Problem 1
Applying CSPs to the 4-queens problem

(a) False, a blank chess board provides no constraints to propagate.

(b) True, a single queen in the first column (third row) provides enough constraint to solve the problem.

Variables: $Q_1, Q_2, Q_3, Q_4$ (queen in each column)
Domain: \{1, 2, 3, 4\} (row)
Constraints: No queens can share a row or diagonal
Solution: $Q_1 = \{3\}$
$Q_2 \rightarrow Q_1$: reduce domain to $Q_2 = \{1\}$
$Q_3 \rightarrow Q_1$: reduce domain to $Q_3 = \{2, 4\}$
$Q_4 \rightarrow Q_1$: reduce domain to $Q_4 = \{1, 2, 4\}$
$Q_3 \rightarrow Q_2$: reduce domain to $Q_3 = \{4\}$
$Q_4 \rightarrow Q_2$: reduce domain to $Q_4 = \{2, 4\}$
$Q_4 \rightarrow Q_3$: reduce domain to $Q_4 = \{2\}$
Remaining arc checks have no effect.

Problem 2
CSP for solving lunch logistics

(a) Variables: $A, B, C, D, E$
Domain: \{Go, ¬Go\}
Shorthand: Writing variable $X$ means $X = Go$ and $¬X$ means $X = ¬Go$
Constraints: $A \lor ¬B, A \lor ¬C, A \lor ¬E, ¬C \lor ¬D, ¬C \lor ¬E, ¬B \lor D \lor E$

(b) Set $B$, arc queue: $A \rightarrow B, C \rightarrow B, D \rightarrow B, E \rightarrow B
A \rightarrow B$: set $A$, add to queue: $B \rightarrow A, C \rightarrow A, D \rightarrow A, E \rightarrow A$
$C \rightarrow B, D \rightarrow B, E \rightarrow B, B \rightarrow A, C \rightarrow A, D \rightarrow A, E \rightarrow A$: no effect

(c) Impossible configuration: $A, B, C, D, E$
For example, $D \rightarrow C$ would make $D = \emptyset$
Problem 3

Logical proofs from a knowledge base

(a) \{ (P \rightarrow Q), (Q \rightarrow R) \} \models (P \rightarrow R) is true

\[
\begin{array}{cccc|c|c|c}
P & Q & R & P \rightarrow Q & Q \rightarrow R & P \rightarrow R \\
\hline
T & T & T & T & T & T \\
T & T & F & T & F & F \\
T & F & T & F & T & T \\
T & F & F & F & T & F \\
F & T & T & T & T & T \\
F & T & F & T & F & T \\
F & F & T & T & T & T \\
F & F & F & T & T & T \\
\end{array}
\]

All lines consistent with the knowledge base make the conclusion true.

(b) \{ (P \rightarrow (Q \lor R)), \neg Q \} \models (P \lor R) is false

\[
\begin{array}{cccc|c|c|c}
P & Q & R & P \rightarrow (Q \lor R) & \neg Q & P \lor R \\
\hline
T & T & T & T & F & T \\
T & T & F & T & F & T \\
T & F & T & T & T & T \\
T & F & F & T & T & T \\
F & T & T & T & F & T \\
F & T & F & T & F & F \\
F & F & T & T & T & T \\
F & F & F & T & T & F \\
\end{array}
\]

The bottom line (all false) is consistent with the knowledge base but not the conclusion.

Problem 4

Converting natural language to FOL

(a) \( \forall x \text{ person}(x) \rightarrow \text{says}(x, \text{cold}) \)

(b) \( \text{person}(x) \land \text{says}(x, \text{cold}) \)

(c) \( \neg \exists x \text{ person}(x) \land \text{says}(x, \text{cold}) \)

(d) \( \forall x \text{ movie}(x) \rightarrow \neg \text{theater}(x) \)

(e) \{ \exists x \text{ movie}(x) \land \neg \text{graphics}(x), \forall x \text{ movie}(x) \rightarrow (\text{director}(x) \land \text{producer}(x)) \} \)

(f) Not exactly FOL: \( \forall x, y (\text{movie}(x) \land \text{movie}(y) \land x \neq y) \rightarrow \text{movie}(x) \neq \text{movie}(y) \)

(g) \{ \exists x \text{ movie}(x) \land \text{boring}(x), \exists x \text{ movie}(x) \land \text{interesting}(x) \} \)
Problem 5

Representation in FOL

- Not all students major in both Computer-science and Statistics.
  \[ \exists s \text{ student}(s) \land \neg (\text{major}(s) = \text{CS} \land \text{major}(s) = \text{Statistics}) \]

- Only one student failed in Statistics.
  \[ \exists s \text{ student}(s) \land \text{failed}(s, \text{Statistics}) \land (\forall t \text{ failed}(t, \text{Statistics}) \implies s = t) \]

- The top score in AI was better than the top score in Graphics.
  \[ \exists s \text{ saiscore}(s) \land \forall t \text{ isgraphicsscore}(t) \implies \text{greaterthan}(s, t) \]

- There is a man who likes every person who is not a Computer-score.
  \[ \exists s \text{ man}(s) \land \forall t \text{ person}(t) \implies \text{likes}(s, t) \]

- Politicians can trick some of the people all the time, and they can trick all of the people some of the time, but they can’t trick all of the people all the time.
  \[ \forall s \text{ politician}(s) \implies ((\exists t \text{ person}(t) \land (\forall u \text{ time}(u) \implies \text{tricks}(s, t, u))) \land (\forall v \text{ person}(v) \implies (\exists w \text{ time}(w) \land \text{tricks}(s, v, w))) \land (\exists w \text{ person}(x) \land (\exists y \text{ time}(y) \land \neg \text{tricks}(s, x, y)))) \]

Problem 6

FOL representation → CNF → proof from knowledge base

FOL representation

Anyone passing his AI exams and winning the jackpot is happy.
\[ \forall s (\text{passes}(s) \land \text{wins}(s)) \implies \text{happy}(s) \]

Anyone who is lucky or studies can pass all his exams.
\[ \forall s (\text{lucky}(s) \lor \text{studies}(s)) \implies \text{passes}(s) \]

Anyone who is lucky wins the jackpot.
\[ \forall s \text{ lucky}(s) \implies \text{wins}(s) \]

Bart did not study, but Bart is lucky.
\[ \neg \text{study}(\text{Bart}) \land \text{lucky}(\text{Bart}) \]

CNF representation

Anyone passing his AI exams and winning the jackpot is happy.
\[ \forall s \neg (\text{passes}(s) \land \text{wins}(s)) \lor \text{happy}(s) \quad \text{i.e.,} \]
\[ \forall s \neg \text{passes}(s) \lor \neg \text{wins}(s) \lor \text{happy}(s) \]
Anyone who is lucky or studies can pass all his exams.

\[ \forall s \neg (\text{lucky}(s) \lor \text{studies}(s)) \lor \text{passes}(s) \]  
\[ \text{i.e.,} \]

\[ \forall s (\neg \text{lucky}(s) \land \neg \text{studies}(s)) \lor \text{passes}(s) \]  
\[ \text{i.e.,} \]

\[ \forall s (\neg \text{lucky}(s) \lor \text{passes}(s)) \land (\neg \text{studies}(s) \lor \text{passes}(s)) \]

Anyone who is lucky wins the jackpot.

\[ \forall s \neg \text{lucky}(s) \lor \text{wins}(s) \]

Bart did not study, but Bart is lucky.

\[ \neg \text{study}(\text{Bart}) \land \text{lucky}(\text{Bart}) \]

Knowledge base

\[ \forall s \neg \text{passes}(s) \lor \neg \text{wins}(s) \lor \text{happy}(s) \]
\[ \forall s \neg \text{lucky}(s) \lor \text{passes}(s) \]
\[ \forall s \neg \text{studies}(s) \lor \text{passes}(s) \]
\[ \forall s \neg \text{lucky}(s) \lor \text{wins}(s) \]
\[ \neg \text{study}(\text{Bart}) \]
\[ \text{lucky}(\text{Bart}) \]

Add negation of Bart is happy:

\[ \neg \text{happy}(\text{Bart}) \]

Resolution refutation

| \neg \text{passes}(s) \lor \neg \text{wins}(s) \lor \text{happy}(s) | \neg \text{happy}(\text{Bart}) |
| \neg \text{lucky}(s) \lor \text{passes}(s) | \neg \text{passes}(\text{Bart}) \lor \neg \text{wins}(\text{Bart}) |
| \neg \text{lucky}(s) \lor \text{wins}(s) | \neg \text{lucky}(\text{Bart}) \lor \neg \text{wins}(\text{Bart}) |
| \text{lucky}(\text{Bart}) | \neg \text{lucky}(\text{Bart}) |