CS540: Artificial Inteliigence
Assignment #3

Solutions

Problem 1

Applying CSPs to the 4-queens problem

(a) False, a blank chess board provides no constraints to propagate.

(b) True, a single queen in the first column (third row) provides enough constraint to solve the problem.
Variables: Q_1, Q_2, Q_3, Q_4 (queen in each column)
Domain: $\{1, 2, 3, 4\}$ (row)
Constraints: No queens can share a row or diagonal
Solution: $Q_1 = \{3\}$
$Q_2 \rightarrow Q_1$: reduce domain to $Q_2 = \{1\}$
$Q_3 \rightarrow Q_1$: reduce domain to $Q_3 = \{2, 4\}$
$Q_4 \rightarrow Q_1$: reduce domain to $Q_4 = \{1, 2, 4\}$
$Q_3 \rightarrow Q_2$: reduce domain to $Q_3 = \{4\}$
$Q_4 \rightarrow Q_2$: reduce domain to $Q_4 = \{2, 4\}$
$Q_4 \rightarrow Q_3$: reduce domain to $Q_4 = \{2\}$
Remaining arc checks have no effect.

Problem 2

CSP for solving lunch logistics

(a) Variables: A, B, C, D, E
Domain: $\{Go, \neg Go\}$
Shorthand: Writing variable X means $X = Go$ and $\neg X$ means $X = \neg Go$
Constraints: $A \vee \neg B$, $A \vee \neg C$, $A \vee \neg E$, $\neg C \vee \neg D$, $\neg C \vee \neg E$, $\neg B \vee D \vee E$

(b) Set B, arc queue: $A \rightarrow B, C \rightarrow B, D \rightarrow B, E \rightarrow B$
$A \rightarrow B$: set A, add to queue: $B \rightarrow A, C \rightarrow A, D \rightarrow A, E \rightarrow A$
$C \rightarrow B, D \rightarrow B, E \rightarrow B, B \rightarrow A, C \rightarrow A, D \rightarrow A, E \rightarrow A$: no effect

(c) Impossible configuration: A, B, C, D, E
For example, $D \rightarrow C$ would make $D = \varnothing$

Problem 3

Logical proofs from a knowledge base

(a) $\{(P \rightarrow Q), (Q \rightarrow R)\} \models (P \rightarrow R)$ is true

P	Q	R	$P \rightarrow Q$	$Q \rightarrow R$	$P \rightarrow R$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

All lines consistent with the knowledge base make the conclusion true.

(b) $\{(P \rightarrow (Q \vee R)), \neg Q\} \models (P \vee R)$ is false

P	Q	R	$P \rightarrow (Q \vee R)$	$\neg Q$	$P \vee R$
T	T	T	T	F	T
T	T	F	T	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	F

The bottom line (all false) is consistent with the knowledge base but not the conclusion.

Problem 4

Converting natural language to FOL

(a) $\forall x$ person$(x) \rightarrow$ says(x, cold)

(b) person$(x) \wedge$ says(x, cold)

(c) $\neg Ex$ person$(x) \wedge$ says(x, cold)

(d) $\forall x$ movie$(x) \rightarrow \neg theater(x)$

(e) $\{Ex$ movie$(x) \wedge \neg$graphics$(x), \forall x$ movie$(x) \rightarrow$ (director$(x) \wedge$ producer$(x))\}$

(f) Not exactly FOL: $\forall x, y$(movie$(x) \wedge$ movie$(y) \wedge x \neq y) \rightarrow$ movie$(x) \neq$ movie(y)

(g) $\{Ex$ movie$(x) \wedge$ boring$(x), Ex$ movie$(x) \wedge$ interesting$(x)\}$

Problem 5

Representation in FOL

- Not all students major in both Computer-science and Statistics.

$$\exists s\, student(s) \land \neg(major(s) = CS \land major(s) = Statistics)$$

- Only one student failed in Statistics.

$$\exists s\, student(s) \land failed(s, Statistics) \land (\forall t\, failed(t, Statistics) \implies s = t)$$

- The top score in AI was better than the top score in Graphics.

$$\exists s\, isaiscore(s) \land \forall t\, isgraphicsscore(t) \implies greaterthan(s, t)$$

- There is a man who likes every person who is not a Computer-score.

$$\exists s\, man(s) \land \forall t\, person(t) \implies likes(s, t)$$

- Politicians can trick some of the people all the time, and they can trick all of the people some of the time, but they can't trick all of the people all the time.

$$\begin{aligned}\forall s\, politician(s) \implies &((\exists t\, person(t) \land (\forall u\, time(u) \implies tricks(s, t, u)))\land\\ &(\forall v\, person(v) \implies (\exists w\, time(w) \land tricks(s, v, w)))\land\\ &(\exists x\, person(x) \land (\exists y\, time(y) \land \neg tricks(s, x, y))))\end{aligned}$$

Problem 6

FOL representation → CNF → proof from knowledge base

<u>FOL representation</u>
Anyone passing his AI exams and winning the jackpot is happy.

$$\forall s\, (passes(s) \land wins(s)) \implies happy(s)$$

Anyone who is lucky or studies can pass all his exams.

$$\forall s\, (lucky(s) \lor studies(s)) \implies passes(s)$$

Anyone who is lucky wins the jackpot.

$$\forall s\, lucky(s) \implies wins(s)$$

Bart did not study, but Bart is lucky.

$$\neg study(Bart) \land lucky(Bart)$$

<u>CNF representation</u>
Anyone passing his AI exams and winning the jackpot is happy.

$$\begin{aligned}&\forall s\, \neg(passes(s) \land wins(s)) \lor happy(s) \quad \text{i.e.,}\\ &\forall s\, \neg passes(s) \lor \neg wins(s) \lor happy(s)\end{aligned}$$

Anyone who is lucky or studies can pass all his exams.

$$\begin{aligned}
&\forall s\, \neg(lucky(s) \vee studies(s)) \vee passes(s) \quad \text{i.e.,} \\
&\forall s\, (\neg lucky(s) \wedge \neg studies(s)) \vee passes(s) \quad \text{i.e.,} \\
&\forall s\, (\neg lucky(s) \vee passes(s)) \wedge (\neg studies(s) \vee passes(s))
\end{aligned}$$

Anyone who is lucky wins the jackpot.

$$\forall s\, \neg lucky(s) \vee wins(s)$$

Bart did not study, but Bart is lucky.

$$\neg study(Bart) \wedge lucky(Bart)$$

Knowledge base

$$\begin{aligned}
&\forall s\, \neg passes(s) \vee \neg wins(s) \vee happy(s) \\
&\forall s\, \neg lucky(s) \vee passes(s) \\
&\forall s\, \neg studies(s) \vee passes(s) \\
&\forall s\, \neg lucky(s) \vee wins(s) \\
&\neg study(Bart) \\
&lucky(Bart)
\end{aligned}$$

Add negation of Bart is happy:

$$\neg happy(Bart)$$

Resolution refutation

$\neg passes(s) \vee \neg wins(s) \vee happy(s)$	$\neg happy(Bart)$
$\neg lucky(s) \vee passes(s)$	$\neg passes(Bart) \vee \neg wins(Bart)$
$\neg lucky(s) \vee wins(s)$	$\neg lucky(Bart) \vee \neg wins(Bart)$
$lucky(Bart)$	$\neg lucky(Bart)$
	$\square$