Stat 571 - HW4 - Nathanael Fillmore

1. > prob.given.bucket1 <- c(black=4, red=5, white=3)/12
> prob.given.bucket2 <- c(black=1, red=4, white=7)/12
> prob.given.bucket3 <- c(black=80, red=0, white=20)/100
> prior <- c(bucket1=0.1, bucket2=0.2, bucket3=0.7)

(a) > prob.black <- prior['bucket1']*prob.given.bucket1['black'] +
+ prior['bucket2']*prob.given.bucket2['black'] +
+ prior['bucket3']*prob.given.bucket3['black']
> cat(sprintf("The probability that the ball is black is %f.", prob.black))
The probability that the ball is black is 0.610000.

(b) > prob.red <- prior['bucket1']*prob.given.bucket1['red'] +
+ prior['bucket2']*prob.given.bucket2['red'] +
+ prior['bucket3']*prob.given.bucket3['red']
> cat(sprintf("The probability that the ball is red is %f.", prob.red))
The probability that the ball is red is 0.108333.

(c) > prob.white <- prior['bucket1']*prob.given.bucket1['white'] +
+ prior['bucket2']*prob.given.bucket2['white'] +
+ prior['bucket3']*prob.given.bucket3['white']
> cat(sprintf("The probability that the ball is white is %f.", prob.white))
The probability that the ball is white is 0.281667.

(d) > prob.bucket1.given.black <- prob.given.bucket1['black']*prior['bucket1']/prob.black
> prob.bucket2.given.black <- prob.given.bucket2['black']*prior['bucket2']/prob.black
> prob.bucket3.given.black <- prob.given.bucket3['black']*prior['bucket3']/prob.black
> cat(sprintf("The probability that the ball came from bucket 1, given that it was black, is %f.", prob.bucket1.given.black))
The probability that the ball came from bucket 1, given that it was black, is 0.054645.

> cat(sprintf("The probability that the ball came from bucket 2, given that it was black, is %f.", prob.bucket2.given.black))
The probability that the ball came from bucket 2, given that it was black, is 0.027322.

> cat(sprintf("The probability that the ball came from bucket 3, given that it was black, is %f.", prob.bucket3.given.black))
The probability that the ball came from bucket 3, given that it was black, is 0.918033.

2. > prob.given.bucket1 <- c(black=17, red=6, white=8)/31
> prob.given.bucket2 <- c(black=40, red=30, white=25)/95
> prior <- c(bucket1=1/2, bucket2=1/2)

(a) > prob.B.eq.1 <- prob.given.bucket1['black']*prior['bucket1'] +
+ prob.given.bucket2['black']*prior['bucket2']
> prob.R.eq.1 <- prob.given.bucket1['red']*prior['bucket1'] +
+ prob.given.bucket2['red']*prior['bucket2']
> prob.W.eq.1 <- prob.given.bucket1['white']*prior['bucket1'] +
+ prob.given.bucket2['white']*prior['bucket2']
> cat(sprintf("$P(B=1)=%f, P(R=1)=%f, P(W=1)=%f$.", prob.B.eq.1,
+ prob.R.eq.1, prob.W.eq.1))

> $P(B=1) = 0.484720,$
> $P(R=1) = 0.254669,$
> $P(W=1) = 0.260611.$

(b) > cat(sprintf("$B$ follows Binomial(n=10, p=%f).", prob.B.eq.1))

$B$ follows Binomial(n=10, p=0.484720).
> cat(sprintf("$R$ follows Binomial(n=10, p=%f).", prob.R.eq.1))

$R$ follows Binomial(n=10, p=0.254669).
> cat(sprintf("$W$ follows Binomial(n=10, p=%f).", prob.W.eq.1))

$W$ follows Binomial(n=10, p=0.260611).
3. > prob.truely.A <- .43
   > prob.truely.B <- .07
   > prob.truely.AB <- .04
   > prob.truely.O <- .46
   > prob.stb.A.given.truely.A <- .88
   > prob.stb.A.given.truely.B <- .04
   > prob.stb.A.given.truely.AB <- .10
   > prob.stb.A.given.truely.O <- .04
     prob.stb.A.given.truely.B * prob.truely.B +
     prob.stb.A.given.truely.AB * prob.truely.AB +
     prob.stb.A.given.truely.O * prob.truely.O
   > cat(sprintf("The probability that a soldier's blood type is $A$ given that he
     was typed as having type $A$ is %f", prob.truely.A.given.stb.A))

The probability that a soldier’s blood type is $A$ given that he was typed as having type $A$ is 0.937562

17. (a) Some plants were tall and had green pods, etc., so “tall” and “green pods” are not mutually exclusive.

(b) > prob.tall.and.green <- 900/1600
   > prob.tall.and.yellow <- 300/1600
   > prob.short.and.green <- 300/1600
   > prob.short.and.yellow <- 100/1600
   > prob.tall <- prob.tall.and.green + prob.tall.and.yellow
   > prob.green <- prob.tall.and.green + prob.short.and.green
   > cat(sprintf("$P(tall)P(green) = %f = %f = P(tall and green)$, so they are
     independent.", prob.tall*prob.green, prob.tall.and.green))

$P(tall)P(green) = 0.562500 = 0.562500 = P(tall and green)$, so they are independent.

19. > prob.G.given.region1 <- .3
   > prob.C.given.region1 <- .3
   > prob.A.given.region1 <- .2
   > prob.T.given.region1 <- .2
   > prob.G.given.region2 <- .25
   > prob.C.given.region2 <- .25
   > prob.A.given.region2 <- .25
   > prob.T.given.region2 <- .25
   (a) > prob.same <- prob.G.given.region1 * prob.G.given.region2 +
     prob.C.given.region1 * prob.C.given.region2 +
     prob.A.given.region1 * prob.A.given.region2 +
     prob.T.given.region1 * prob.T.given.region2
   > cat(sprintf("The probability that they are the same is %f", prob.same))

The probability that they are the same is 0.250000

(b) “Success” means “the two nucleotides we just picked are the same”, so $P(success) = .25$. We get 3 successes in
3 trials.

   > prob.success <- .25
   > prob.3.successes <- dbinom(3, size=3, prob=.25) # or just prob.success^3
   > cat(sprintf("The probability that the triplets are the same is %f", prob.3.successes))

The probability that the triplets are the same is 0.015625
21. (a)  

```
/answer /
/ \heads/.5 no .5
/ coin /
\ \tails\/.5  yes .2
\answer /
\ no .8
```

(b)  

```r
> prob.heads.and.yes <- .5*.5
> prob.tails.and.yes <- .5*.2
> prob.yes <- prob.heads.and.yes + prob.tails.and.yes
> cat(sprintf("P(%$yes$)=%.4f", prob.yes))
P(yes) = 0.350000
```