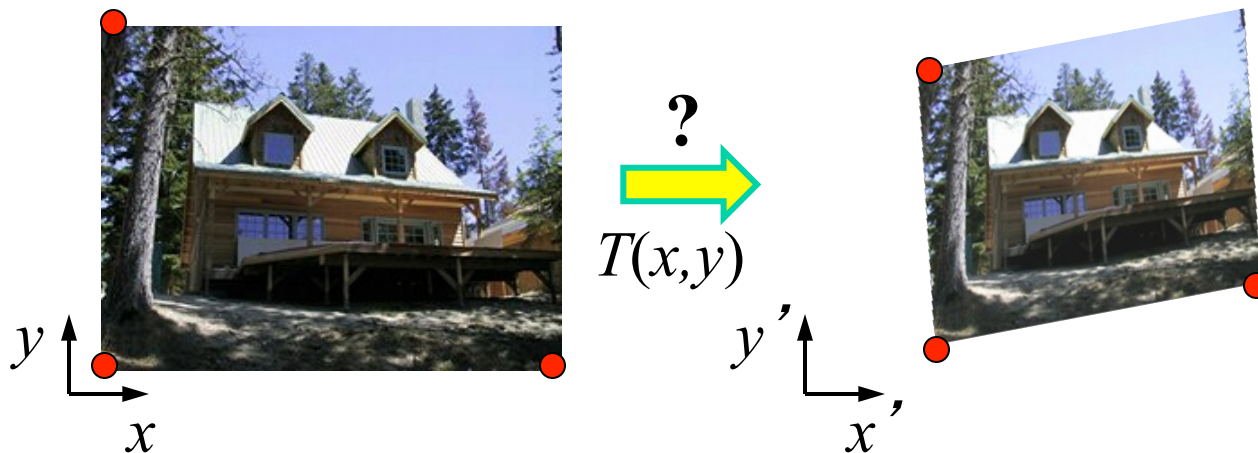
$$\mathbf{M} = \begin{bmatrix} 1 & 0 & p'_x - p_x \\ 0 & 1 & p'_y - p_y \\ 0 & 0 & 1 \end{bmatrix}$$

Euclidian: # correspondences?



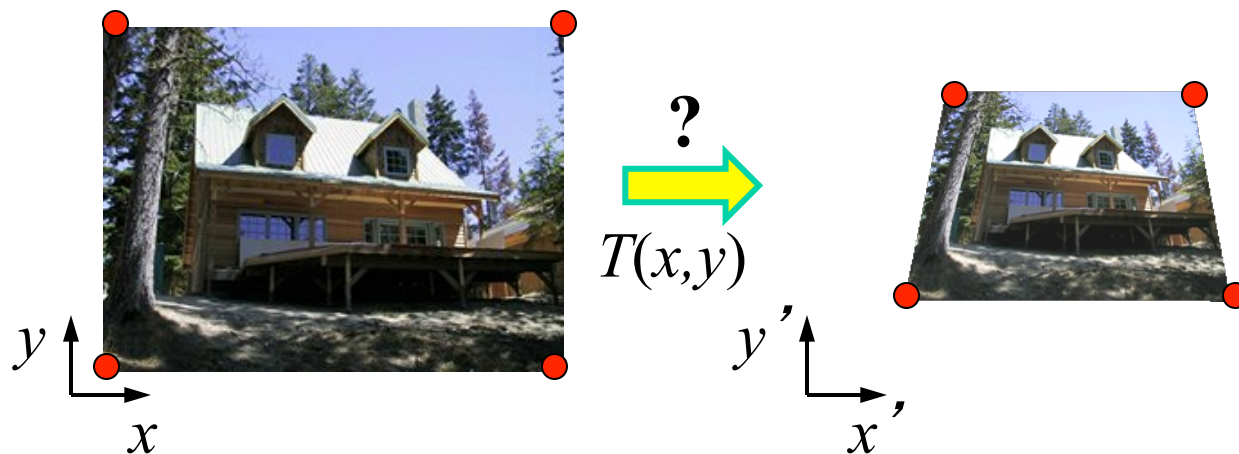
- How many correspondences needed for translation+rotation?
- How many DOF?

Affine: # correspondences?



- How many correspondences needed for affine?
- How many DOF?

Projective: # correspondences?



- How many correspondences needed for projective?
- How many DOF?