CS 514: Wavelet Analysis of Musical Signals

Luke Tokheim

http://www.cs.wisc.edu/~ltokheim/cs514/

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Contents

1 Input Signals

Musical signals were chosen for simplicity in feature finding. The simplest signal chosen is a single tone at a known frequency, particularily A at 440 Hz. It was sampled at 44100 Hz. The second simplest signal is the middle C Major scale. The frequency of each note is known, each note is at a constant gain, and the notes do not overlap. It was sampled at 22050 Hz. The third simplest signal chosen is a harp *arpeggio*, an ascending chord of notes played in fast succession. The signal covers a wide range of frequency and shows attack and decay features. This signal was sampled at 11025 Hz. A flute music single was also used in one example, it was sampled at 44100 Hz.

2 Wavelets Used

2.1 Daubechies Wavelets

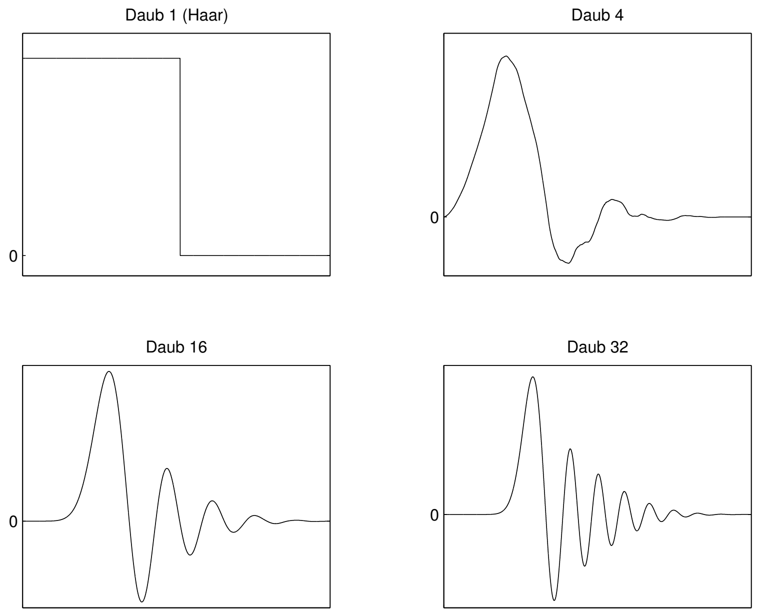


Figure 1: Daub1, Daub4, Daub16, Daub32 on time domain.

Daubechies wavelet systems are orthonormal. As the order N of the mother wavelet ψ becomes larger:

1. support on time domain increases, the interval of support for $\psi(t)$ becomes larger.

2. the mother wavelet becomes *smoother*, more times differentiable. $\psi(t) \in \mathrm{C}^n$ for some n related to the order of the wavelet.

3. mother wavelet ψ has more vanishing moments, k. $\hat{\psi}^{(n)}(0) = 0,\ n = 0, \ldots, k-1$.

These all lead to better frequency localization. Visually, we can see the size of the interval of support on the time domain and the improvements in smoothness. But what about the vanishing moments?

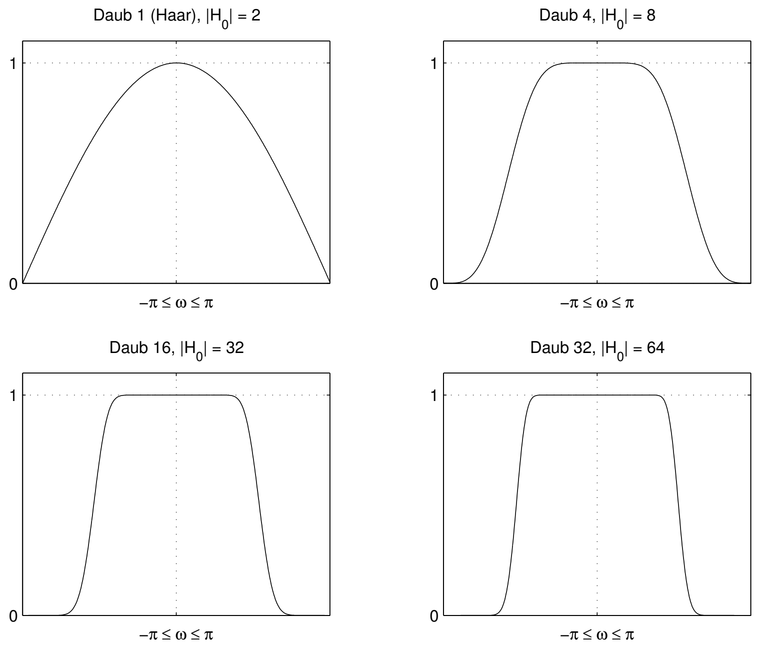


Figure 2: Daub1, Daub4, Daub16, Daub32 low-pass filters on frequency domain.

The graphs on the frequency domain of the low-pass filters of the Daubechies wavelet systems used show a flattening of the function over $\omega = 0$, exactly where we need many derivatives to be 0. [See figure 2] It would be flat at the origin and at $\pm\pi$ and $H_0(0) = 1$, $H_0(\pm\pi) = 0$. As the length of the low-pass filter H_0 increases it becomes closer to the *ideal* low-pass filter. $H_0(\omega)$ flattens about $\omega = 0$ which leads to more 0 derivatives at the origin. Ideally, the filter would be piecewise constant, $H_0(\omega) = 1$ if $-\pi \leq \omega \leq \pi$ and 0 otherwise. [2]

For Daubechies wavelet ψ of order N, with $|H_0| = 2N$, the length of the interval of support of $\psi(t) = 2N - 1$ and ψ has N vanishing moments. For large N, $\psi \in \mathrm{C}^{\mu N}$, for constant $\mu \simeq 0.2$. [1]

2.2 Coiflets

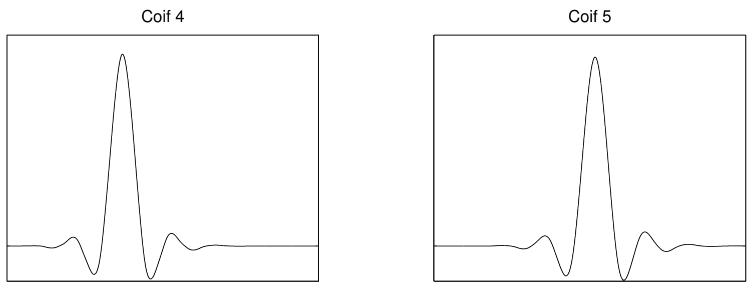


Figure 3: Coif4 and Coif5 on time domain.

Coiflet wavelets were also built by Daubechies, and are orthogonal. Coiflets are more symmetrical than Daubechies wavelets. The interval of support of Coiflet ψ of order N is $6N-1$, comparable to Daubchies wavelet of order $3N$. ψ of order N has $2N$ vanishing moments. [1]

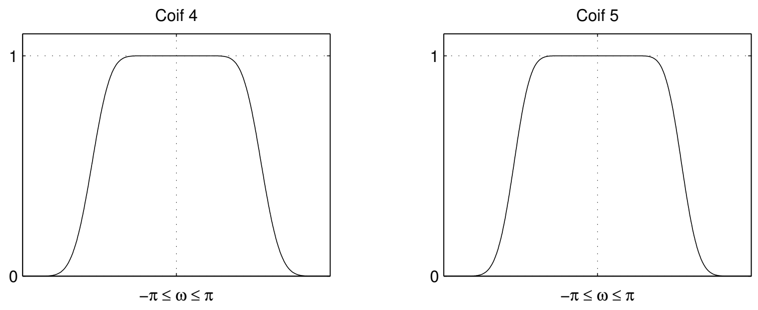


Figure 4: Coif4 and Coif5 low-pass filters on frequency domain.

2.3 Biorthogonal Wavelet Systems

Biorthogonal wavelet systems use different mother wavelets for decomposition and reconstruction. This allows the mother wavelets to be symmetric and still have perfect reconstruction. The size

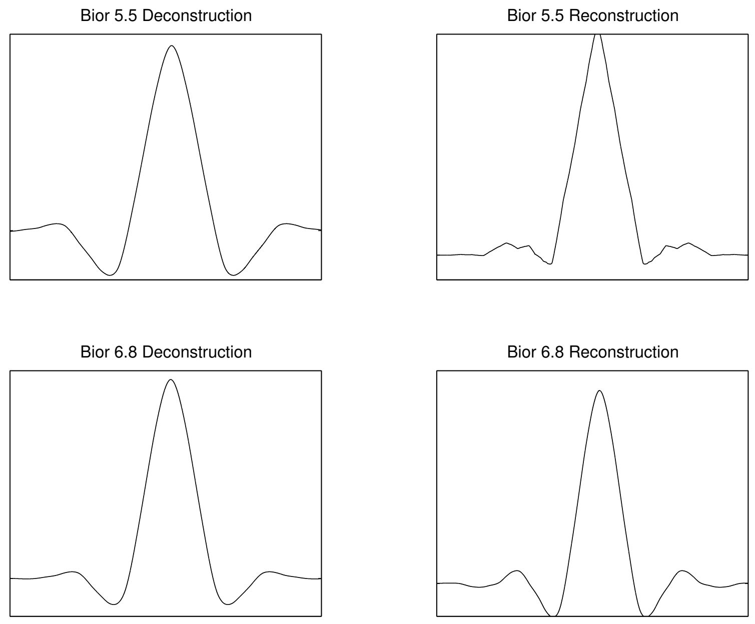


Figure 5: Bior5.5 and Bior6.8 on time domain.

of the support of $\psi(t)$ of order N is $2N - 1$. The lengths of the decomposition filters, and the reconstruction filters can be different and are denoted as $BiorR.D$, where R is the order of the reconstruction wavelet and D is the order of the deconstruction wavelet. ψ for decomposition has $N - 1$ vanishing moments. [1]

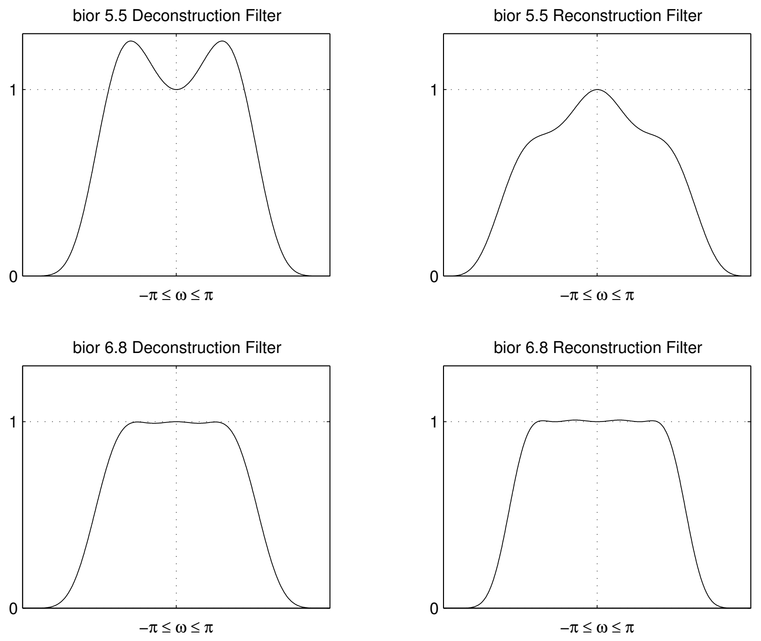


Figure 6: Bior5.5 and Bior6.8 low-pass filters on frequency domain.

3 Signal Attack

3.1 Introduction

The *attack* of a musical signal is a feature that occurs at the beginning of a tone, or note. It is characterized on the time domain as a sudden increase in the gain of the signal, where gain is the magnitude of the signal value. The wavelet coefficients represent attack similarily to the signal on the time domain. A sudden increase in the magnitude of the wavelet coefficients represents an attack feature in that frequency window of the signal.

While the signal over time only shows that there is attack happening over all frequencies, the wavelet coefficients also show which frequency windows the attack is active in. In the context of music, this means that the wavelet coefficients show which pitch range and, equivalently, frequency ranges are contributing the attack to the musical signal. The fourier transform of the signal also shows which frequencies are active in the signal. Unfortunately, the fourier transform shows nothing about where in time the frequencies occur. The wavelet coefficients show the signal with respect to time and frequency. [See figure 7]

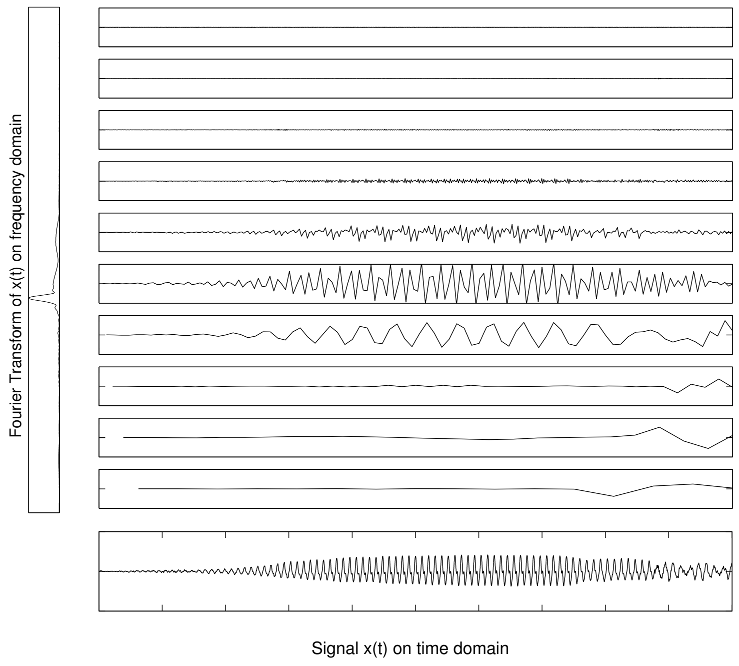


Figure 7: An example of *attack* for a single tone of a flute. The center are the wavelet coefficients of decomposition with Daubechies with $|H_0| = 8$ for 10 levels. This shows the time-frequency representation of wavelet analysis.

3.2 Harp Arpeggio

The wavelet decomposition and analysis of the harp arpeggio signal shows the attack and decay pattern clearly. The attack feature causes a jump in one or for frequency windows and then is followed by slow decay. As the arpeggio progresses to higher notes, the frequency windows that are active also also progress to higher frequencies. [See figures 8 through 11]

The *Daub1* wavelet system does not show attack features well because the mother wavelet is not well localized on frequency domain. The attack features shown by the *Daub1* system manifest themselves as smooth bumps with slow increase in magnitude of the wavelet coefficients, or in

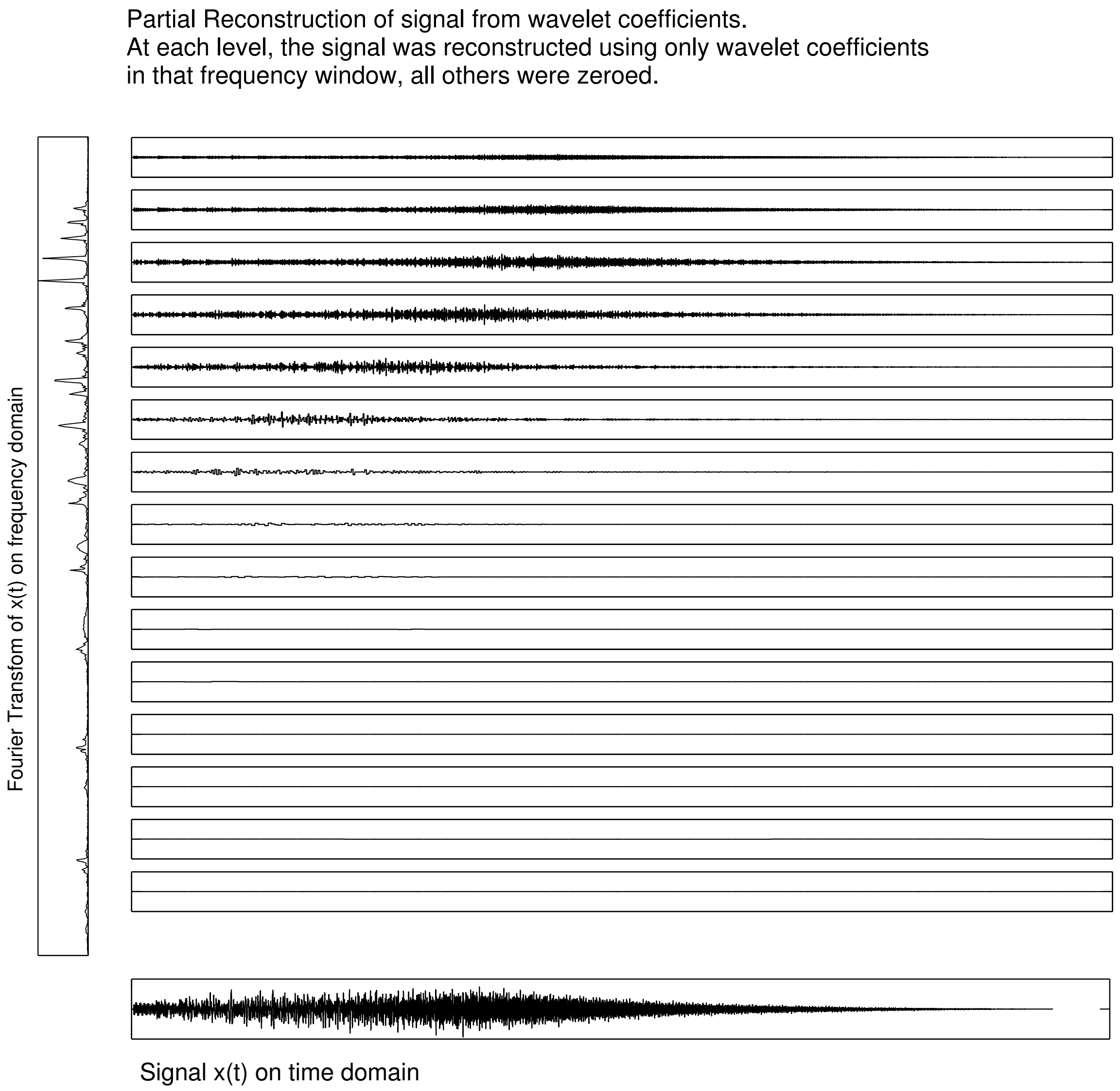


Figure 8: Analysis of arpeggio signal with Daub1. Smooth attack features.

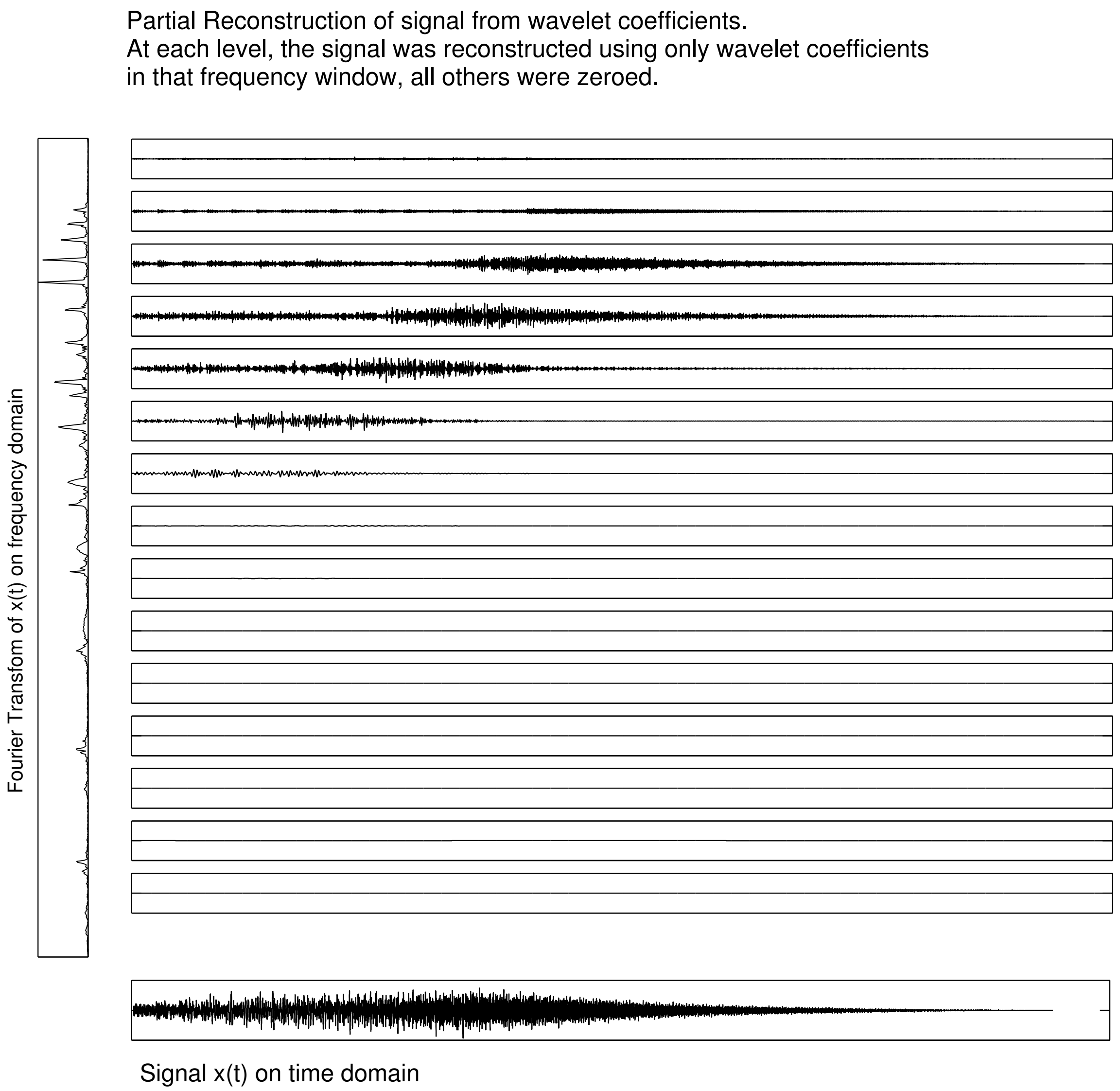


Figure 9: Analysis of arpeggio signal with Daub4. Yields better results than than Daub1, but still inferior to the other wavelet systems with more vanishing moments.

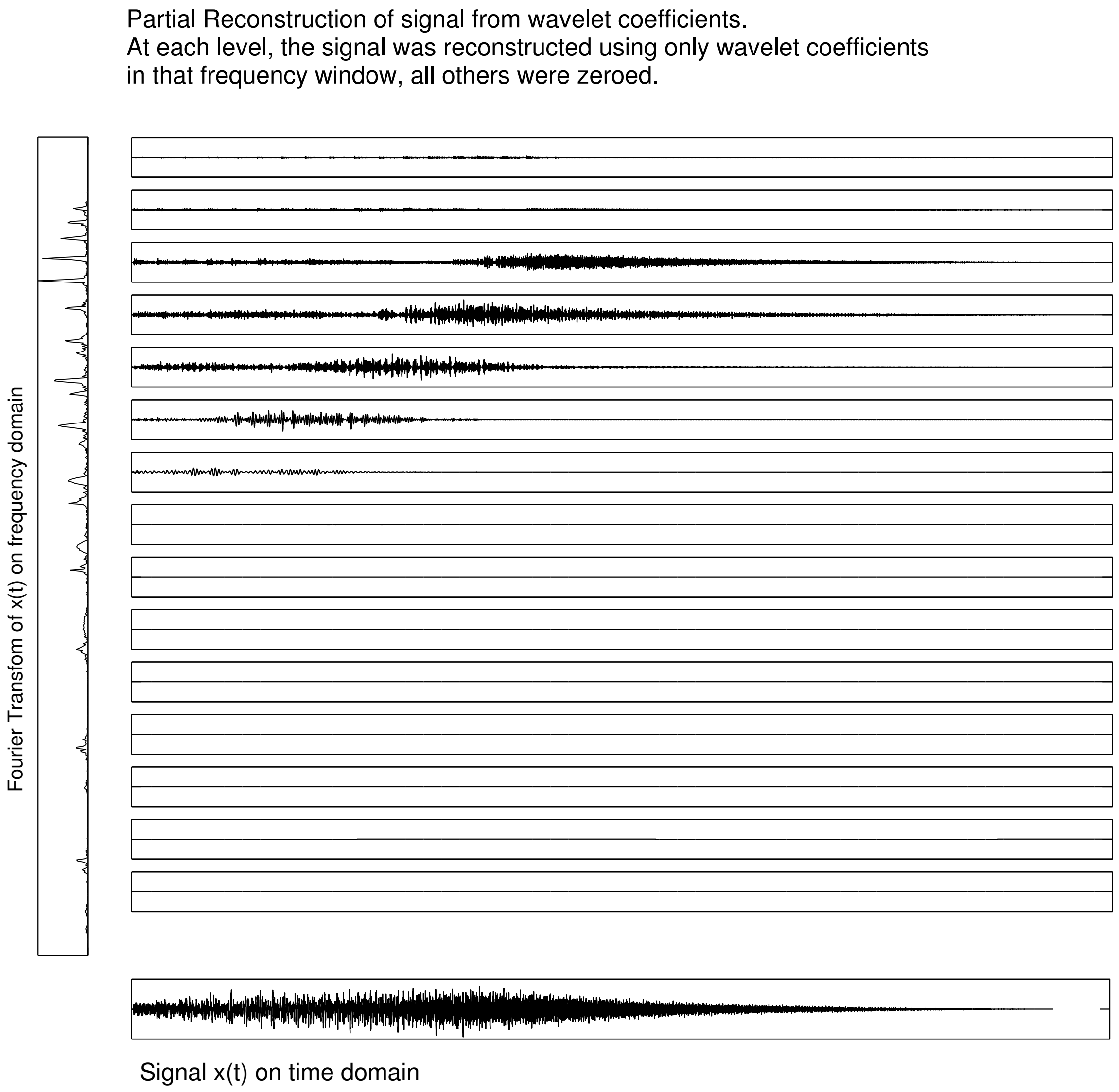


Figure 10: Analysis of arpeggio signal with Coif5.

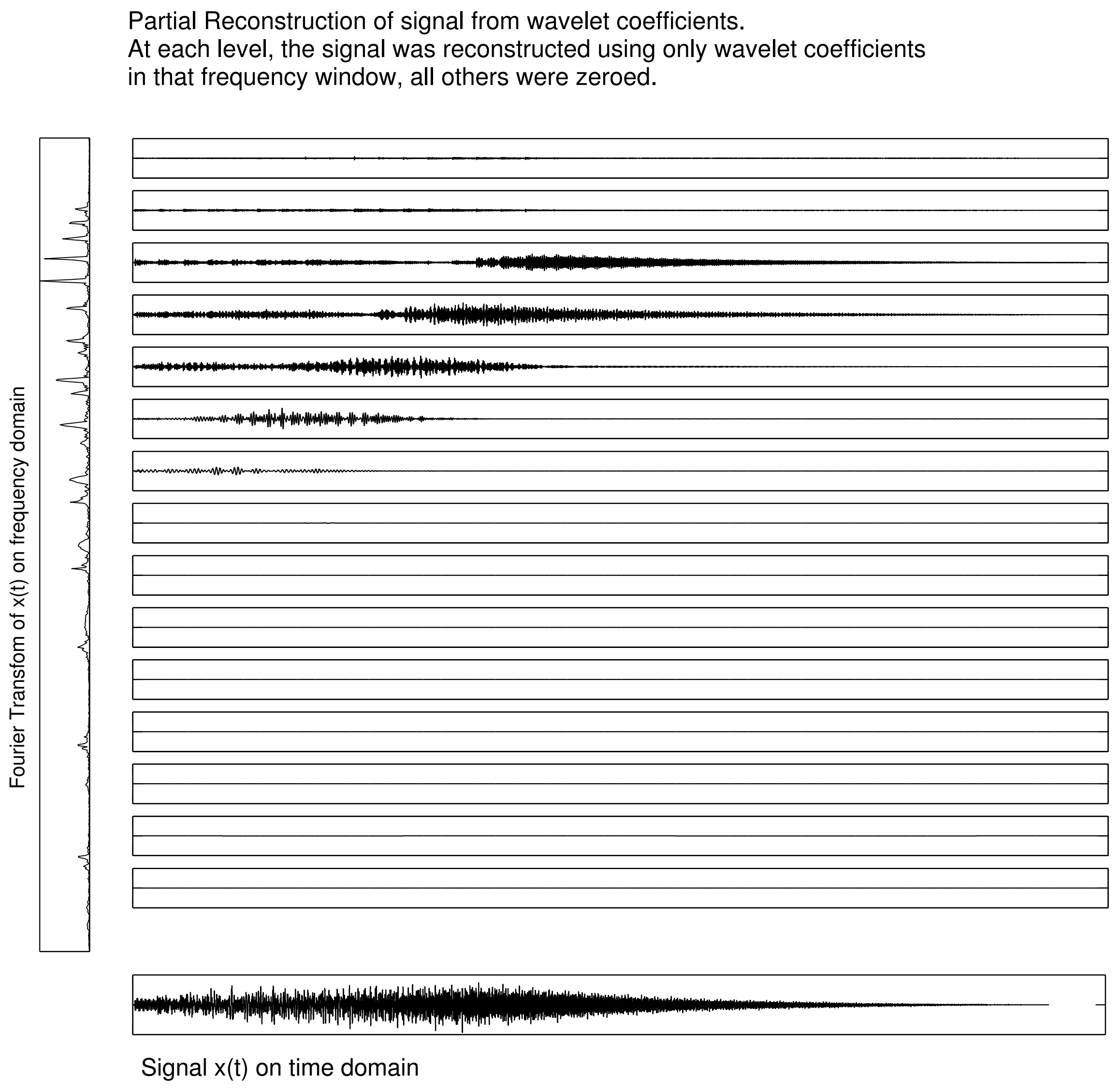


Figure 11: Analysis of arpeggio signal with Daub32.

the gain of the partially reconstructed signal. [See figure 8] The attack and decay of the partially reconstructed signal are, visually, almost symmetric. This is not faithful to the actual behavior of the signal. The attack is sudden, and followed by a gradual decay. Wavelet systems with better localization on the frequency domain and greater smoothness on the time domain show the sudden attack followed by gradual decay that the *Daub1* system does not. [See figure 12]

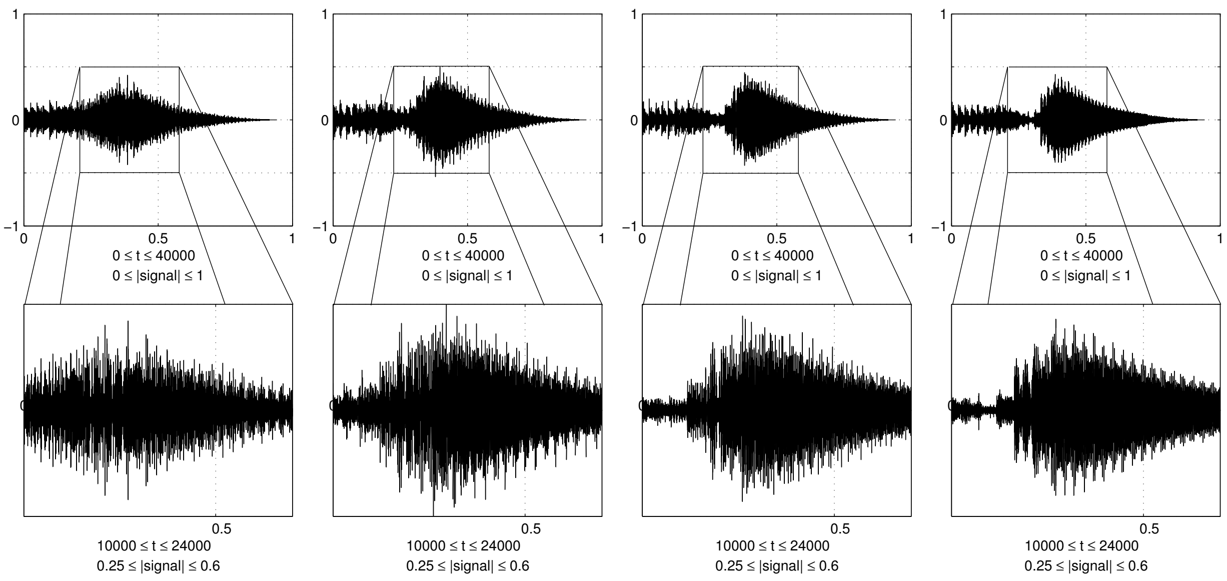


Figure 12: Partially reconstructed harp arpeggio signal, all wavelet coefficients thresholded to 0 except for level three, which is not scaled. From left to right: Daub1, Daub4, Coif5, Daub32.

Notice the difference between the response to attack from the *Daub1* and the other mother wavelets. The *Daub1* shows a smooth bump, while the others, especially *Coif5* and *Daub32*, show a sharp increase in the gain followed by a slow decay. However, even the *daub4* shows the attack feature fairly well. The wavelet systems with more vanishing moments, and differentiable more times, and therefore with better localization on the frequency domain, show better attack feature detection results. [See figure 12]

4 Frequency Window

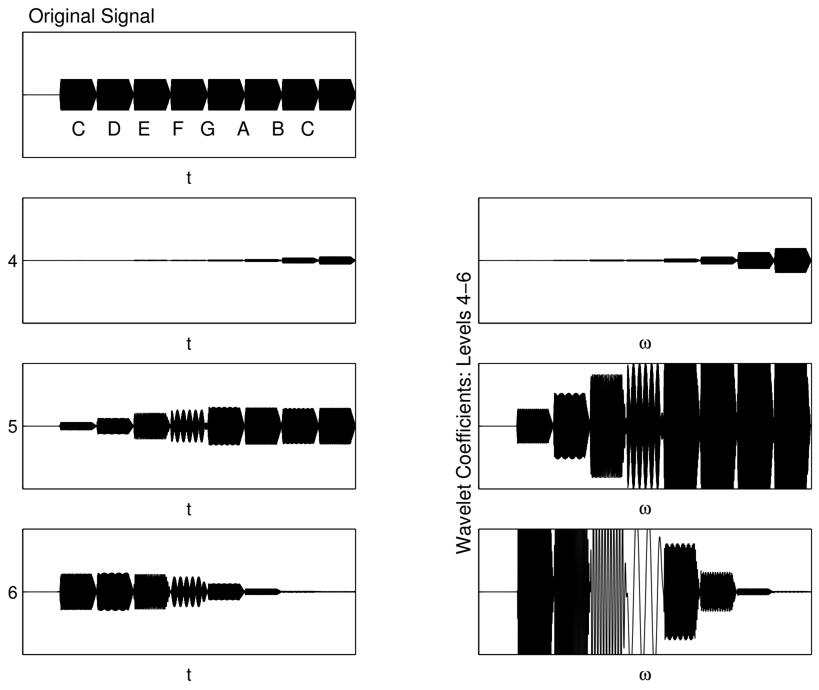
4.1 Introduction

The wavelet decompostion of a musical signal shows the amount of activity for the given signal, in a given frequency range. The musical signal, if viewed on the time domain, only makes the overall gain of the signal apparent. No judgment can be made about the pitch, or notes that make up the signal. The fourier tranform of the signal, the representation of the signal on the frequency domain, would show the amount of a certain frequency present in the signal, but is invariant in time. There is no way to say that a certain frequency is present at a certain time. However, with the wavelet decomposition, the amount of a certain level of pitch, or frequency, in a signal at a given time becomes apparent.

The wavelet decomposition separates the musical signal into pitch or frequency windows. Partial reconstruction of the signal from only one frequency window shows the signal over time, and limited to a certain frequency band. By reconstructing for many levels, (I needed at most 10), the musical signal can be represented in time and frequency simultaneously. This is limited in that the frequency window is somewhat coarse. For example, the C major scale spans only three frequency windows. [See figure 13]

4.2 C Major Scale

The eight notes in the C Major scale fall into only three frequency windows. We can draw a conclusion as to generally what frequency levels are active, such as “The pitches around A440 are active,” but not exactly which note is active at a given time.



Frequency (Hz)	Note
523.25	C, *DO*
493.88	B, *TI*
440.00	A, *LA*
392.00	G, *SO*
349.23	F, *FA*
329.63	E, *MI*
293.66	D, *RE*
261.63	C, *DO*

Figure 13: C Major scale. Left: Original musical signal and partial reconstruction from one frequency window for three levels. Center: The three frequency windows that contain all pitches in the scale. Right: The actual frequencies of each note in the scale.

4.3 Single A440 Note

Partial reconstruction shows which frequency windows are active. Different wavelet systems will separate the frequency windows more effectively. The wavelet systems with better frequency localization will not show response in a frequency window that is not actually active. Figure 14 shows ten levels of partial reconstruction from a single level of wavelet decomposition coefficients of a very simple signal, a digitally generated A440 single tone, with no change in gain. All eight wavelet systems presented are used to analyse this signal. By looking at the fourier transform of the signal we can see that all of the signal is at exactly one frequency, and the time domain would show us the amount of gain over time. The wavelet decomposition and partial reconstruction are not particularily useful, except to see how the wavelet systems perform with respect to frequency

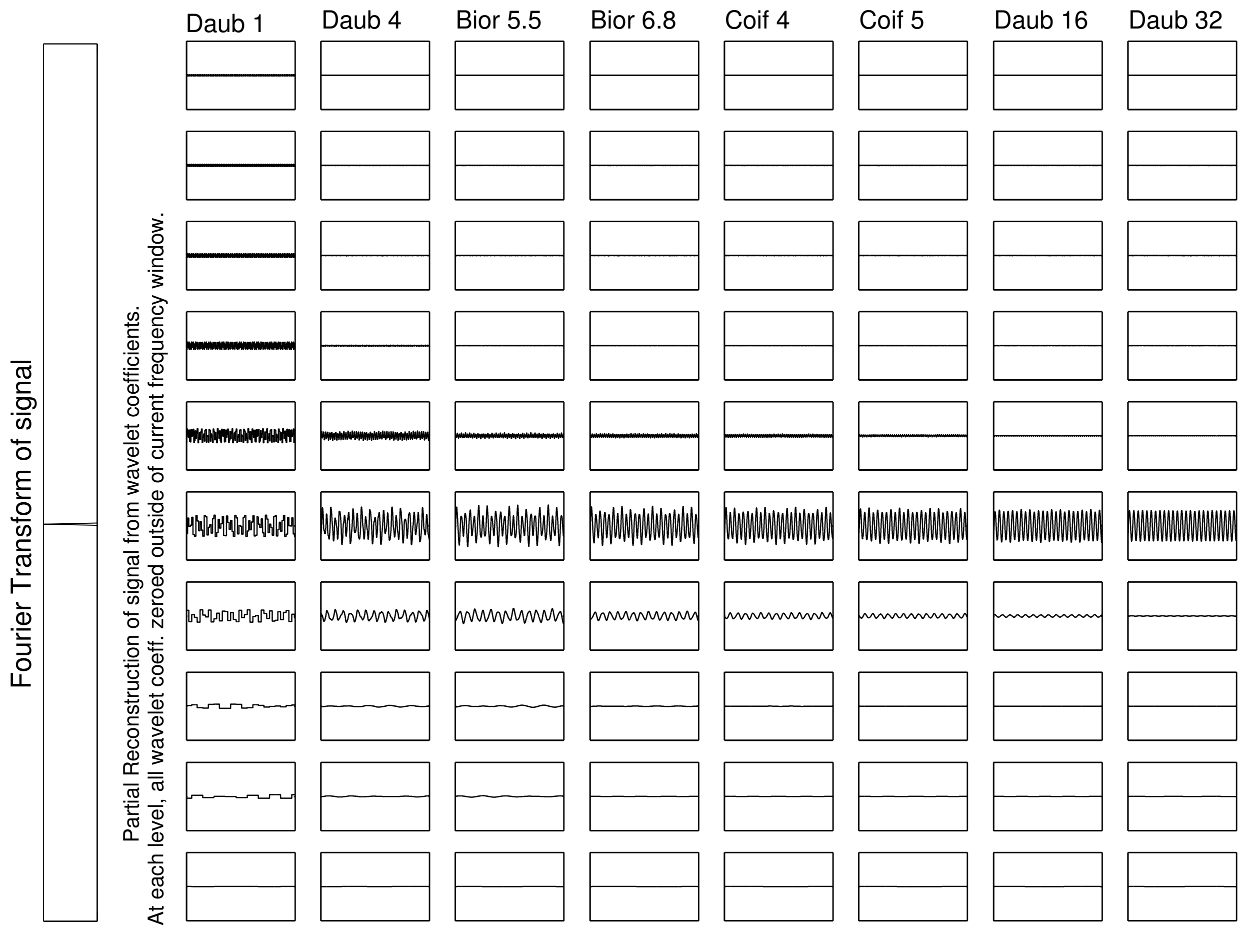


Figure 14: Ten levels of partial reconstruction of A440 note signal ordered by either number of vanishing moments of mother wavelet ψ or, equivalently, visually.

localization.

We already have a judgement of the frequncy localization in the number of vanishing moments of the wavelets. Figure 14 orders the wavelet analyses from left to right by increasing vanishing moments and support on time domain. The results confirm that the performance of the wavelets are ordered as expected.

The results of partial reconstruction differs substantially between the chosen wavelet systems. The wavelet systems with less vanishing moments show response in frequency windows that should not. This is because the wavelet system is not local enough in frequency. The wavelet system is spreading frequency information between the frequency windows. As the number of vanishing moments of the wavelet system increases, the *quality* of the partial reconstruction improves. [See figure 14] The table of partial reconstructions is ordered by number of vanishing moments of mother wavelet ψ, but it is also ordered as quality of results. They imply each other.

5 Conclusions

Wavelet analysis of musical signals is useful to show features in both time and frequency simultaneously. This is advantageous because the benefits of both domains can be taken advantage of. Having only access to the signal on the time domain and the fourier transform of that signal on the frequency domain is not very useful unless there is a way to relate them.

The time-frequency representation of the signal shows the signal as neither time nor frequency could show independently. The frequency windows allow the independent analysis of a particular frequency range with respect to time. Without the time-frequency representation we could not draw a conclusion like, "There was an attack at time t in frequency range $[a, b]$." Whether or not this is useful for actual applications is unclear. The application must take advantage of the ability to look time and frequency simultaneously to reap the benefits of wavelet signal analysis.

References

[1] Matlab. Online help and command *waveinfo*. Matlab refers to I. Daubechies.

[2] Ron, Amos. CS 514 online lecture notes (2002).

A Matlab Code

A.1 Plotting Wavelets

biorPlot.m

```
function biorPlot
clf;

[y dy] = plotBiorWavelet(10,'bior5.5');
subplot(2,2,1); plot(y);
axis([2000 8000 -.0004 .0016])
title('Bior 5.5 Deconstruction');
set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');

subplot(2,2,2); plot(dy);
axis([1500 8500 -.0001 .0009]);
title('Bior 5.5 Reconstruction');
set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');

[y dy] = plotBiorWavelet(10,'bior6.8');
subplot(2,2,3); plot(y);
axis([1500 8500 -.0002 .0011])
title('Bior 6.8 Deconstruction');
set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');

subplot(2,2,4); plot(dy);
axis([4000 12000 -.0002 .0013])
title('Bior 6.8 Reconstruction');
set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');
```

coifPlot.m

```
function coifPlot
clf;

y = plotCoifWavelet(10,'coif4');
subplot(2,2,1); plot(y); axis([3000 17000 -.0002 .0012])
title('Coif 4');
set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');

y = plotCoifWavelet(10,'coif5');
subplot(2,2,2); plot(y); axis([3000 17000 -.0002 .0012])
title('Coif 5');
set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');
```

daubPlot.m

```
function daubPlot
clf;

y = plotDaubWavelet(10,'db1');
subplot(2,2,1); plot(y); axis([0 2000 -0.0001 .0011]);
title('Daub 1 (Haar)');
set(gca,'ytick',[0 2000],'xtick',0,'yticklabel',[0 2000],'xticklabel','');

y = plotDaubWavelet(10,'db4');
subplot(2,2,2); plot(y); axis([0 6000 -.0004 .00125]);
title('Daub 4');
set(gca,'ytick',[0 6000],'xtick',0,'yticklabel',[0 6000],'xticklabel','');

y = plotDaubWavelet(10,'db16');
subplot(2,2,3); plot(y); axis([0 13500 -.0005 .0009]);
title('Daub 16');
set(gca,'ytick',[0 13500],'xtick',0,'yticklabel',[0 13500],'xticklabel','');

y = plotDaubWavelet(10,'db32');
subplot(2,2,4); plot(y);  axis([0 24000 -.0005 .0008]);
title('Daub 32');
set(gca,'ytick',[0 24000],'xtick',0,'yticklabel',[0 24000],'xticklabel','');
```

graphBiorHs.m

```
function graphBiorHs

clf;
x = linspace(-pi,pi,512);

[h rh] = biorwavf('bior5.5');
h = fftshift(fft(h,512));
subplot(2,2,1);
plot(x,abs(h));
title('bior 5.5 Deconstruction Filter');
axis([-pi pi 0 1.3]);
set(gca,'ytick',[0 1],'xtick',0,'yticklabel',[0 1],'xticklabel','')
set(gca,'ygrid','on','xgrid','on');
xlabel('-\pi \leq \omega \leq \pi');

rh = fftshift(fft(rh,512));
subplot(2,2,2);
plot(x,abs(rh));
title('bior 5.5 Reconstruction Filter');
axis([-pi pi 0 1.3]);
set(gca,'ytick',[0 1],'xtick',0,'yticklabel',[0 1],'xticklabel','');
set(gca,'ygrid','on','xgrid','on');
```

```
xlabel('-\pi \leq \omega \leq \pi');

[h rh] = biorwavf('bior6.8');
h = fftshift(fft(h,512));
subplot(2,2,3);
plot(x,abs(h));
title('bior 6.8 Deconstruction Filter');
axis([-pi pi 0 1.3]);
set(gca,'ytick',[0 1],'xtick',0,'yticklabel',[0 1],'xticklabel','');
set(gca,'ygrid','on','xgrid','on');
xlabel('-\pi \leq \omega \leq \pi');

rh = fftshift(fft(rh,512));
subplot(2,2,4);
plot(x,abs(rh));
title('bior 6.8 Reconstruction Filter');
axis([-pi pi 0 1.3]);
set(gca,'ytick',[0 1],'xtick',0,'yticklabel',[0 1],'xticklabel','');
set(gca,'ygrid','on','xgrid','on');
xlabel('-\pi \leq \omega \leq \pi');
```

graphCoifHs.m

```
function graphCoifs

clf
x = linspace(-pi,pi,512);

h = coifwavf('coif4'); h = fftshift(fft(h,512));
subplot(2,2,1);
plot(x,abs(h));
title('Coif 4');
axis([-pi pi 0 1.1]);
set(gca,'ytick',[0 1],'xtick',0,'yticklabel',[0 1],'xticklabel','');
set(gca,'xgrid','on','ygrid','on');
xlabel('-\pi \leq \omega \leq \pi');

h = coifwavf('coif5'); h = fftshift(fft(h,512));
subplot(2,2,2);
plot(x,abs(h));
title('Coif 5');
axis([-pi pi 0 1.1]);
set(gca,'ytick',[0 1],'xtick',0,'yticklabel',[0 1],'xticklabel','');
set(gca,'xgrid','on','ygrid','on');
xlabel('-\pi \leq \omega \leq \pi');
```

graphDaubHs.m

```
function graphDaubHs

clf
x = linspace(-pi,pi,512);

h = dbwavf('db1'); h = fftshift(fft(h,512));
subplot(2,2,1); plot(x,abs(h));
title('Daub 1 (Haar), |H_{0}| = 2');
axis([-pi pi 0 1.1]);
set(gca,'ytick',[0 1],'xtick',0,'yticklabel',[0 1],'xticklabel','');
set(gca,'xgrid','on','ygrid','on');
xlabel('-\pi \leq \omega \leq \pi');


h = dbwavf('db4'); h = fftshift(fft(h,512));
subplot(2,2,2); plot(x,abs(h));
title('Daub 4, |H_{0}| = 8');
axis([-pi pi 0 1.1]);
set(gca,'ytick',[0 1],'xtick',0,'yticklabel',[0 1],'xticklabel','');
set(gca,'xgrid','on','ygrid','on');
xlabel('-\pi \leq \omega \leq \pi');

h = dbwavf('db16'); h = fftshift(fft(h,512));
subplot(2,2,3); plot(x,abs(h));
title('Daub 16, |H_{0}| = 32');
axis([-pi pi 0 1.1]);
set(gca,'ytick',[0 1],'xtick',0,'yticklabel',[0 1],'xticklabel','');
set(gca,'xgrid','on','ygrid','on');
xlabel('-\pi \leq \omega \leq \pi');

h = dbwavf('db32'); h = fftshift(fft(h,512));
subplot(2,2,4); plot(x,abs(h));
title('Daub 32, |H_{0}| = 64');
axis([-pi pi 0 1.1]);
set(gca,'ytick',[0 1],'xtick',0,'yticklabel',[0 1],'xticklabel','');
set(gca,'xgrid','on','ygrid','on');
xlabel('-\pi \leq \omega \leq \pi');
```

plotBiorWavelet.m

```
function [y,dy] = plotBiorWavelet(iterations, name)
% plot a bior wavelet on time domain
    y = [1 0];
    dy = [1 0];
    [h dh] = biorwavf(name);
    for i=1:iterations,
        y = upsample(y);
        dy = upsample(dy);
        y = conv(h, y);
        dy = conv(dh, dy);
    end
```

plotCoifWavelet.m

```
function y = plotCoifWavelet(iterations, name)
% plot a coiflet on time domain
    y = [1 0];
    h = coifwavf(name);
    for i=1:iterations,
        y = upsample(y);
        y = conv(h, y);
    end
```

plotDaubWavelet.m

```
function y = plotDaubWavelet(iterations, name)
% plot a daub wavelet on time domain
    y = [1 0 0];
    h = dbwavf(name);
    for i=1:iterations,
        y = upsample(y);
        y = conv(h, y);
    end
```

upsample.m

```
function y = upsample(x)
    for i=1:length(x),
     y(2*i-1) = x(i);
    end
```

A.2 Decomposition and Reconstruction

graphAttack.m

```
function graphAttack(x,wname)
% plot a detail of signal with an attack feature

   clf;

   n = 8;
   for i=1:n,
     [y detail] = partialRec(x,i,wname);
    subplot(n, 2, (2*i));
    plot(detail);
    axis([(length(detail)/5) (length(detail)/2) -1 1]);
        set(gca,'ytick',0,'xtick',0,'yticklabel',i,'xticklabel','');
    end
```

graphPartialRec.m

```
function graphPartialRec(x,n,wname)
% plot a table of n partial reconstructions

    clf;

    n = (n+1);

    h = subplot(n, 2, 1);
    plot(x); axis([0 length(x) -1 1]);
    set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');

    for i=2:n,
     [y detail] = partialRec(x,(i-1),wname);
    subplot(n, 2, (2*i-1));
    plot(y);
    axis([0 length(y) -1 1]);
        set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');
    subplot(n, 2, (2*i));
    plot(detail);
    axis([0 length(detail) -1 1]);
        set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');
    end
```

graphPartialRecAllWavelets.m

```
function graphPartialRecAllWavelets(x,n,m,wname)
% plot a table of n partial reconstructions
% at column m

    num = 1; % number wavelet systems

    for i=1:n,
     [y detail] = partialRec(x,i,wname);
    subplot(n, num, (num*(i-1)+m));
    plot(y);
        axis([0 length(y) -1 1]);
        set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');
    end
```

graphPartialRecRange.m

```
function graphPartialRecRange(x,start,n,wname)
% plot a range of n partial reconstructions
% starting at index start

    clf;

    n = (n+1);

    h = subplot(n, 2, 1);
    plot(x); axis([0 length(x) -1 1]);
    xlabel('t');
    set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');

    for i=2:n,
     [y detail] = partialRec(x,(i-1+start),wname);
    subplot(n, 2, (2*i-1));
    plot(y); xlabel('t');
    axis([0 length(y) -1 1]);
        set(gca,'ytick',0,'xtick',0,'yticklabel',(i-1+start),'xticklabel','');
    subplot(n, 2, (2*i));
    plot(detail); xlabel('\omega');
    axis([0 length(detail) -1 1]);
        set(gca,'ytick',0,'xtick',0,'yticklabel','','xticklabel','');
    end
```

partialRec.m

```
function [y, detail] = partialRec(x, level, name)
% signal [y, detail] = partialRec(signal x, int level, string waveletname)
% do deconstruction with specified wavelets to level 'level', and then do
% partial reconstruction, zeroing all levels > 'level'
%
% code based on Assignment 5 solution

    cD = cell(level,1);

    cA = x;

    for i=1:level,
     [cA cD{i}] = dwt(cA, name);
    end

    detail = cD{level};

    cA = zeros(length(cD{level}), 1);
    cA = idwt(cA, cD{level}, name);
    for i=(level-1):-1:1,
cA = cA([1:length(cD{i})]);
     cA = idwt(cA, zeros(length(cD{i}), 1), name);
    end

    y = cA;
```