A: SIZE BOUNDS FOR JOINS [25%]

1. [15%] Compute the maximum possible output size for the following join queries, when all relations have size at most $N$.

   (a) $q(x, y, z) : -R(x), T(y), U(z), S(x, y, z)$.
   (b) $q(x, y, z, w, t) : -R(x, y), S(y, z), T(z, w), U(w, x), V(x, t)$.
   (c) $q(x, y, z) : -R(x, y, z), S(x, z, w), T(x, y, w), U(y, z, w)$.

2. [10%] Suppose that relation $R_i$ has size $N_i$ (in tuples). Compute the maximum output size for the following query. (Hint: there will be different cases depending on the given $N_i$).

   $q(x_1, x_2, x_3, x_4, x_5) : -R_1(x_1, x_2), R_2(x_2, x_3), R_3(x_3, x_4), R_4(x_4, x_5)$.

B: DATALOG [75%]

1. [15%] Is the following Datalog program equivalent to a UCQ query? If so, write the query. If not, prove why it is not the case.

   $B(X, Y) : - L(X, Y)$.
   $B(X, Y) : - T(X), B(Z, Y)$.

2. [20%] A Datalog program $P$ with a single recursive predicate is said to be bounded if there is a positive integer $n_0$ such that, on every database instance $I$, the bottom-up evaluation of $P$ terminates within at most $n_0$ steps. Otherwise, we say that $P$ is unbounded.

   (a) Prove that transitive closure is unbounded.
   (b) Give an example of a Datalog program that is bounded and has at least one recursive predicate.

3. [10%] Consider the following Datalog program:
\begin{verbatim}
T(x, y) :- R(x, y).
T(x, t) :- T(x, y), T(y, z), T(z, w), R(w, t).

Can you write an equivalent linear Datalog program? If yes, provide the program; otherwise, explain why this is not the case.

4. [20\%] Perform the magic set transformation for the following Datalog program:

\begin{verbatim}
T(x, y) :- F(x, y).
T(x, y) :- up(x, z1), T(z1, z2), F(z2, z3), T(z3, z4), down(z4, y).
q(y)    :- T(a, y).
\end{verbatim}

5. [10\%] Find all possible stratifications for the following Datalog program with negation:

\begin{verbatim}
T(x)    :- S(x), not R(x).
S(x)    :- T(x), not R(x).
U(x)    :- R(x), not T(x), not S(x).
V(x, y) :- V(x, y), not U(x), not U(y).
\end{verbatim}

\textbf{Deliverables}

Submit a single PDF document using Canvas (Homework 2). It is strongly suggested to use LaTeX to write your solution.
\end{verbatim}