

3D Modeling from a photograph



<https://research.microsoft.com/vision/cambridge/3d/3dart.htm>

A different way of modeling

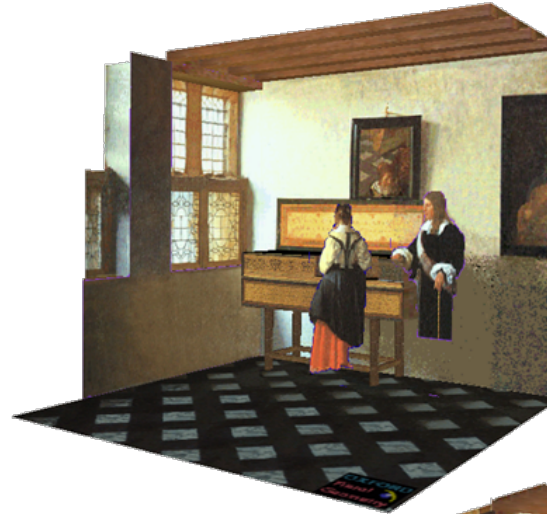
- <http://mit.edu/jxiao/museum/>

Last topic

- Single View Modeling



Vermeer's *Music Lesson*



Reconstructions by Criminisi et al.

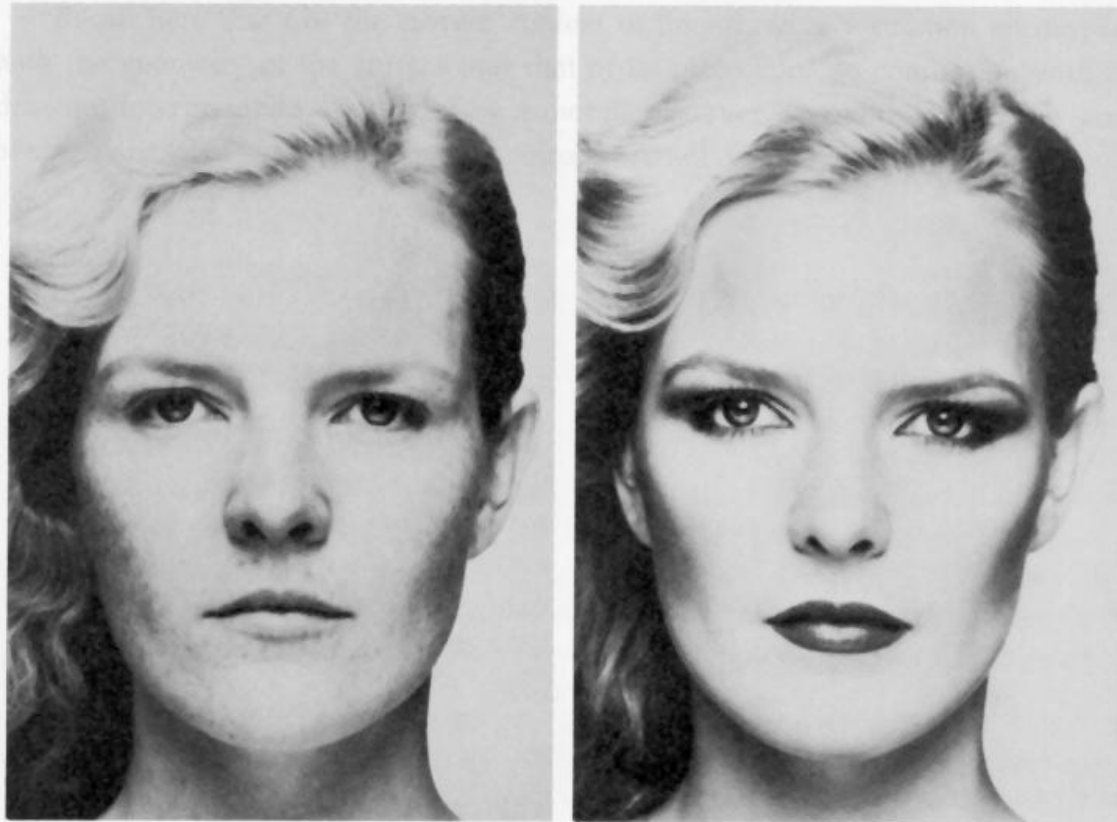
Next

- Photometric Stereo
- Separate Global and Direct Illumination

Photometric Stereo



Photometric Stereo

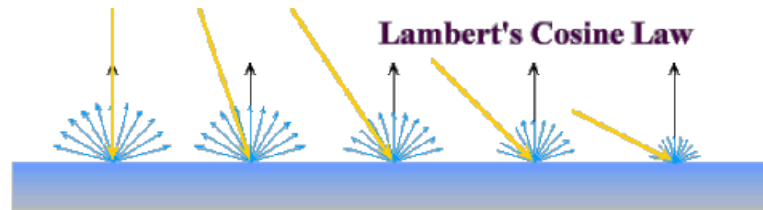
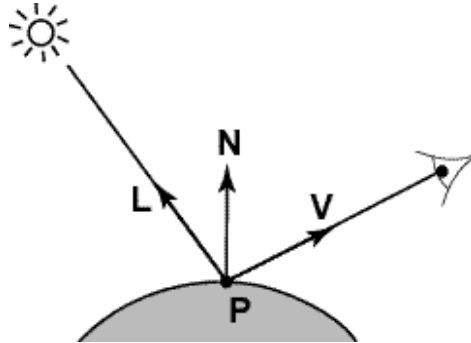


Merle Norman Cosmetics, Los Angeles

Readings

- R. Woodham, *Photometric Method for Determining Surface Orientation from Multiple Images*. Optical Engineering 19(1)139-144 (1980). ([PDF](#))

Diffuse reflection



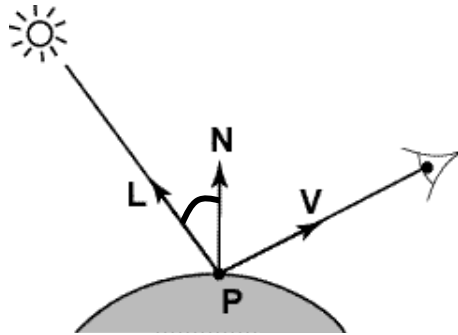
$$R_e = k_d \mathbf{N} \cdot \mathbf{L} R_i$$

image intensity of $\mathbf{P} \longrightarrow I = k_d \mathbf{N} \cdot \mathbf{L}$

Simplifying assumptions

- $I = R_e$: camera response function f is the identity function:
 - can always achieve this in practice by solving for f and applying f^{-1} to each pixel in the image
- $R_i = 1$: light source intensity is 1
 - can achieve this by dividing each pixel in the image by R_i

Shape from shading



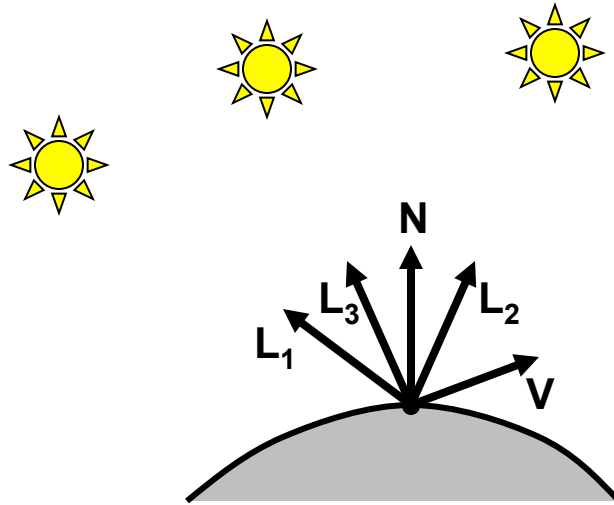
Suppose $k_d = 1$

$$\begin{aligned} I &= k_d \mathbf{N} \cdot \mathbf{L} \\ &= \mathbf{N} \cdot \mathbf{L} \\ &= \cos \theta_i \end{aligned}$$

You can directly measure angle between normal and light source

- Not quite enough information to compute surface shape
- But can be if you add some additional info, for example
 - assume a few of the normals are known (e.g., along silhouette)
 - smoothness
- Hard to get it to work well in practice
 - plus, how many real objects have constant albedo?

Photometric stereo



$$I_1 = k_d \mathbf{N} \cdot \mathbf{L}_1$$

$$I_2 = k_d \mathbf{N} \cdot \mathbf{L}_2$$

$$I_3 = k_d \mathbf{N} \cdot \mathbf{L}_3$$

Can write this as a matrix equation:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = k_d \begin{bmatrix} \mathbf{L}_1^T \\ \mathbf{L}_2^T \\ \mathbf{L}_3^T \end{bmatrix} \mathbf{N}$$

Solving the equations

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{\substack{\mathbf{I} \\ 3 \times 1}} = \underbrace{\begin{bmatrix} \mathbf{L}_1^T \\ \mathbf{L}_2^T \\ \mathbf{L}_3^T \end{bmatrix}}_{\substack{\mathbf{L} \\ 3 \times 3}} \underbrace{k_d \mathbf{N}}_{\substack{\mathbf{G} \\ 3 \times 1}}$$

$$\mathbf{G} = \mathbf{L}^{-1} \mathbf{I}$$

$$k_d = \|\mathbf{G}\|$$

$$\mathbf{N} = \frac{1}{k_d} \mathbf{G}$$

More than three lights

Get better results by using more lights

$$\begin{bmatrix} I_1 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1 \\ \vdots \\ \mathbf{L}_n \end{bmatrix} k_d \mathbf{N}$$

Least squares solution:

$$\mathbf{I} = \mathbf{L}\mathbf{G}$$

$$\mathbf{L}^T \mathbf{I} = \mathbf{L}^T \mathbf{L} \mathbf{G}$$

$$\mathbf{G} = (\mathbf{L}^T \mathbf{L})^{-1} (\mathbf{L}^T \mathbf{I})$$

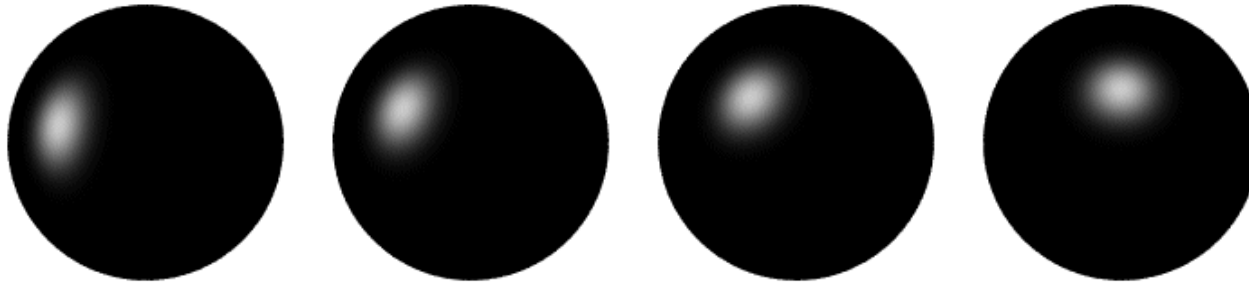
Solve for \mathbf{N} , k_d as before

What's the size of $\mathbf{L}^T \mathbf{L}$?



Computing light source directions

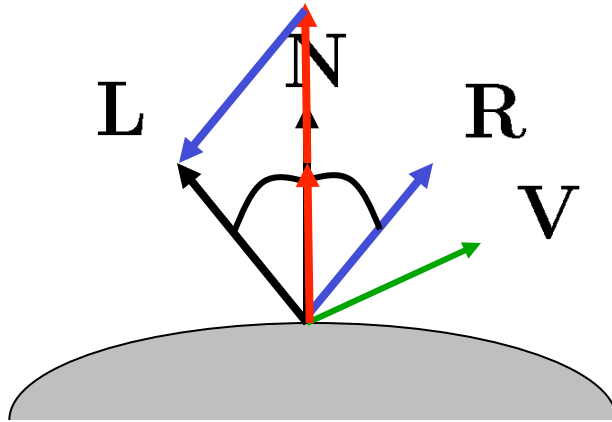
Trick: place a chrome sphere in the scene



- the location of the highlight tells you where the light source is

Recall the rule for specular reflection

For a perfect mirror, light is reflected about **N**



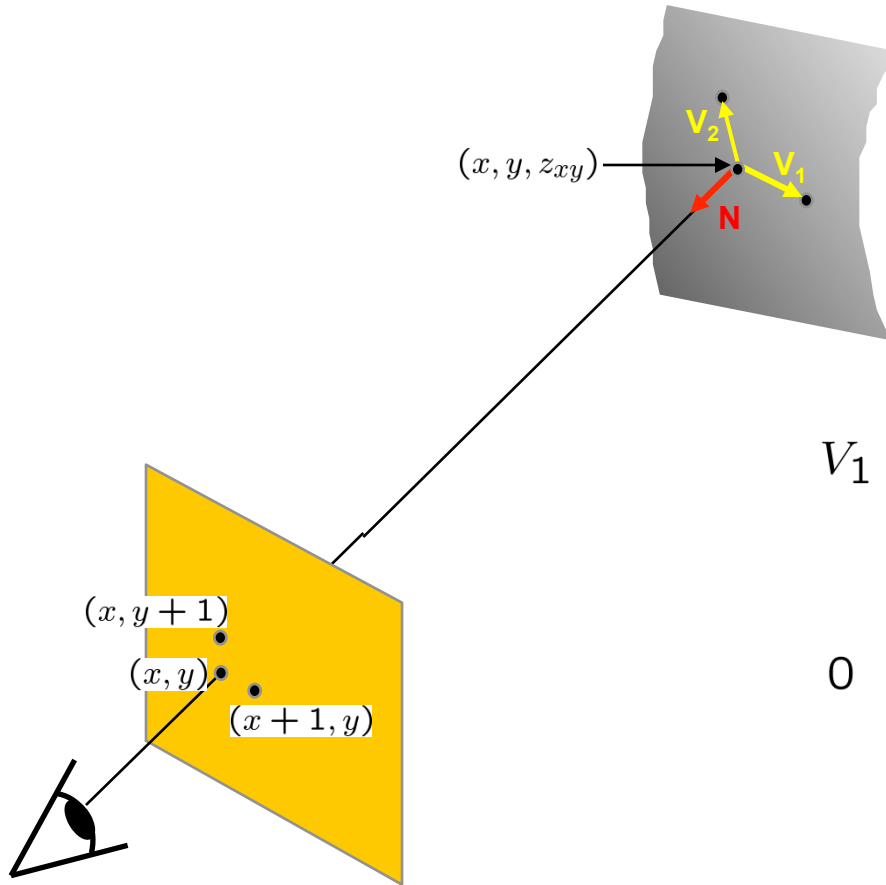
$$R_e = \begin{cases} R_i & \text{if } \mathbf{V} = \mathbf{R} \\ 0 & \text{otherwise} \end{cases}$$

We see a highlight when **V = R**

- then **L** is given as follows:

$$\mathbf{L} = 2(\mathbf{N} \cdot \mathbf{R})\mathbf{N} - \mathbf{R}$$

Depth from normals



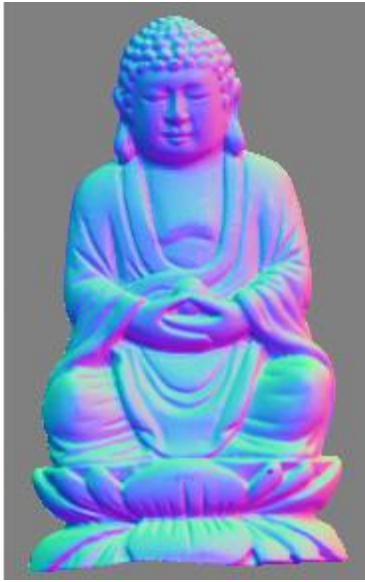
$$\begin{aligned} V_1 &= (x+1, y, z_{x+1,y}) - (x, y, z_{xy}) \\ &= (1, 0, z_{x+1,y} - z_{xy}) \end{aligned}$$

$$\begin{aligned} 0 &= N \cdot V_1 \\ &= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\ &= n_x + n_z(z_{x+1,y} - z_{xy}) \end{aligned}$$

Get a similar equation for \mathbf{V}_2

- Each normal gives us two linear constraints on z
- compute z values by solving a matrix equation

Example



What if we don't have mirror ball?

- Hayakawa, Journal of the Optical Society of America, 1994,
[Photometric stereo under a light source with arbitrary motion.](#)