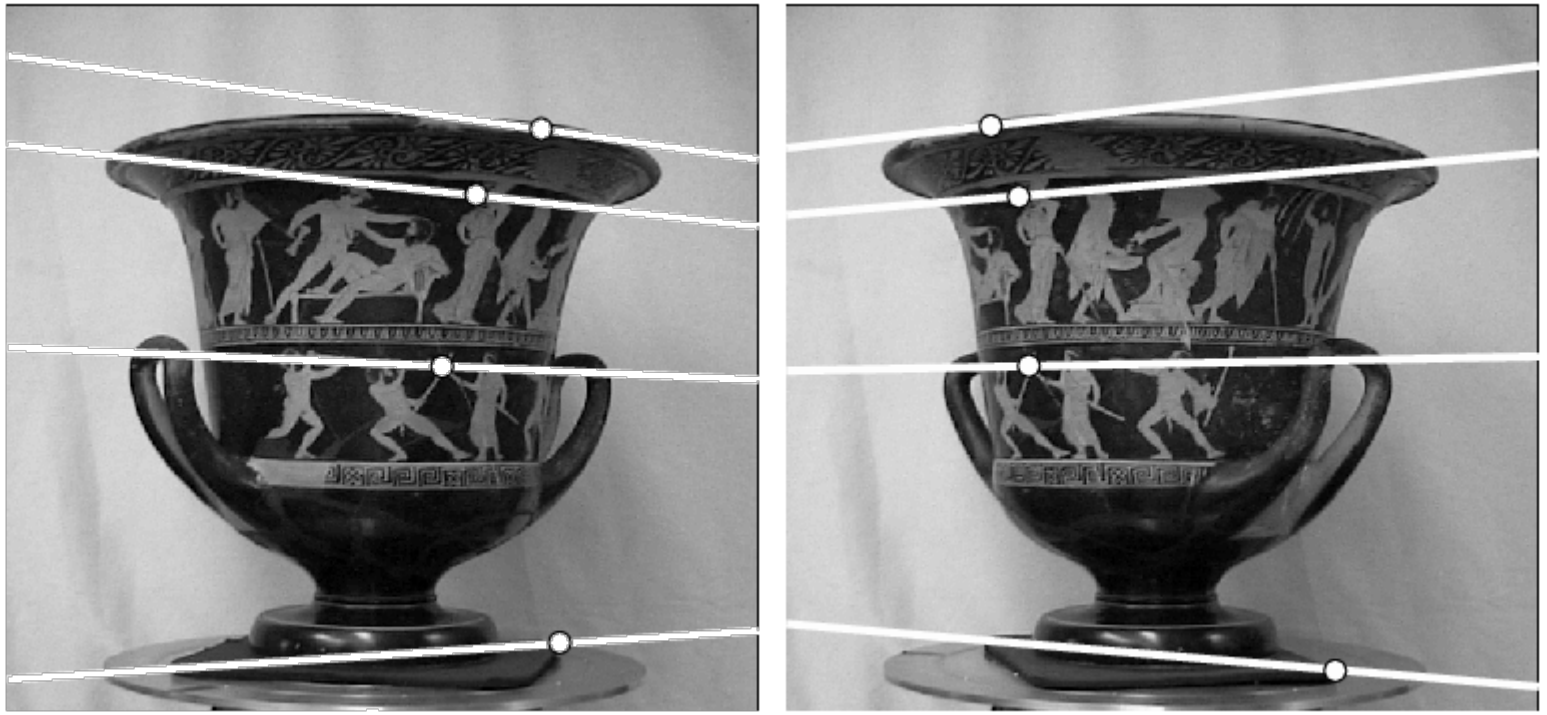
The fundamental matrix F



- It can be used for
  - Simplifies matching
  - Allows to detect wrong matches

Estimation of F — 8-point algorithm

- The fundamental matrix F is defined by

$$\mathbf{x}'^{\mathrm{T}}\mathbf{F}\mathbf{x} = 0$$

for any pair of matches x and x' in two images.

- Let x=$(u,v,1)^{\mathrm{T}}$ and x'=$(u',v',1)^{\mathrm{T}}$, $\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$

each match gives a linear equation

$$uu'f_{11} + vu'f_{12} + u'f_{13} + uv'f_{21} + vv'f_{22} + v'f_{23} + uf_{31} + vf_{32} + f_{33} = 0$$

8-point algorithm

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_n' & v_nu_n' & u_n' & u_nv_n' & v_nv_n' & v_n' & u_n & v_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

$$\mathbf{Af} = 0$$

8-point algorithm

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_n' & v_nu_n' & u_n' & u_nv_n' & v_nv_n' & v_n' & u_n & v_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

- In reality, instead of solving $\mathbf{Af} = 0$, we seek $\mathbf{f}$ to minimize $\left\|\mathbf{Af}\right\|$, least eigenvector of $\mathbf{A}^{\mathrm{T}}\mathbf{A}$.

8-point algorithm

- To enforce that F is of rank 2, F is replaced by F' that minimizes $\|\mathbf{F} - \mathbf{F}'\|$ subject to $\det \mathbf{F}' = 0$.

8-point algorithm

- To enforce that F is of rank 2, F is replaced by F' that minimizes $\|\mathbf{F} - \mathbf{F}'\|$ subject to $\det \mathbf{F}' = 0$.

- It is achieved by SVD. Let $\mathbf{F} = \mathbf{U}\Sigma\mathbf{V}^{\mathrm{T}}$, where

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}, \text{ let } \quad \Sigma' = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

then $\mathbf{F}' = \mathbf{U}\Sigma'\mathbf{V}^{\mathrm{T}}$ is the solution.

8-point algorithm

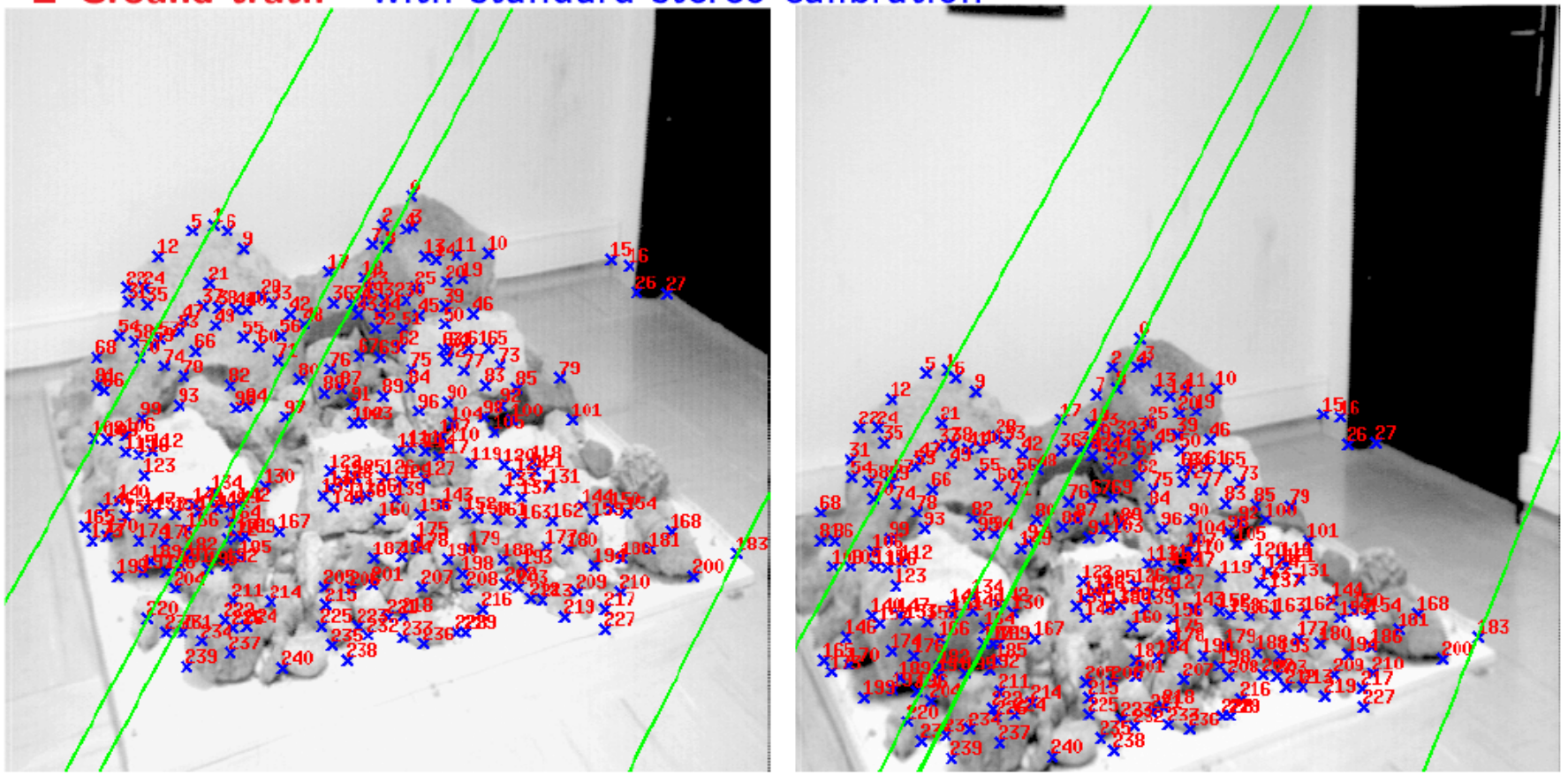
```
% Build the constraint matrix
   A = [x2(1,:)‘.*x1(1,:)'   x2(1,:)'.*x1(2,:)'  x2(1,:)' ...
        x2(2,:)'.*x1(1,:)'   x2(2,:)'.*x1(2,:)'  x2(2,:)' ...
        x1(1,:)'             x1(2,:)'            ones(npts,1) ];

   [U,D,V] = svd(A);

% Extract fundamental matrix from the column of V
% corresponding to the smallest singular value.
   F = reshape(V(:,9),3,3)';

% Enforce rank2 constraint
   [U,D,V] = svd(F);
   F = U*diag([D(1,1) D(2,2) 0])*V';
```

8-point algorithm

- Pros: it is linear, easy to implement and fast
- Cons: susceptible to noise

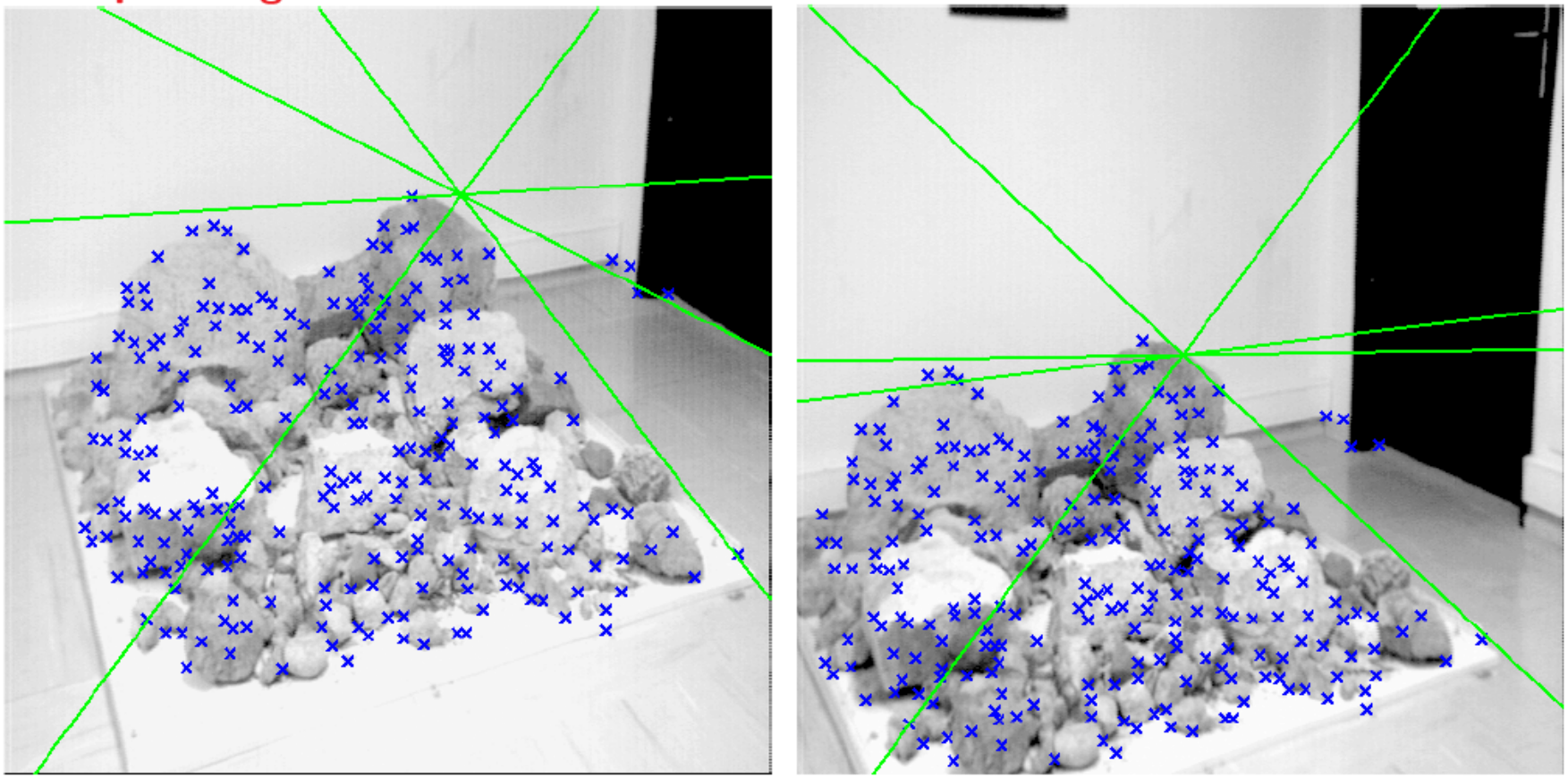
Results (ground truth)

- **Ground truth** with standard stereo calibration



Results (8-point algorithm)

- 8-point algorithm



Problem with 8-point algorithm

$$\begin{bmatrix} u_1u_1' & v_1u_1' & u_1' & u_1v_1' & v_1v_1' & v_1' & u_1 & v_1 & 1 \\ u_2u_2' & v_2u_2' & u_2' & u_2v_2' & v_2v_2' & v_2' & u_2 & v_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_nu_n' & v_nu_n' & u_n' & u_nv_n' & v_nv_n' & v_n' & u_n & v_n & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix} = 0$$

~10000 ~10000 ~100 ~10000 ~10000 ~100 ~100 ~100 1

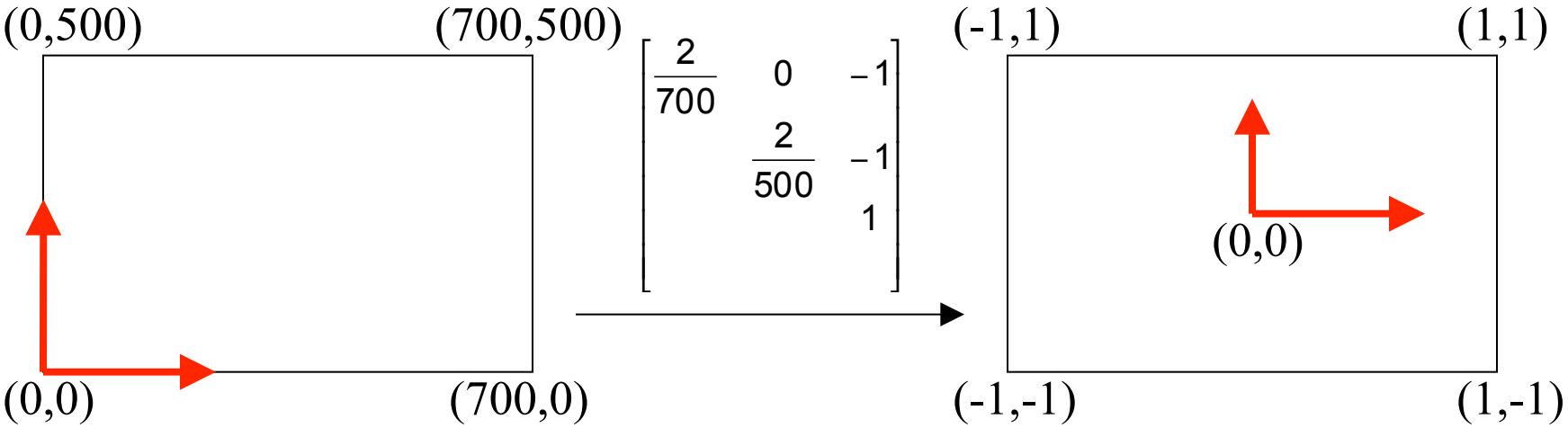
Orders of magnitude difference between column of data matrix

→ least-squares yields poor results

Normalized 8-point algorithm

normalized least squares yields good results

Transform image to ~[-1,1]x[-1,1]



Normalized 8-point algorithm

1. Transform input by $\hat{\mathbf{x}}_i = \mathbf{T}\mathbf{x}_i, \hat{\mathbf{x}}'_i = \mathbf{T}\mathbf{x}'_i$
2. Call 8-point on $\hat{\mathbf{x}}_i, \hat{\mathbf{x}}'_i$ to obtain $\hat{\mathbf{F}}$
3. $\mathbf{F} = \mathbf{T}'^{\mathrm{T}}\hat{\mathbf{F}}\mathbf{T}$

$$\mathbf{x}'^{\mathrm{T}}\mathbf{F}\mathbf{x} = 0$$

$$\hat{\mathbf{x}}'^{\mathrm{T}}\mathbf{T}'^{-\mathrm{T}}\mathbf{F}\mathbf{T}^{-1}\hat{\mathbf{x}} = 0$$

$$\hat{\mathbf{F}}$$

Normalized 8-point algorithm

```
[x1, T1] = normalise2dpts(x1);
[x2, T2] = normalise2dpts(x2);
  A = [x2(1,:)‘.*x1(1,:)'   x2(1,:)'.*x1(2,:)'  x2(1,:)' ...
       x2(2,:)'.*x1(1,:)'   x2(2,:)'.*x1(2,:)'  x2(2,:)' ...
       x1(1,:)'             x1(2,:)'            ones(npts,1) ];

  [U,D,V] = svd(A);

  F = reshape(V(:,9),3,3)';

  [U,D,V] = svd(F);
  F = U*diag([D(1,1) D(2,2) 0])*V';

% Denormalise
   F = T2'*F*T1;
```

Normalization

```
function [newpts, T] = normalise2dpts(pts)

    c = mean(pts(1:2,:)')';    % Centroid
    newp(1,:) = pts(1,:)-c(1); % Shift origin to centroid.
    newp(2,:) = pts(2,:)-c(2);

    meandist = mean(sqrt(newp(1,:).^2 + newp(2,:).^2));
    scale = sqrt(2)/meandist;

    T = [scale      0    -scale*c(1)
            0     scale  -scale*c(2)
            0       0          1      ];
    newpts = T*pts;
```

RANSAC

repeat

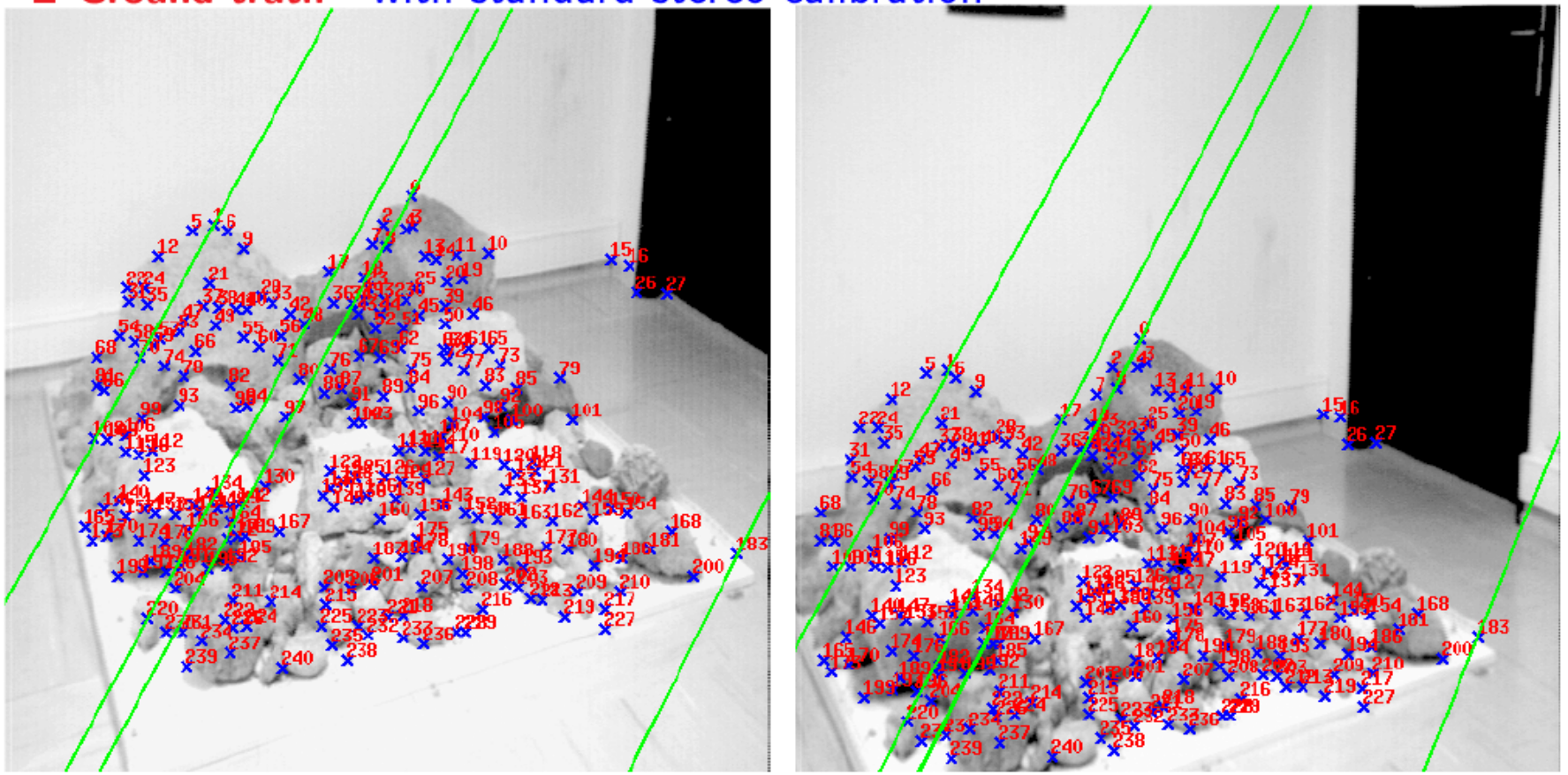
- select minimal sample (8 matches)
- compute solution(s) for F
- determine inliers

until Γ(*#inliers*,*#samples*)>95% or too many times

compute F based on all inliers

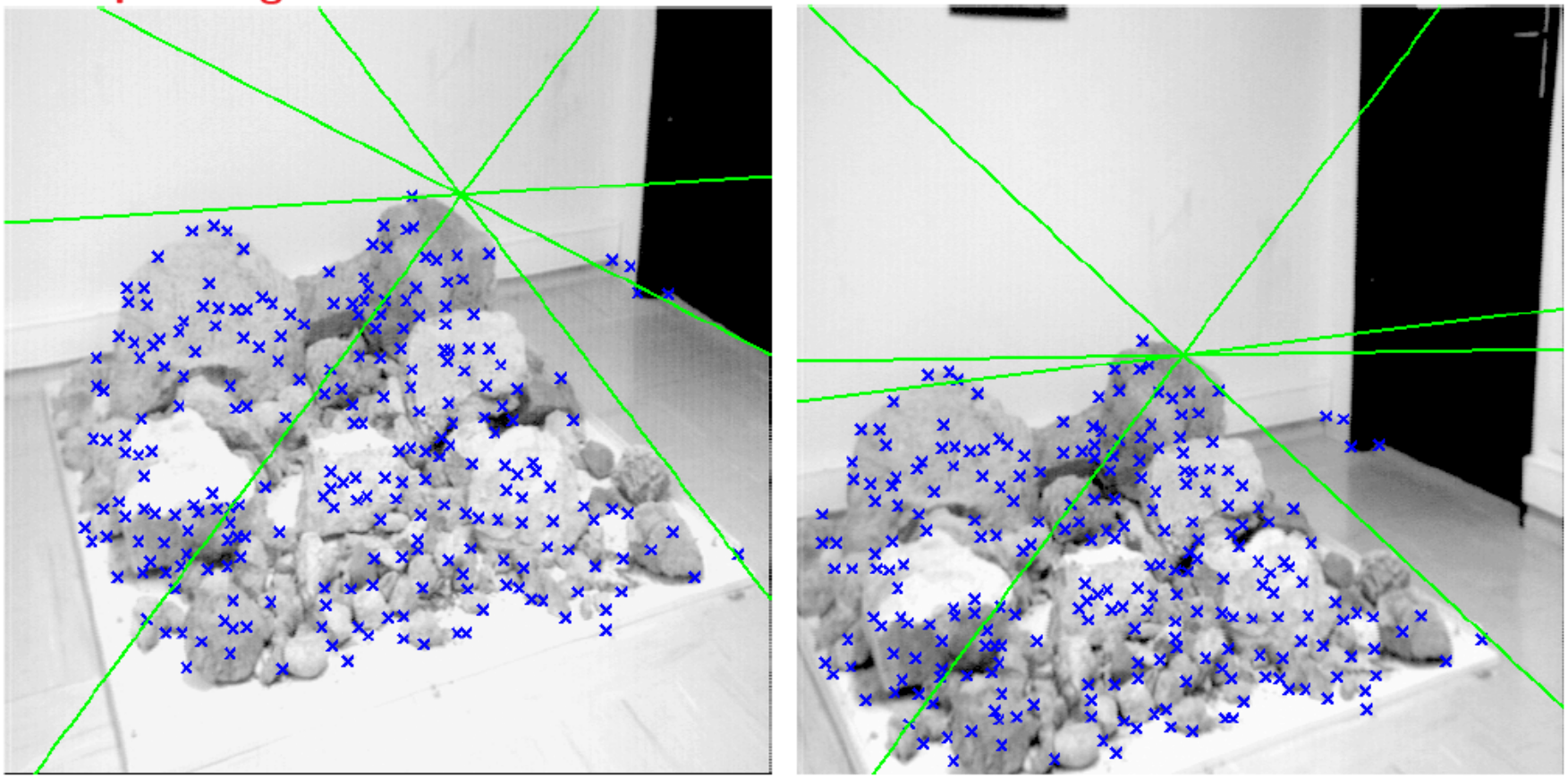
Results (ground truth)

- **Ground truth** with standard stereo calibration



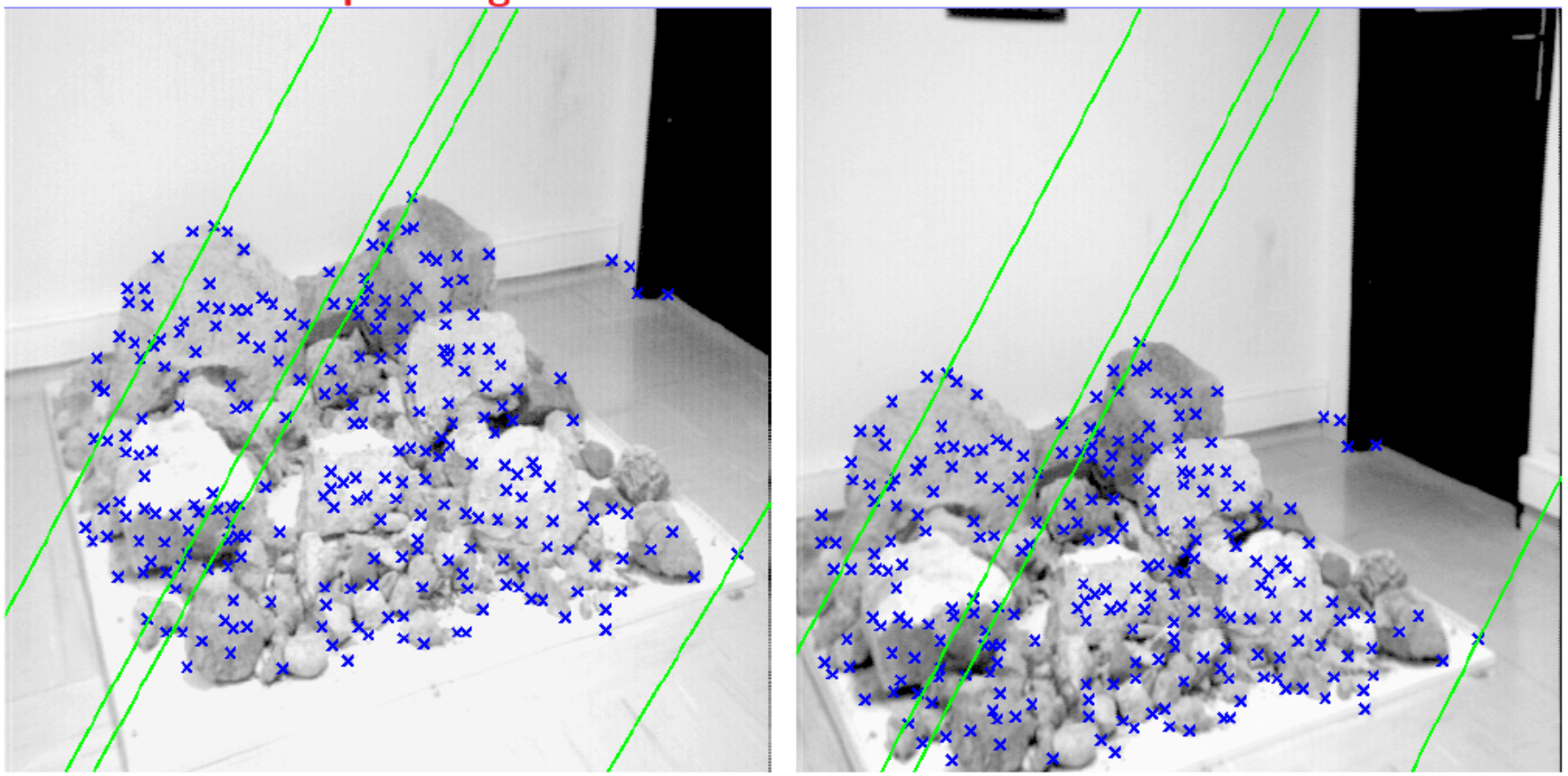
Results (8-point algorithm)

- 8-point algorithm



Results (normalized 8-point algorithm)

- Normalized 8-point algorithm



From F to R, T

$$\mathbf{x}'^{\mathrm{T}}\mathbf{F}\mathbf{x} = 0$$

$$\mathbf{x}'^{\mathrm{T}}\mathbf{K}'^{-\mathrm{T}}\mathbf{E}\mathbf{K}^{-1}\mathbf{x} = 0$$

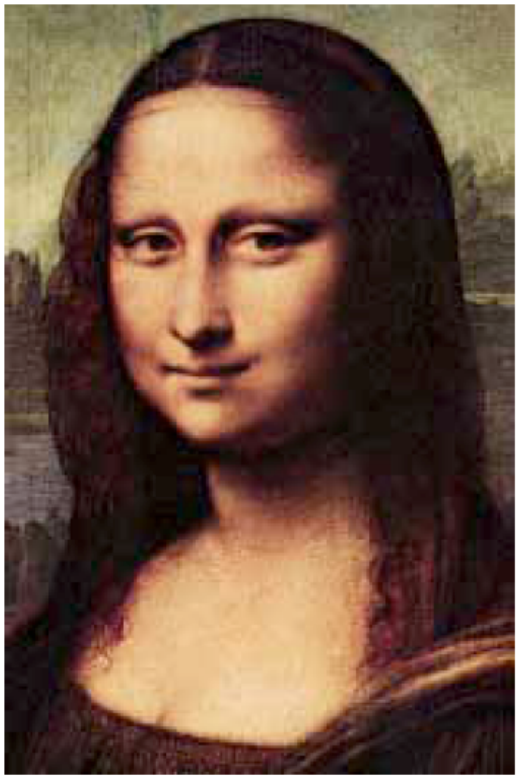
$$\mathbf{E} = \mathbf{K}'^{\mathrm{T}}\mathbf{F}\mathbf{K}$$

If we know camera parameters

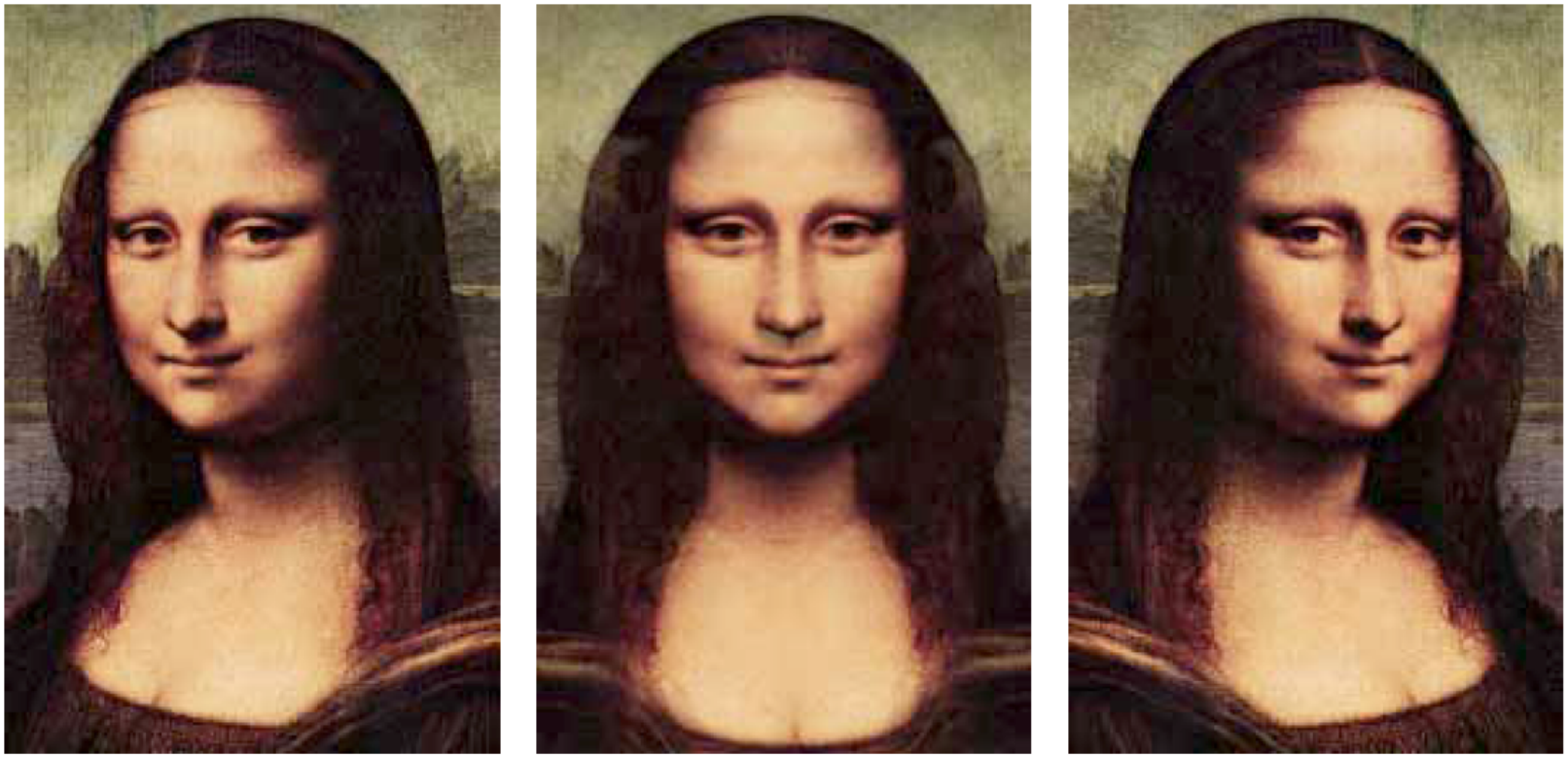
$$\mathbf{E} = \mathbf{R}[\mathbf{T}]_{\times}$$

Hartley and Zisserman, Multiple View Geometry, 2nd edition, pp 259

Application: View morphing



Application: View morphing



Problem with morphing

- Without rectification

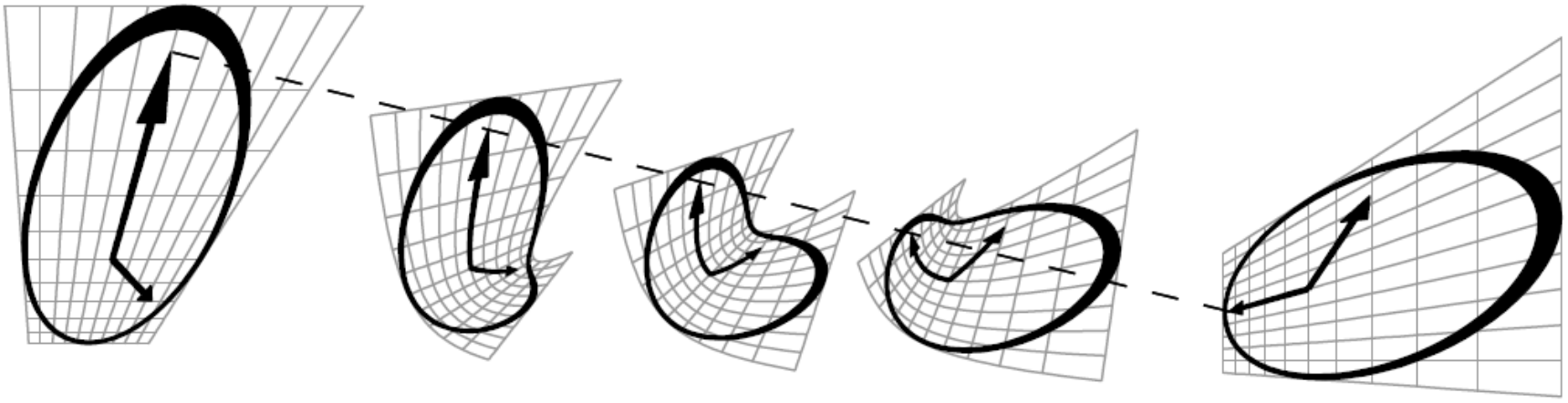


Figure 2: A Shape-Distorting Morph. Linearly interpolating two perspective views of a clock (far left and far right) causes a geometric bending effect in the in-between images. The dashed line shows the linear path of one feature during the course of the transformation. This example is indicative of the types of distortions that can arise with image morphing techniques.

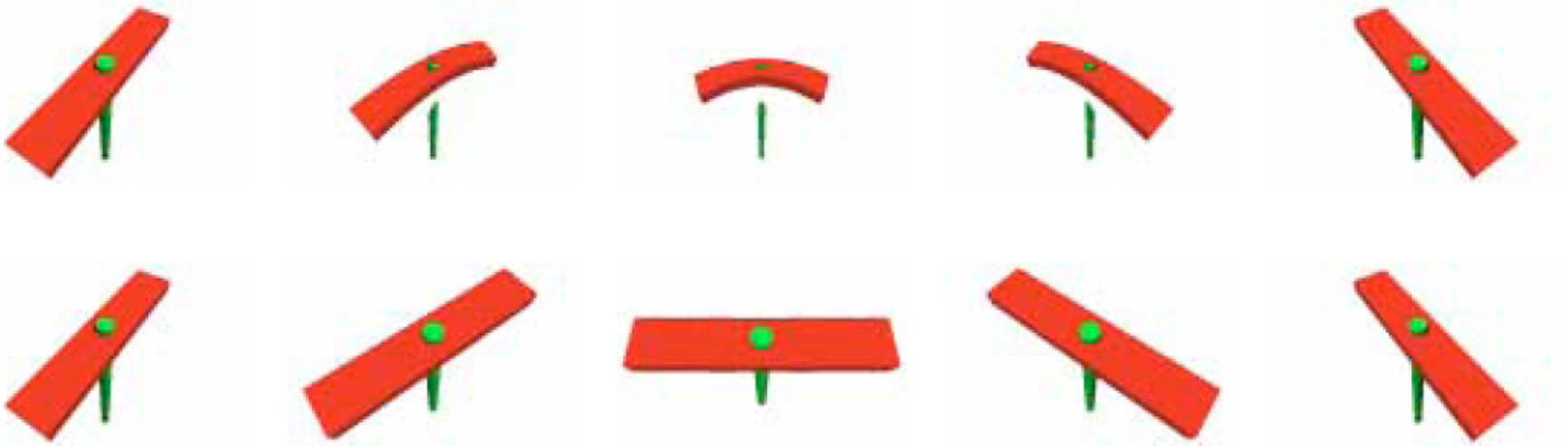


Figure 10: Image Morphing Versus View Morphing. Top: image morph between two views of a helicopter toy causes the in-between images to contract and bend. Bottom: view morph between the same two views results in a physically consistent morph. In this example the image morph also results in an extraneous hole between the blade and the stick. Holes can appear in view morphs as well.

Main trick

- Prewarp with a homography to rectify images
- So that the two views are parallel
  - Because linear interpolation works when views are parallel

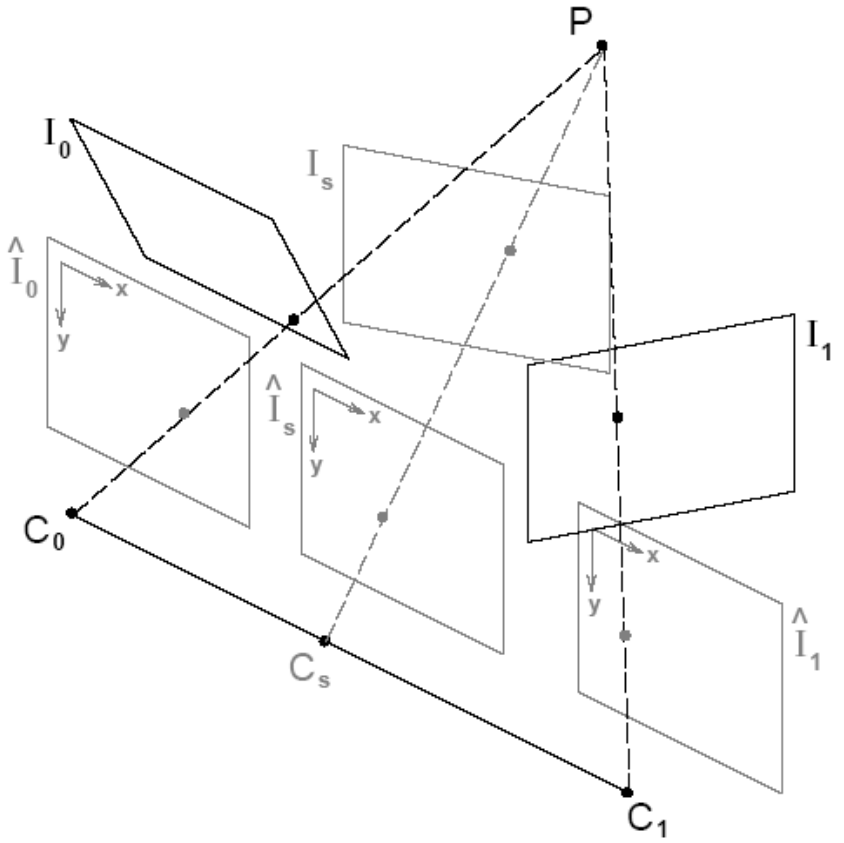


Figure 4: View Morphing in Three Steps. (1) Original images $\mathcal{I}_0$ and $\mathcal{I}_1$ are prewarped to form parallel views $\hat{\mathcal{I}}_0$ and $\hat{\mathcal{I}}_1$. (2) $\hat{\mathcal{I}}_s$ is produced by morphing (interpolating) the prewarped images. (3) $\hat{\mathcal{I}}_s$ is postwarped to form $\mathcal{I}_s$.

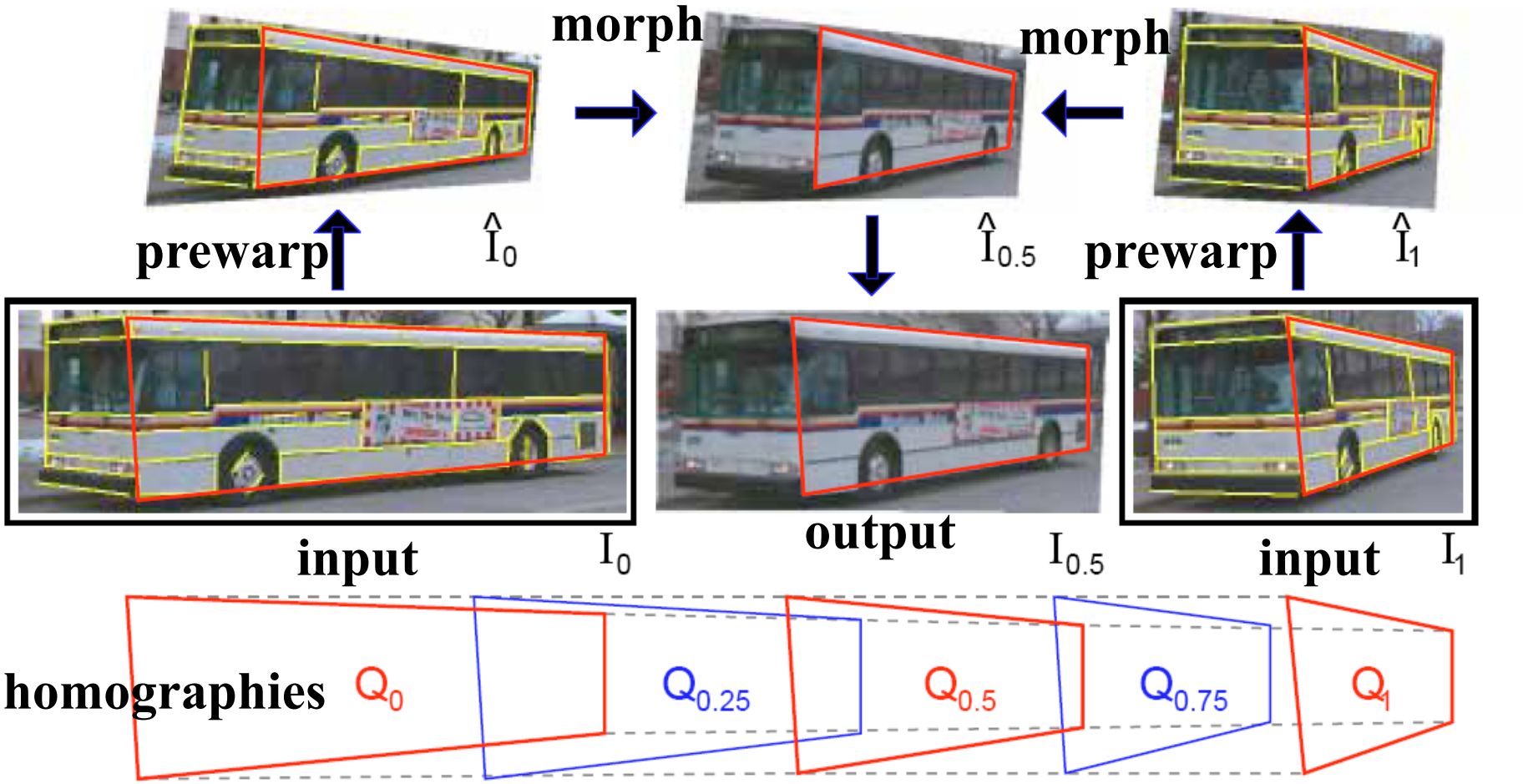


Figure 6: View Morphing Procedure: A set of features (yellow lines) is selected in original images $\mathcal{I}_0$ and $\mathcal{I}_1$. Using these features, the images are automatically prewarped to produce $\hat{\mathcal{I}}_0$ and $\hat{\mathcal{I}}_1$. The prewarped images are morphed to create a sequence of in-between images, the middle of which, $\hat{\mathcal{I}}_{0.5}$, is shown at top-center. $\hat{\mathcal{I}}_{0.5}$ is interactively postwarped by selecting a quadrilateral region (marked red) and specifying its desired configuration, $Q_{0.5}$, in $\mathcal{I}_{0.5}$. The postwarps for other in-between images are determined by interpolating the quadrilaterals (bottom).

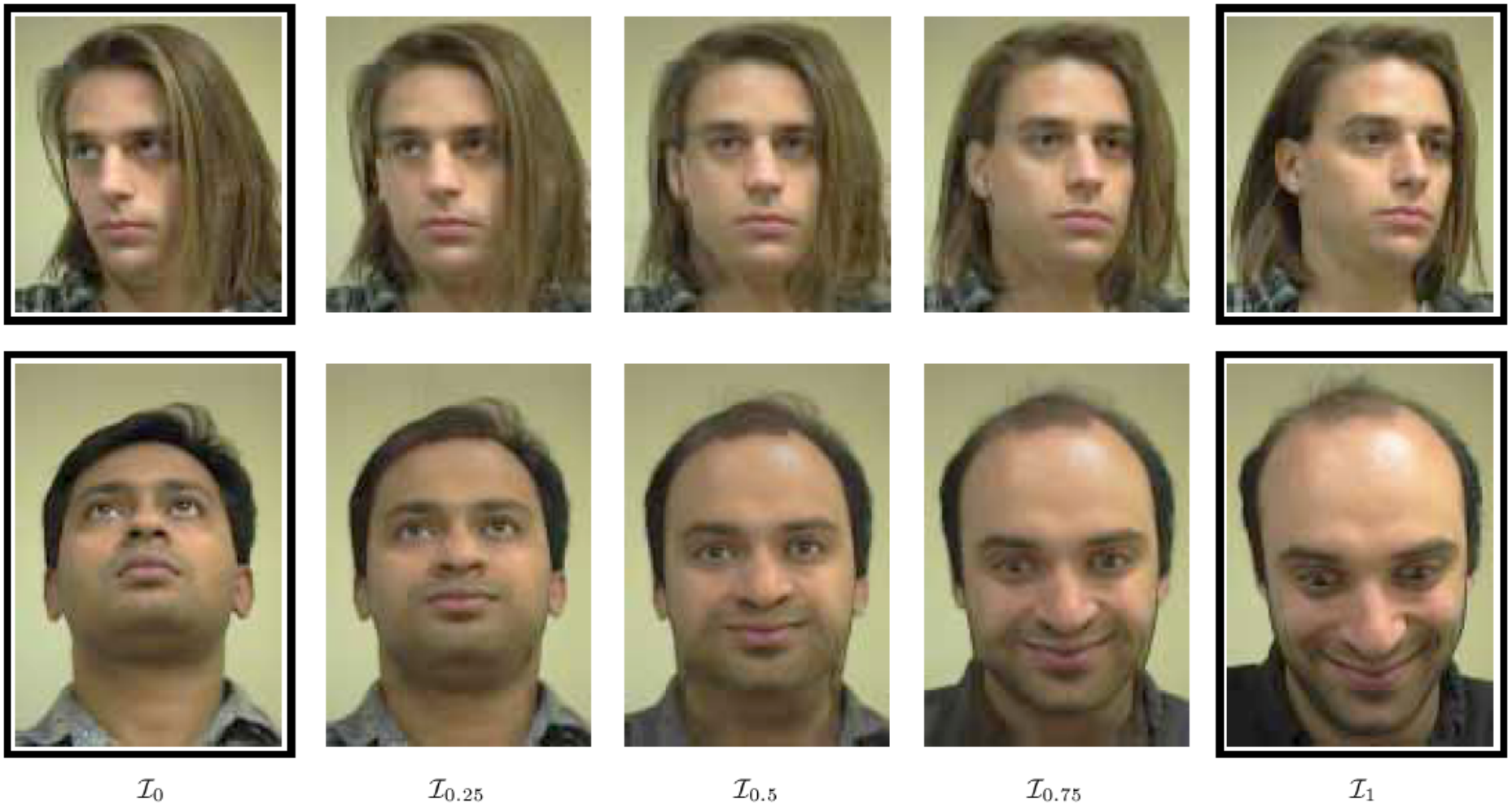


Figure 7: Facial View Morphs. Top: morph between two views of the same person. Bottom: morph between views of two different people. In each case, view morphing captures the change in facial pose between original images $\mathcal{I}_0$ and $\mathcal{I}_1$, conveying a natural 3D rotation.

Video demo

Triangulation

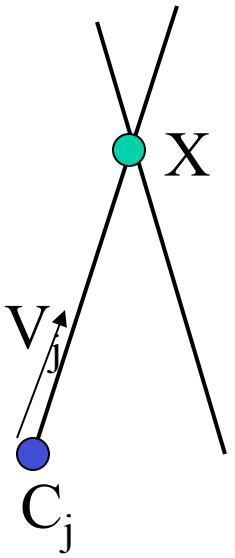
- Problem: Given some points in *correspondence* across two or more images (taken from calibrated cameras), $\{(u_j,v_j)\}$, compute the 3D location $\mathbf{X}$

Triangulation

- **Method I**: intersect viewing rays in 3D, minimize:

$$\arg\min_{\mathbf{X},\{s_j\}} \sum_j \left\| \mathbf{C} + s_j \mathbf{V}_j - \mathbf{X} \right\|$$

  - **X** is the unknown 3D point
  - $\mathbf{C}_j$ is the optical center of camera *j*
  - $\mathbf{V}_j$ is the *viewing ray* for pixel (u_j, v_j)
  - s_j is unknown distance along $\mathbf{V}_j$
- Advantage: geometrically intuitive



Triangulation

- **Method II**: solve linear equations in **X**
  - advantage: very simple

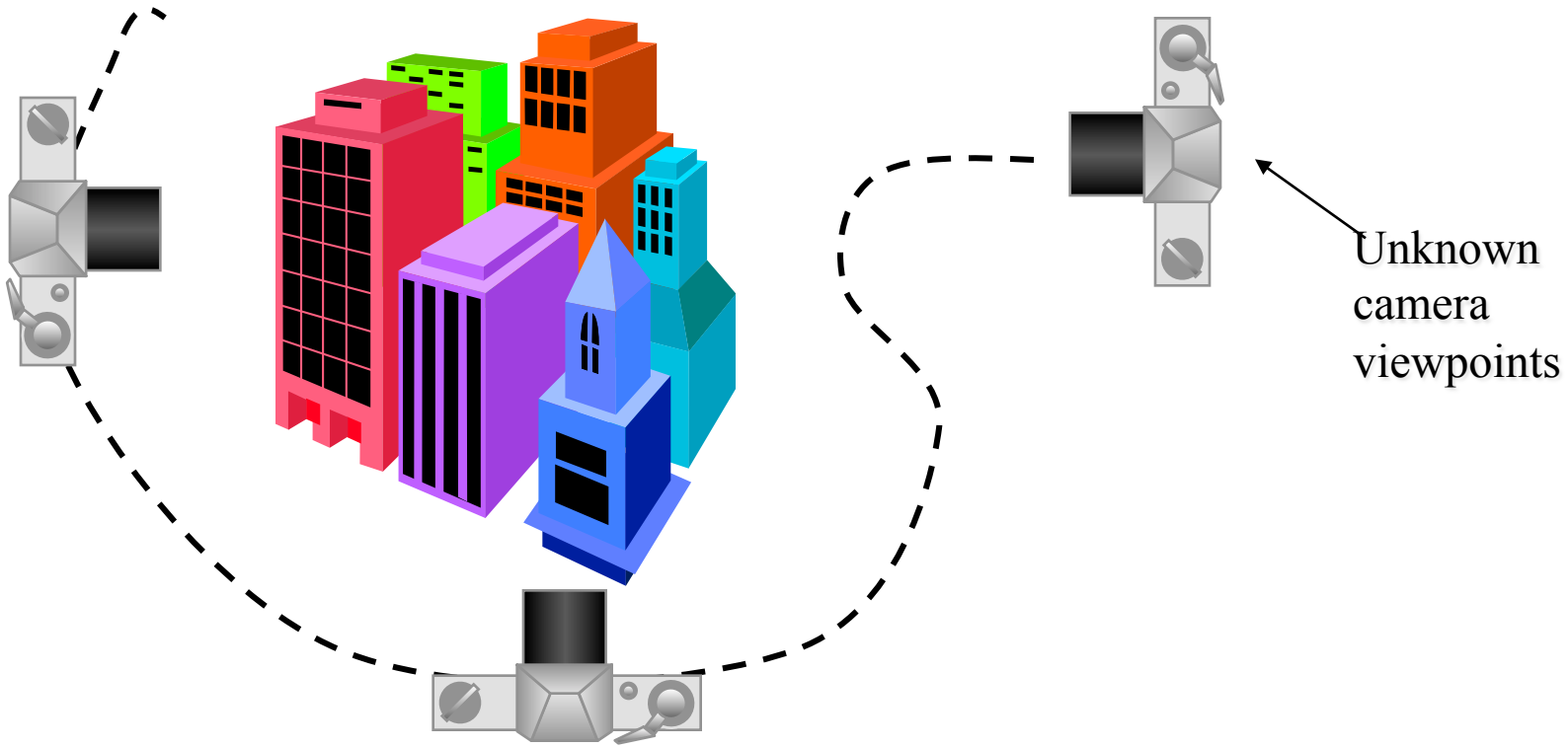
$$u_i = \frac{m_{00}^i X + m_{01}^i X + m_{02}^i X + m_{03}^i}{m_{20}^i X + m_{21}^i X + m_{22}^i X + 1}$$

$$v_i = \frac{m_{10}^i X + m_{11}^i X + m_{12}^i X + m_{13}^i}{m_{20}^i X + m_{21}^i X + m_{22}^i X + 1}$$

- **Method III**: non-linear minimization
  - advantage: most accurate (image plane error)

Structure from motion

Structure from motion

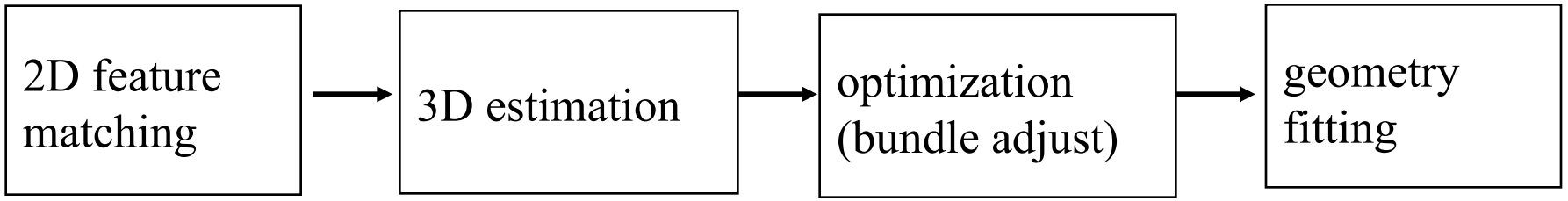


structure from motion: automatic recovery of **camera motion** and **scene structure** from two or more images. It is a self calibration technique and called *automatic camera tracking* or *matchmoving*.

Applications

- For computer vision, multiple-view environment reconstruction, novel view synthesis and autonomous vehicle navigation.
- For film production, seamless insertion of CGI into live-action backgrounds

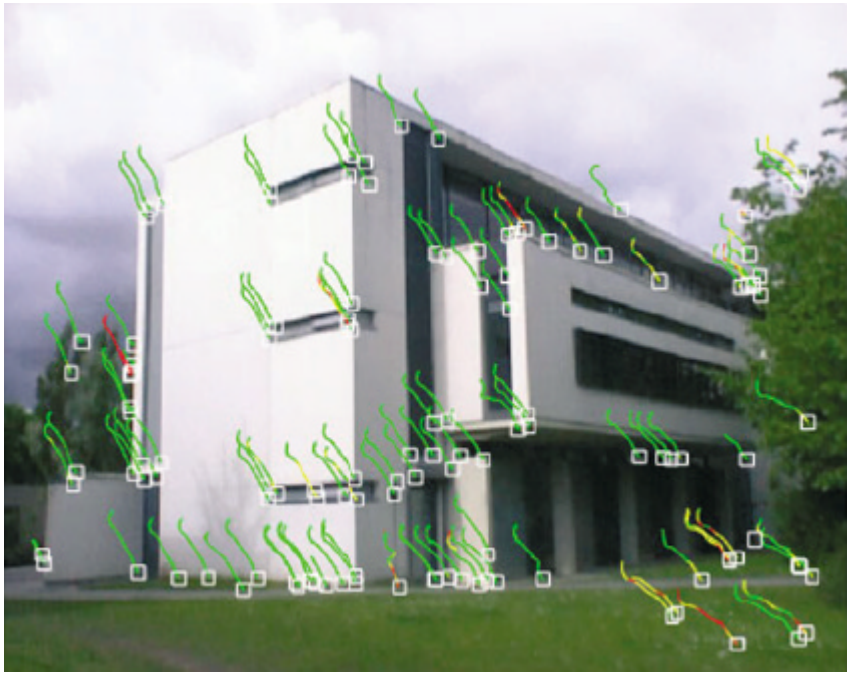
Structure from motion



SFM pipeline

Structure from motion

- Step 1: Track Features
  - Detect good features, Shi & Tomasi, SIFT
  - Find correspondences between frames
    - Lucas & Kanade-style motion estimation
    - window-based correlation
    - SIFT matching

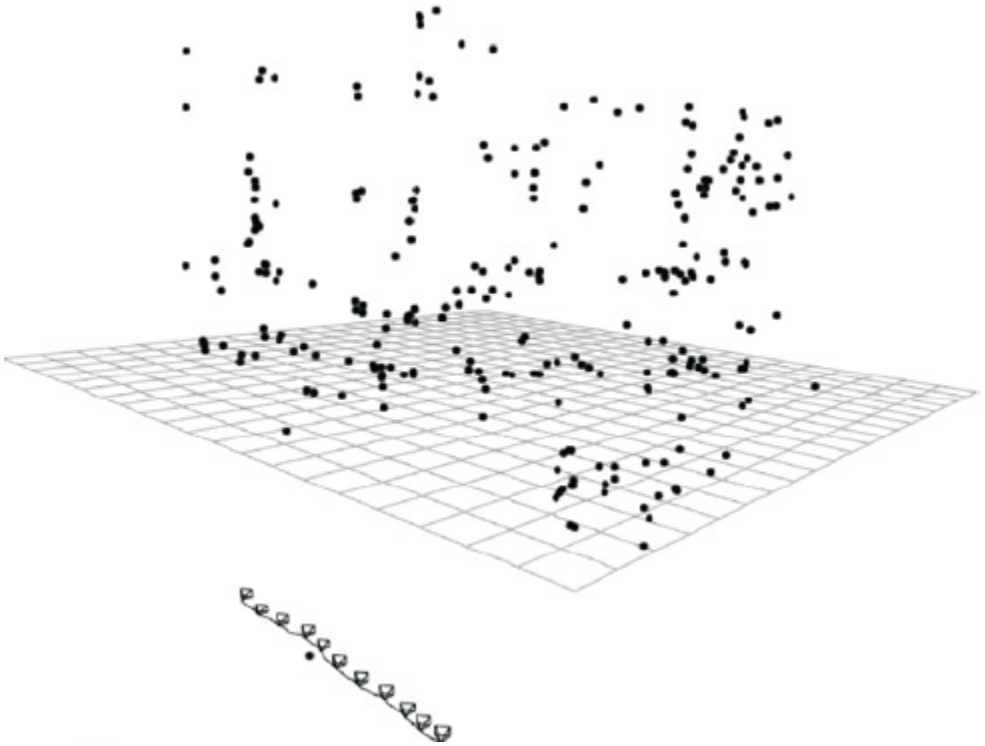


Structure from Motion

- Step 2: Estimate Motion and Structure
  - Simplified projection model, e.g., **[Tomasi 92]**
  - 2 or 3 views at a time **[Hartley 00]**

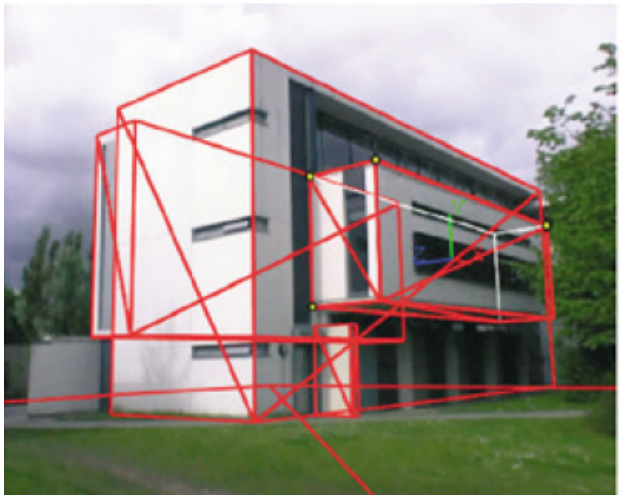
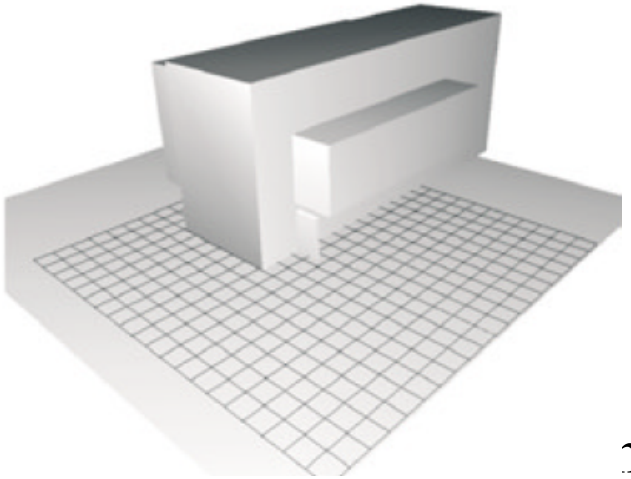
Structure from Motion

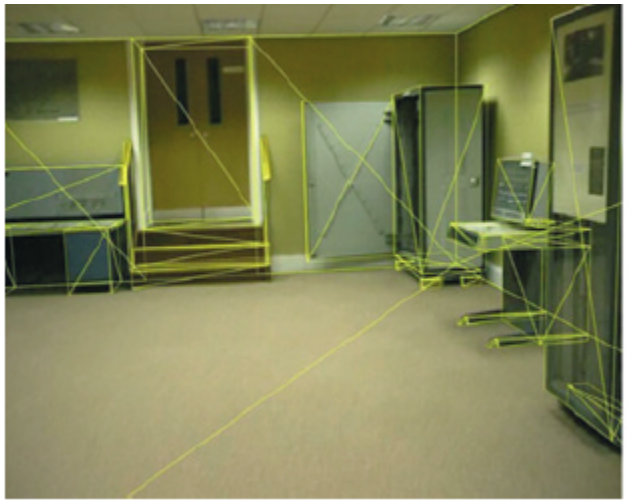
- Step 3: Refine estimates
  - “Bundle adjustment” in photogrammetry
  - Other iterative methods

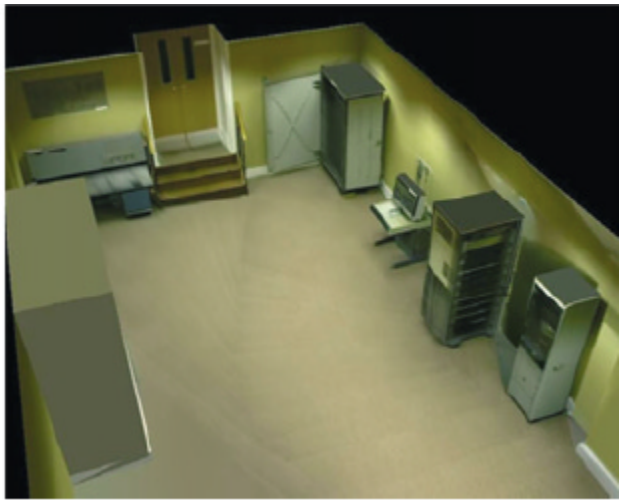


Structure from Motion

- Step 4: Recover surfaces (image-based triangulation, silhouettes, stereo…)

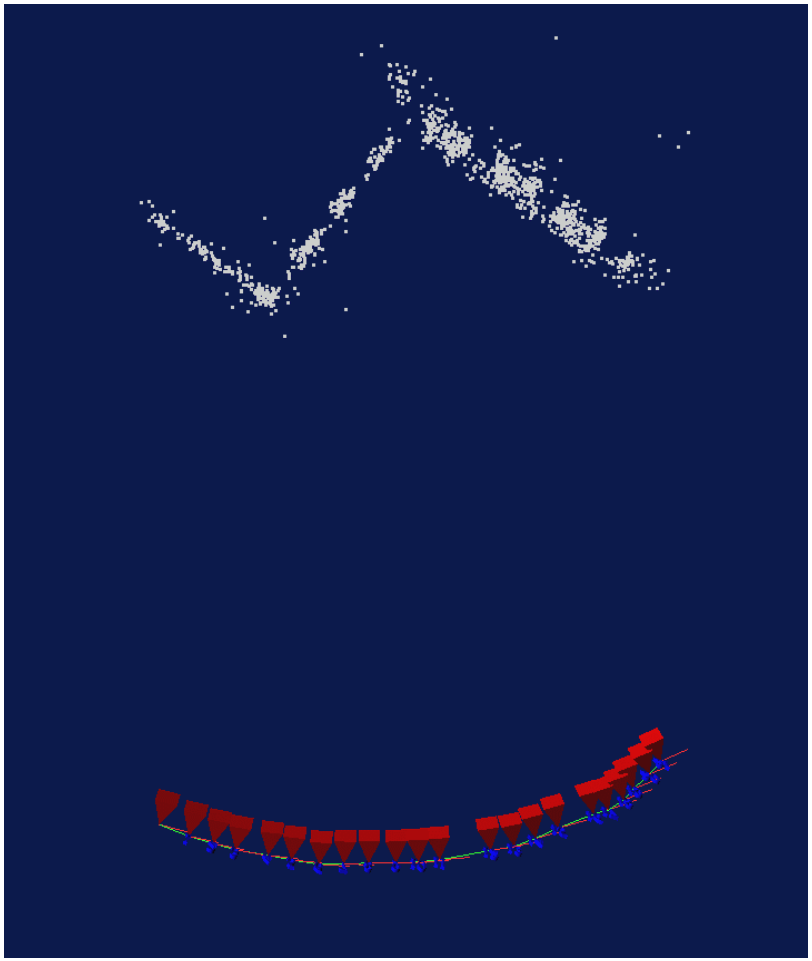




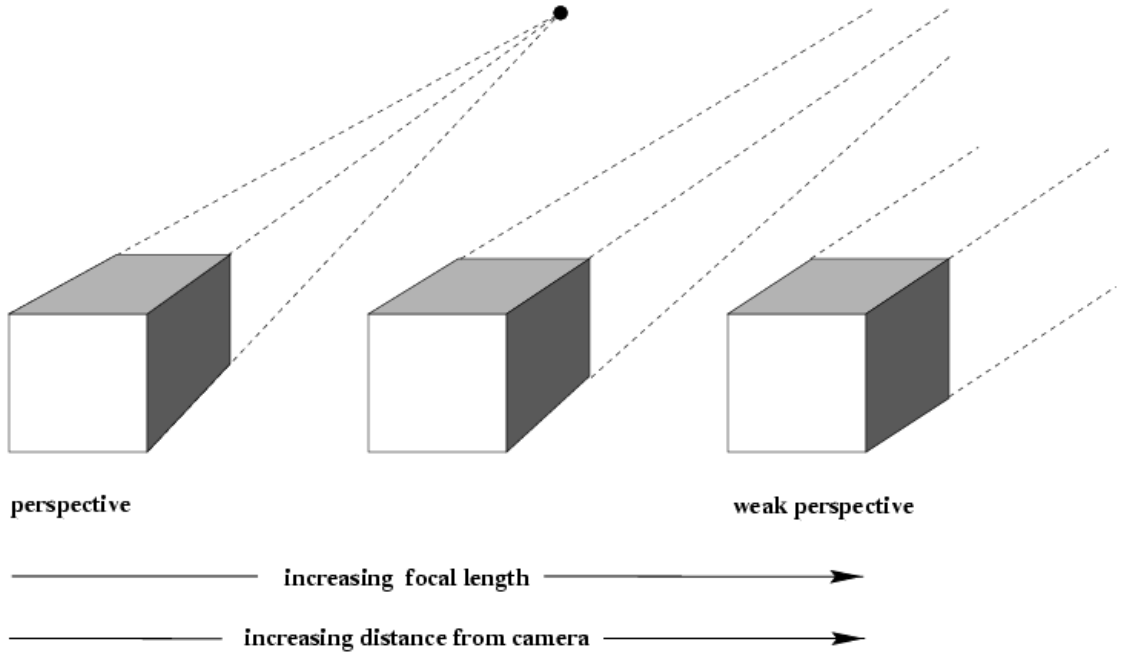
Example : Photo Tourism

Factorization methods

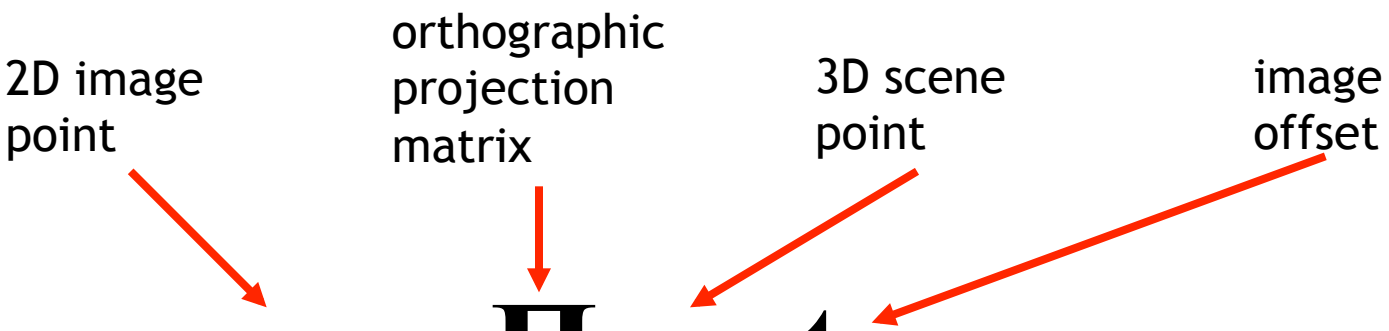
Problem statement



Other projection models



SFM under orthographic projection



- Trick
  - Choose scene origin to be centroid of 3D points
  - Choose image origins to be centroid of 2D points
  - Allows us to drop the camera translation:

$$\mathbf{q} = \Pi\mathbf{p}$$

factorization (Tomasi & Kanade)

projection of *n* features in one image:

$$\underset{2\times n}{\begin{bmatrix}\mathbf{q}_1 & \mathbf{q}_2 & \cdots & \mathbf{q}_n\end{bmatrix}} = \underset{2\times 3}{\prod} \underset{3\times n}{\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_n\end{bmatrix}}$$

projection of *n* features in *m* images

$$\underset{2m\times n}{\begin{bmatrix}\mathbf{q}_{11} & \mathbf{q}_{12} & \cdots & \mathbf{q}_{1n}\\ \mathbf{q}_{21} & \mathbf{q}_{22} & \cdots & \mathbf{q}_{2n}\\ \vdots & \vdots & \ddots & \vdots\\ \mathbf{q}_{m1} & \mathbf{q}_{m2} & \cdots & \mathbf{q}_{mn}\end{bmatrix}} = \underset{2m\times 3}{\begin{bmatrix}\mathbf{\Pi}_1\\ \mathbf{\Pi}_2\\ \vdots\\ \mathbf{\Pi}_m\end{bmatrix}} \underset{3\times n}{\begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_n\end{bmatrix}}$$

W measurement M motion S shape

Key Observation: *rank*(W) <= 3