

BLOCK CIPHERS

Permutations and Inverses

A function $f: \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$ is a **permutation** if there is an inverse function $f^{-1}: \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$ satisfying

$$\forall x \in \{0, 1\}^\ell : f^{-1}(f(x)) = x$$

This means f must be one-to-one and onto, meaning for every $y \in \{0, 1\}^\ell$ there is a unique $x \in \{0, 1\}^\ell$ such that $f(x) = y$.

Permutations and Inverses

x	00	01	10	11
$f(x)$	01	11	00	10

A permutation

x	00	01	10	11
$f(x)$	01	11	11	10

Not a permutation

Permutations and Inverses

x	00	01	10	11
$f(x)$	01	11	00	10

A permutation

x	00	01	10	11
$f^{-1}(x)$	10	00	11	01

Its inverse

Let

$$E: \{0, 1\}^k \times \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$$

be a function taking a key K and input x to return output $E(K, x)$. For each key K we let $E_K: \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$ be the function defined by

$$E_K(x) = E(K, x) .$$

We say that E is a block cipher if

- $E_K: \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$ is a permutation for every K , meaning has an inverse E_K^{-1} ,
- E, E^{-1} are efficiently computable,

where $E^{-1}(K, x) = E_K^{-1}(x)$.

Example

The table entry corresponding to the key in row K and input in column x is $E_K(x)$.

	00	01	10	11
00	00	01	10	11
01	01	00	11	10
10	10	11	00	01
11	11	10	01	00

In this case, the inverse cipher E^{-1} is given by the same table: the table entry corresponding to the key in row K and output in column y is $E_K^{-1}(y)$.

Block Ciphers: Example

Let $\ell = k$ and define $E: \{0, 1\}^k \times \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$ by

$$E_K(x) = E(K, x) = K \oplus x$$

Then E_K has inverse E_K^{-1} where

$$E_K^{-1}(y) = K \oplus y$$

Why? Because

$$E_K^{-1}(E_K(x)) = E_K^{-1}(K \oplus x) = K \oplus K \oplus x = x$$

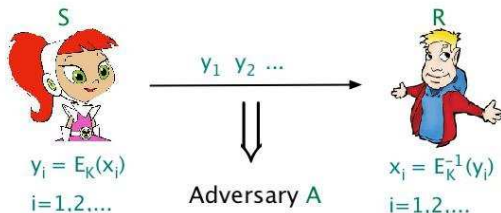
The inverse of block cipher E is the block cipher E^{-1} defined by

$$E^{-1}(K, y) = E_K^{-1}(y) = K \oplus y$$

Block cipher usage

- $K \xleftarrow{s} \{0, 1\}^k$
- K (magically) given to parties S, R, but not to A.
- S,R use E_K

Algorithm E is public! Think of E_K as encryption under key K .



Leads to security requirements like:

- Hard to get K from y_1, y_2, \dots
- Hard to get x_i from y_i

1972 – NBS (now NIST) asked for a block cipher for standardization

1974 – IBM designs Lucifer

Lucifer eventually evolved into DES.

Widely adopted as a standard including by ANSI and American Bankers association

Used in ATM machines

Replaced (by AES) only a few years ago

Key Length $k = 56$

Block length $\ell = 64$

So,

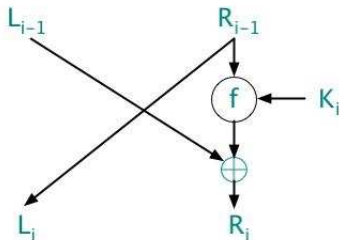
$$\text{DES}: \{0, 1\}^{56} \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$$

$$\text{DES}^{-1}: \{0, 1\}^{56} \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$$

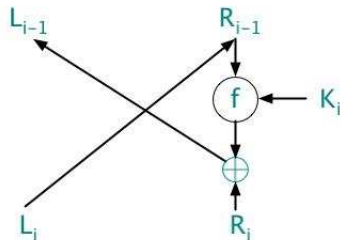
DES Construction

```
function DESK(M) // |K| = 56 and |M| = 64
  (K1, ..., K16) ← KeySchedule(K) // |Ki| = 48 for 1 ≤ i ≤ 16
  M ← IP(M)
  Parse M as L0 || R0 // |L0| = |R0| = 32
  for i = 1 to 16 do
    Li ← Ri-1 ; Ri ← f(Ki, Ri-1) ⊕ Li-1
  C ← IP-1(L16 || R16)
  return C
```

Round i:



Invertible given K_i :



DES Construction

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  for i = 1 to 16 do
    Li ← Ri-1 ; Ri ← f(Ki, Ri-1) ⊕ Li-1
  C ← IP-1(L16 || R16)
  return C
```

```
function DESK-1(C) // |K| = 56 and |M| = 64
  (K1, ..., K16) ← KeySchedule(K) // |Ki| = 48 for 1 ≤ i ≤ 16
  C ← IP(C)
  Parse C as L16 || R16
  for i = 16 downto 1 do
    Ri-1 ← Li ; Li-1 ← f(Ki, Ri-1) ⊕ Ri
  M ← IP-1(L0 || R0)
  return M
```

DES Construction

```
function DESK(M) // |K| = 56 and |M| = 64
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IP

IP⁻¹

58	50	42	34	26	18	10	2	40	8	48	16	56	24	64	32
60	52	44	36	28	20	12	4	39	7	47	15	55	23	63	31
62	54	46	38	30	22	14	6	38	6	46	14	54	22	62	30
64	56	48	40	32	24	16	8	37	5	45	13	53	21	61	29
57	49	41	33	25	17	9	1	36	4	44	12	52	20	60	28
59	51	43	35	27	19	11	3	35	3	43	11	51	19	59	27
61	53	45	37	29	21	13	5	34	2	42	10	50	18	58	26
63	55	47	39	31	23	15	7	33	1	41	9	49	17	57	25

DES Construction

function $f(J, R)$ // $|J| = 48$ and $|R| = 32$

$R \leftarrow E(R)$; $R \leftarrow R \oplus J$

Parse R as $R_1 \parallel R_2 \parallel R_3 \parallel R_4 \parallel R_5 \parallel R_6 \parallel R_7 \parallel R_8$ // $|R_i| = 6$ for $1 \leq i$
for $i = 1, \dots, 8$ do

$R_i \leftarrow S_i(R_i)$ // Each S-box returns 4 bits

$R \leftarrow R_1 \parallel R_2 \parallel R_3 \parallel R_4 \parallel R_5 \parallel R_6 \parallel R_7 \parallel R_8$ // $|R| = 32$ bits

$R \leftarrow P(R)$

return R

E						P			
32	1	2	3	4	5	16	7	20	21
4	5	6	7	8	9	29	12	28	17
8	9	10	11	12	13	1	15	23	26
12	13	14	15	16	17	5	18	31	10
16	17	18	19	20	21	2	8	24	14
20	21	22	23	24	25	32	27	3	9
24	25	26	27	28	29	19	13	30	6
28	29	30	31	32	1	22	11	4	25

S-boxes

		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
S_1 :	0 0	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0
	0 1	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3
	1 0	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5
	1 1	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
S_2 :	0 0	15	1	8	14	6	11	3	4	9	7	2	13	12	0	5
	0 1	3	13	4	7	15	2	8	14	12	0	1	10	6	9	11
	1 0	0	14	7	11	10	4	13	1	5	8	12	6	9	3	2
	1 1	13	8	10	1	3	15	4	2	11	6	7	12	0	5	14
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
S_3 :	0 0	10	0	9	14	6	3	15	5	1	13	12	7	11	4	2
	0 1	13	7	0	9	3	4	6	10	2	8	5	14	12	11	15
	1 0	13	6	4	9	8	15	3	0	11	1	2	12	5	10	14
	1 1	1	10	13	0	6	9	8	7	4	15	14	3	11	5	2

Figure: The DES S-boxes.

Cryptanalysis: Key Recovery Attacks on Block Ciphers

Adversary A knows $E: \{0,1\}^k \times \{0,1\}^\ell \rightarrow \{0,1\}^\ell$

$T \xleftarrow{\$} \{0,1\}^k$ is the target key.

Given: $(M_1, C_1), \dots, (M_q, C_q)$ where $C_i = E(T, M_i)$ for $i = 1, \dots, q$
and M_1, \dots, M_q are distinct.

Find: T

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Find: T

Certainly A should be given C_1, \dots, C_q . But why does A know M_1, \dots, M_q ?

- A posteriori **revelation** of data
- A **priori knowledge** of context

Good to be **conservative!**

A posteriori revelation of data

- S, R share key K
- On January 10, S encrypts

$M = \text{Let's meet tomorrow at 5 pm}$

and sends ciphertext C to R .

- Adversary captures C
- On January 11, adversary observes S, R meeting at 5 pm and deduces that M is as above
- Adversary knows C and its decryption M

A priori knowledge of context

- S, R share key K
- E-mails always begin with the keyword “From”
- S encrypts an email
- Adversary gets ciphertext C
- Since it knows part of the plaintext (“From”) it may have an input-output example of the block cipher under K

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Types of attacks

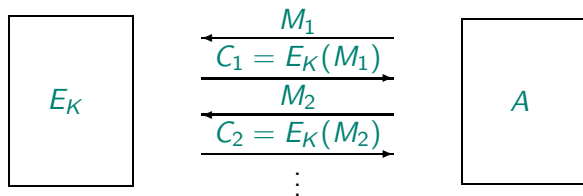
Given: $(M_1, C_1), \dots, (M_q, C_q)$ where $C_i = E(T, M_i)$ for $i = 1, \dots, q$ and M_1, \dots, M_q are distinct.

Known Message Attack: M_1, \dots, M_q arbitrary, not chosen by A.

Types of attacks

Given: $(M_1, C_1), \dots, (M_q, C_q)$ where $C_i = E(T, M_i)$ for $i = 1, \dots, q$ and M_1, \dots, M_q are distinct.

Chosen Message Attack: A can pick M_1, \dots, M_q , even **adaptively**, meaning pick M_i as a function of $(M_1, C_1), \dots, (M_{i-1}, C_{i-1})$ for $i = 1, \dots, q$.



Examples:

- A sends S e-mails which S encrypts and forwards to R
- S is a router encrypting any packet it receives

Cryptanalysis: Key Recovery Attacks on Block Ciphers

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Find: T

Exhaustive Key Search

Let T_1, \dots, T_{2^k} be a list of all k bit keys. Let $T \xleftarrow{\$} \{0, 1\}^k$ be the target key and let (M_1, C_1) satisfy $E_T(M_1) = C_1$.

```
algorithm  $EKS_E(M_1, C_1)$   
  for  $i = 1, \dots, 2^k$  do  
    if  $E(T_i, M_1) = C_1$  then return  $T_i$ 
```

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Let S be the set of all keys consistent with (M_1, C_1) . Then EKS_E finds some key in S .

Fact: If $\ell \geq k$ then T is “usually” the only key in S .

Exhaustive Key Search

Let T_1, \dots, T_{2^k} be a list of all k bit keys. Let $T \xleftarrow{\$} \{0, 1\}^k$ be the target key and let (M_1, C_1) satisfy $E_T(M_1) = C_1$.

```
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  for  $i = 1, \dots, 2^k$  do
    if  $E(T_i, M_1) = C_1$  then return  $T_i$ 
```

Does this find the target key T ? **Yes, usually.**

Increasing likelihood of getting target key

Let T_1, \dots, T_{2^k} be a list of all k bit keys. Let $T \xleftarrow{\$} \{0, 1\}^k$ be the target key and let $(M_1, C_1), \dots, (M_q, C_q)$ satisfy $E_T(M_i) = C_i$ for all $1 \leq i \leq q$.

```
algorithm  $EKS_E((M_1, C_1), \dots, (M_q, C_q))$   
  for  $i = 1, \dots, 2^k$  do  
    if (  $E(T_i, M_1) = C_1$  and  $\dots$  and  $E(T_i, M_q) = C_q$  ) then  
      return  $T_i$ 
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```


How long does exhaustive key search take?

DES can be computed at 1.6 Gbits/sec in hardware.

DES plaintext = 64 bits

Chip can perform $(1.6 \times 10^9)/64 = 2.5 \times 10^7$ DES computations per second

Expect *EKS* to succeed in 2^{55} DES computations, so it takes time

$$\begin{aligned}\frac{2^{55}}{2.5 \times 10^7} &\approx 1.4 \times 10^9 \text{ seconds} \\ &\approx 45 \text{ years!}\end{aligned}$$

Key Complementation \Rightarrow 22.5 years

But this is prohibitive.

Does this mean DES is secure?

Differential and linear cryptanalysis

Exhaustive key search is a generic attack: Did not attempt to “look inside” DES and find/exploit weaknesses.

Method	when	q	Type of attack
Differential cryptanalysis	1992	2^{47}	Chosen-message
Linear cryptanalysis	1993	2^{44}	Known-message

Differential and linear cryptanalysis

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Differential cryptanalysis	1992	2^{47}	Chosen-message
Linear cryptanalysis	1993	2^{44}	Known-message

But merely storing 2^{44} input-output pairs requires 281 Tera-bytes.

In practice these attacks are prohibitively expensive.

EKS revisited

Let T_1, \dots, T_{2^k} be a list of all k bit keys. Let $T \xleftarrow{\$} \{0, 1\}^k$ be the target key and let (M_1, C_1) satisfy $E_T(M_1) = C_1$.

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Observation: The E computations can be performed in parallel.

EKS revisited

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Observation: The E computations can be performed in parallel.

- Wiener 1993:
 - \$1 million
 - 57 chips
 - Finds key in 3.5 hours
- EFF
 - \$250,000
 - Finds key in 56 hours

DES is considered broken because its short key size permits rapid key-search.

But DES is a very strong design as evidenced by the fact that there are no practical attacks that exploit its structure.

Block cipher $2DES : \{0, 1\}^{112} \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$ is defined by

$$2DES_{K_1 K_2}(M) = DES_{K_2}(DES_{K_1}(M))$$

- Exhaustive key search takes 2^{112} DES computations, which is too much even for machines
- Resistant to differential and linear cryptanalysis.

Meet-in-the-middle attack on 2DES

Suppose K_1K_2 is a target 2DES key and adversary has M, C such that

$$C = 2DES_{K_1K_2}(M) = DES_{K_2}(DES_{K_1}(M))$$

Then

$$DES_{K_2}^{-1}(C) = DES_{K_1}(M)$$

Meet-in-the-middle attack on 2DES

Suppose $DES_{K_2}^{-1}(C) = DES_{K_1}(M)$ and T_1, \dots, T_N are all possible DES keys, where $N = 2^{56}$.

T_1	$DES(T_1, M)$
T_i	$DES(T_i, M)$
T_N	$DES(T_N, M)$

Table L

$DES^{-1}(T_1, C)$	T_1
$DES^{-1}(T_j, C)$	T_j
$DES^{-1}(T_N, C)$	T_N

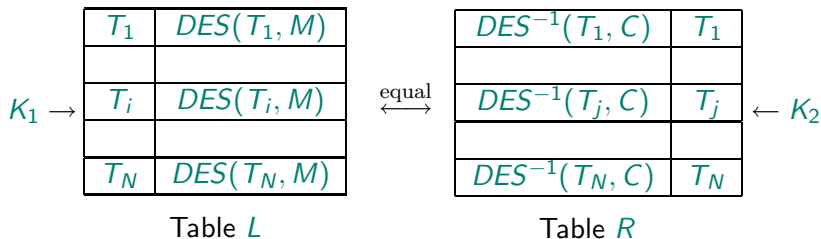
Table R

Attack idea:

- Build L,R tables

Meet-in-the-middle attack on 2DES

Suppose $DES_{K_2}^{-1}(C) = DES_{K_1}(M)$ and T_1, \dots, T_N are all possible DES keys, where $N = 2^{56}$.



Attack idea:

- Build L,R tables
- Find i, j s.t. $L[i] = R[j]$
- Guess that $K_1 K_2 = T_i T_j$

Meet-in-the-middle attack on 2DES

Let $T_1, \dots, T_{2^{56}}$ denote an enumeration of DES keys.

$MinM_{2DES}(M_1, C_1)$

for $i = 1, \dots, 2^{56}$ do $L[i] \leftarrow \text{DES}(T_i, M_1)$

for $j = 1, \dots, 2^{56}$ do $R[j] \leftarrow \text{DES}^{-1}(T_j, C_1)$

$S \leftarrow \{ (i, j) : L[i] = R[j] \}$

Pick some $(l, r) \in S$ and return $T_l \parallel T_r$

Attack takes about 2^{57} DES/DES⁻¹ computations.

Interesting, but not practical.

Block ciphers

$$3DES3 : \{0, 1\}^{168} \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$$

$$3DES2 : \{0, 1\}^{112} \times \{0, 1\}^{64} \rightarrow \{0, 1\}^{64}$$

are defined by

$$3DES3_{K_1 \parallel K_2 \parallel K_3}(M) = DES_{K_3}(DES_{K_2}^{-1}(DES_{K_1}(M)))$$

$$3DES2_{K_1 \parallel K_2}(M) = DES_{K_2}(DES_{K_1}^{-1}(DES_{K_2}(M)))$$

Meet-in-the-middle attack on **3DES3** reduces its “effective” key length to **112**.

$$DESX_{KK_1K_2}(M) = K_2 \oplus DES_K(K_1 \oplus M)$$

- Key length = $56 + 64 + 64 = 184$
- “effective” key length = 120 due to a 2^{120} time meet-in-middle attack
- No more resistant than DES to linear or differential cryptanalysis

Good practical replacement for DES that has lower computational cost than 2DES or 3DES.

Block size limitation

Later we will see “birthday” attacks that “break” a block cipher $E : \{0, 1\}^k \times \{0, 1\}^\ell \rightarrow \{0, 1\}^\ell$ in time $2^{\ell/2}$

For DES this is $2^{64/2} = 2^{32}$ which is small, and this is **unchanged** for 2DES and 3DES.

Would like a larger block size.

1998: NIST announces competition for a new block cipher

- key length 128
- block length 128
- faster than DES in software

Submissions from all over the world: MARS, Rijndael, Two-Fish, RC6, Serpent, Loki97, Cast-256, Frog, DFC, Magenta, E2, Crypton, HPC, Safer+, Deal

1998: NIST announces competition for a new block cipher

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2001: NIST selects Rijndael to be AES.

```
function AESK(M)
  (K0, ..., K10) ← expand(K)
  s ← M ⊕ K0
  for r = 1 to 10 do
    s ← S(s)
    s ← shift-rows(s)
    if r ≤ 9 then s ← mix-cols(s) fi
    s ← s ⊕ Kr
  end for
  return s
```

- Fewer tables than DES
- Finite field operations

No key-recovery attack better than **EKS** is known, and latter is prohibitive for 128 bit keys.

Adversary A knows $E: \{0,1\}^k \times \{0,1\}^\ell \rightarrow \{0,1\}^\ell$

$T \xleftarrow{\$} \{0,1\}^k$ is the target key.

Given: $(M_1, C_1), \dots, (M_q, C_q)$ where $C_i = E(T, M_i)$ for $i = 1, \dots, q$
and M_1, \dots, M_q are distinct.

Find: T

So far, a block cipher has been viewed as secure if it resists key recovery, namely if there is no efficient way to solve the above problem.

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$$A : \{0, 1\}^{128} \times \{0, 1\}^{128} \rightarrow \{0, 1\}^{128}$$

that is guaranteed to resist key recovery. Would you use it encrypt?

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The cipher is:

$$A_K(M) = M$$

- Impossible to find key from input-output examples, but
- Encryption is insecure because given ciphertext I know plaintext.

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Answer: It tells us that security against key recovery is not, as a block-cipher property, sufficient for security of uses of the block cipher.

As designers and users we want to know what properties of a block cipher give us security when the block cipher is used.

So what is a “good” block cipher?

Possible Properties	Necessary?	Sufficient?
security against key recovery	YES	

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⋮		

We can't define or understand security well via some such (indeterminable) list.

We want a single “master” property of a block cipher that is sufficient to ensure security of common usages of the block cipher.