Stat 571 - HW4 - Nathanael Fillmore

1.
```
> prob.given.bucket1 <- c(black=4,  red=5, white=3)/12
> prob.given.bucket2 <- c(black=1,  red=4, white=7)/12
> prob.given.bucket3 <- c(black=80, red=0, white=20)/100
> prior <- c(bucket1=0.1, bucket2=0.2, bucket3=0.7)
```

(a)
```
> prob.black <- prior["bucket1"]*prob.given.bucket1["black"] +
+               prior["bucket2"]*prob.given.bucket2["black"] +
+               prior["bucket3"]*prob.given.bucket3["black"]
> cat(sprintf("The probability that the ball is black is %f.", prob.black))
```

The probability that the ball is black is 0.610000.

(b)
```
> prob.red   <- prior["bucket1"]*prob.given.bucket1["red"] +
+               prior["bucket2"]*prob.given.bucket2["red"] +
+               prior["bucket3"]*prob.given.bucket3["red"]
> cat(sprintf("The probability that the ball is red is %f.", prob.red))
```

The probability that the ball is red is 0.108333.

(c)
```
> prob.white <- prior["bucket1"]*prob.given.bucket1["white"] +
+               prior["bucket2"]*prob.given.bucket2["white"] +
+               prior["bucket3"]*prob.given.bucket3["white"]
> cat(sprintf("The probability that the ball is white is %f.", prob.white))
```

The probability that the ball is white is 0.281667.

(d)
```
> prob.bucket1.given.black <- prob.given.bucket1["black"]*prior["bucket1"]/prob.black
> prob.bucket2.given.black <- prob.given.bucket2["black"]*prior["bucket2"]/prob.black
> prob.bucket3.given.black <- prob.given.bucket3["black"]*prior["bucket3"]/prob.black
> cat(sprintf("The probability that the ball came from bucket 1, given that it
+ was black, is %f.", prob.bucket1.given.black))
```

The probability that the ball came from bucket 1, given that it was black, is 0.054645.

```
> cat(sprintf("The probability that the ball came from bucket 2, given that it
+ was black, is %f.", prob.bucket2.given.black))
```

The probability that the ball came from bucket 2, given that it was black, is 0.027322.

```
> cat(sprintf("The probability that the ball came from bucket 3, given that it
+ was black, is %f.", prob.bucket3.given.black))
```

The probability that the ball came from bucket 3, given that it was black, is 0.918033.

2.
```
> prob.given.bucket1 <- c(black=17, red=6, white=8)/31
> prob.given.bucket2 <- c(black=40, red=30, white=25)/95
> prior <- c(bucket1=1/2, bucket2=1/2)
```

(a)
```
> prob.B.eq.1 <- prob.given.bucket1["black"]*prior["bucket1"] +
+                prob.given.bucket2["black"]*prior["bucket2"]
> prob.R.eq.1 <- prob.given.bucket1["red"]*prior["bucket1"] +
+                prob.given.bucket2["red"]*prior["bucket2"]
> prob.W.eq.1 <- prob.given.bucket1["white"]*prior["bucket1"] +
+                prob.given.bucket2["white"]*prior["bucket2"]
> cat(sprintf("$P(B=1)=%f,\\\\P(R=1)=%f,\\\\P(W=1)=%f$.", prob.B.eq.1,
+ prob.R.eq.1, prob.W.eq.1))
```

$P(B=1)=0.484720,$
$P(R=1)=0.254669,$
$P(W=1)=0.260611.$

(b)
```
> cat(sprintf("$B$ follows Binomial$(n=10, p=%f)$.", prob.B.eq.1))
```

B follows Binomial$(n=10, p=0.484720)$.

```
> cat(sprintf("$R$ follows Binomial$(n=10, p=%f)$.", prob.R.eq.1))
```

R follows Binomial$(n=10, p=0.254669)$.

```
> cat(sprintf("$W$ follows Binomial$(n=10, p=%f)$.", prob.W.eq.1))
```

W follows Binomial$(n=10, p=0.260611)$.

3.
```
> prob.truely.A <- .43
> prob.truely.B <- .07
> prob.truely.AB <- .04
> prob.truely.O <- .46
> prob.stb.A.given.truely.A <- .88
> prob.stb.A.given.truely.B <- .04
> prob.stb.A.given.truely.AB <- .10
> prob.stb.A.given.truely.O <- .04
> prob.stb.A <- prob.stb.A.given.truely.A * prob.truely.A +
+               prob.stb.A.given.truely.B * prob.truely.B +
+               prob.stb.A.given.truely.AB * prob.truely.AB +
+               prob.stb.A.given.truely.O * prob.truely.O
> prob.truely.A.given.stb.A <- prob.stb.A.given.truely.A * prob.truely.A / prob.stb.A
> cat(sprintf("The probability that a soldier's blood type is $A$ given that he
+ was typed as having type $A$ is %f", prob.truely.A.given.stb.A))
```

The probability that a soldier's blood type is A given that he was typed as having type A is 0.937562

17.

(a) Some plants were tall and had green pods, etc., so "tall" and "green pods" are not mutually exclusive.

(b)
```
> prob.tall.and.green <- 900/1600
> prob.tall.and.yellow <- 300/1600
> prob.short.and.green <- 300/1600
> prob.short.and.yellow <- 100/1600
> prob.tall <- prob.tall.and.green + prob.tall.and.yellow
> prob.green <- prob.tall.and.green + prob.short.and.green
> cat(sprintf("$P($tall$)P($green$) = %f = %f = P($tall and green$)$, so they are
+ independent.", prob.tall*prob.green, prob.tall.and.green))
```

$P(\text{tall})P(\text{green}) = 0.562500 = 0.562500 = P(\text{tall and green})$, so they are independent.

19.
```
> prob.G.given.region1 <- .3
> prob.C.given.region1 <- .3
> prob.A.given.region1 <- .2
> prob.T.given.region1 <- .2
> prob.G.given.region2 <- .25
> prob.C.given.region2 <- .25
> prob.A.given.region2 <- .25
> prob.T.given.region2 <- .25
```

(a)
```
> prob.same <- prob.G.given.region1 * prob.G.given.region2 +
+              prob.C.given.region1 * prob.C.given.region2 +
+              prob.A.given.region1 * prob.A.given.region2 +
+              prob.T.given.region1 * prob.T.given.region2
> cat(sprintf("The probability that they are the same is %f", prob.same))
```

The probability that they are the same is 0.250000

(b) "Success" means "the two nucleotides we just picked are the same", so $P(\text{success}) = .25$. We get 3 successes in 3 trials.

```
> prob.success <- .25
> prob.3.successes <- dbinom(3, size=3, prob=.25) # or just prob.success^3
> cat(sprintf("The probability that the triplets are the same is %f", prob.3.successes))
```

The probability that the triplets are the same is 0.015625

21. (a)

```
                      yes .5
             /answer /
           /         \
    heads/.5          no .5
       /
coin /
     \
       \
    tails\.5
           \          yes .2
             \answer /
                     \
                      no .8
```

(b)

```
> prob.heads.and.yes <- .5*.5
> prob.tails.and.yes <- .5*.2
> prob.yes <- prob.heads.and.yes + prob.tails.and.yes
> cat(sprintf("$P($yes$)=%f$", prob.yes))
```

$P(\text{yes}) = 0.350000$