CS 540, HW2 Solution, Spring 2010

## Problem 1

1. BFS is complete in general. BFS is optimal in this case, because edge costs are 1.
2. DFS is not complete, because the search space is infinite. DFS is not optimal; for example, if the start state is $(0,0)$ and the goal state is $(2,1)$, then $(0,0) \rightarrow(2,1)$ is an optimal path, but BFS could find $(0,0) \rightarrow(-1,2) \rightarrow(1,3) \rightarrow(2,1)$.
3. The Manhattan distance heuristic is not admissible. For example, say the goal state is $(0,0)$. Let $s$ be the state $(2,1)$. Denote the heuristic by $h$ and the true cost to the goal by $h^{*}$. Then $h(s)=$ $|2-0|+|1-0|=3 \not \leq 1=h^{*}(s)$.
4. Yes, the heuristic

$$
h(s)= \begin{cases}0 & \text { if } s \text { is a goal node } \\ 1 & \text { otherwise }\end{cases}
$$

is consistent, since for any state $s$ and successor $s^{\prime}$, we have $h(s) \leq 1 \leq 1+h\left(s^{\prime}\right)=c\left(s, s^{\prime}\right)+h\left(s^{\prime}\right)$, where $c\left(s, s^{\prime}\right)=1$ denotes the cost of moving from $s$ to $s^{\prime}$.

## Problem 2.

1. The goal state $\mathrm{G}_{3}$ is reached after expanding the states S A B C D E G $\mathrm{G}_{3}$. Details:

| Pop | Expand? | Queue |
| :---: | :---: | :---: |
| S | Y | A B |
| A | Y | B ; B C D |
| B | Y | B C D ; D E G ${ }_{3}$ |
| B | N | C D ; D E G ${ }_{3}$ |
| C | Y | $\mathrm{D} ; \mathrm{DE} \mathrm{G} \mathrm{E}_{3} ; \mathrm{S}$ |
| D | Y | D E G $\mathrm{F}_{3} ; \mathrm{S} ; \mathrm{G}_{1} \mathrm{G}_{2}$ |
| D | N | $\mathrm{E} \mathrm{G}_{3} ; \mathrm{S} ; \mathrm{G}_{1} \mathrm{G}_{2}$ |
| E | Y | $\mathrm{G}_{3} ; \mathrm{S} ; \mathrm{G}_{1} \mathrm{G}_{2} ; \mathrm{D}$ |
| $\mathrm{G}_{3}$ | Y |  |

2. The goal state $\mathrm{G}_{1}$ is reached after expanding the states $\mathrm{S} A \mathrm{~B} D \mathrm{G}_{1}$. Details:

| Pop | Expand? | Queue |
| :---: | :---: | :--- |
| S | Y | A B |
| A | Y | B C D $; \mathrm{B}_{\mathrm{B}}^{\mathrm{Y}}$ |
| Y | $\mathrm{D} \mathrm{E} \mathrm{G}_{3} ; \mathrm{C} \mathrm{D} ; \mathrm{B}$ |  |
| $\mathrm{G}_{1}$ | Y | $\mathrm{G}_{1} \mathrm{G}_{2} ; \mathrm{E} \mathrm{G}_{3} ; \mathrm{CD} ; \mathrm{B}$ |

(We also accepted $\mathrm{G}_{2}$ as the goal state reached.)
3. The goal state $G_{3}$ is reached after expanding the states $S$ A B in the first iteration and S A B C D B $E \mathrm{G}_{3}$ in the second iteration. Details:

| Iter | Pop | Expand? | Queue (length of path from S) |
| :---: | :---: | :---: | :--- |
| 1 | S | Y | $\mathrm{A} \mathrm{(1)} \mathrm{~B} \mathrm{(1)}$ |
|  | A | $\mathrm{Y}^{*}$ | $\mathrm{~B}(1)$ |
|  | B | $\mathrm{Y}^{*}$ |  |
| 2 | S | Y | $\mathrm{A}(1) \mathrm{B}(1)$ |
|  | A | Y | $\mathrm{B}(2) \mathrm{C}(2) \mathrm{D}(2) ; \mathrm{B}(1)$ |
|  | B | $\mathrm{Y}^{*}$ | $\mathrm{C}(2) \mathrm{D}(2) ; \mathrm{B}(1)$ |
|  | C | $\mathrm{Y}^{*}$ | $\mathrm{D}(2) ; \mathrm{B}(1)$ |
|  | D | $\mathrm{Y}^{*}$ | $\mathrm{~B}(1)$ |
|  | B | Y | $\mathrm{D}(2) \mathrm{E}(2) \mathrm{G}_{3}(2)$ |
|  | D | N | $\mathrm{E}(2) \mathrm{G}_{3}(2)$ |
|  | E | $\mathrm{Y}^{*}$ | $\mathrm{G}_{3}(2)$ |
|  | $\mathrm{G}_{3}$ | Y |  |

(*Because the maximum path length is reached, no successors are added, and the state is not added to the closed list. We accepted several variations on this answer.)
4. The goal state $\mathrm{G}_{2}$ is reached after expanding the states S A C D $\mathrm{G}_{2}$. Details:

| Pop | Expand? | Queue $(g+h)$ |
| :---: | :---: | :--- |
| S | Y | $\mathrm{A}(1+10=11) \mathrm{B}(7+23=30)$ |
| A | Y | $\mathrm{C}((1+1)+1=3) \mathrm{D}((1+15)+3=19) \mathrm{B}((1+2)+23=26) \mathrm{B}(7+23=30)$ |
| C | Y | $\mathrm{D}((1+15)+3=19) \mathrm{B}((1+2)+23=26) \mathrm{B}(7+23=30)$ <br> $\mathrm{S}((1+1+2)+100=104)$ |
| D | Y | $\mathrm{G}_{2}((1+15+3)+0=19) \mathrm{G}_{1}((1+15+5)+0=21) \mathrm{B}((1+2)+23=26)$ <br>  <br> $\mathrm{G}_{2}$ |
|  | Y | $\mathrm{B}(7+23=30) \mathrm{S}((1+1+2)+100=104)$ |

