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Selection of beam orientations in intensity-modulated radiation therapy using single-beam indices and integer programming

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Abstract

While the process of IMRT planning involves optimization of the dose distribution, the procedure for selecting the beam inputs for this process continues to be largely trial-and-error. We have developed an integer programming (IP) optimization method to optimize beam orientation using mean organ-at-risk (MOD) data from single-beam plans. Two test cases were selected in which one organ-at-risk (OAR) and four OARs were simulated, respectively, along with a PTV. Beam orientation space was discretized in 10° increments. For each beam orientation, a single-beam plan without intensity modulation and without constraints on OAR dose was generated and normalized to yield a mean PTV dose of 2 Gy and the corresponding MOD was calculated. The degree of OAR sparing was related to the average OAR MODs resulting from the beam orientations utilized with improvements of up to 10% at some dose levels. On the other hand, OAR DVHs in the IMRT plans were insensitive to beam numbers (in the 6–9 range) for similar average single-beam MODs. These MOD data were input to an IP optimization process, which then selected specified numbers of beam angles as inputs to a treatment planning system. Our results show that sets of beam angles with lower average single-beam MODs produce IMRT plans with better OAR sparing than manually selected beam angles. To optimize beam orientations, weights were assigned to each OAR following MOD input to the IP which was subsequently solved using the branch-and-cut algorithm. Seven-beam orientations obtained from solving the IP were applied to the test case with four OARs and the resulting plan with a dose prescription of 63 Gy was compared with an equi-spaced beam plan. The IP selected beams produced dose–volume improvements of up to 40% for OARs proximal to the PTV. Further improvement in the DVH can be obtained by increasing the weights assigned to these OARs but at the expense of the remaining OARs.

1. Introduction

Intensity-modulated radiation therapy (IMRT) involves the delivery of radiation dose from multiple-beam orientations around the patient in which the radiation beam from each orientation is modulated to deliver a highly conformal dose to the tumour while reducing the dose to surrounding organs-at-risk (OARs). While the process of IMRT plan generation involves inverse planning (in which the planner specifies dose–volume constraints for the tumour and OARs, and the objective function measures deviations from these constraints), the selection of beam orientation continues to be one of trial and error. If the obtained plan is not satisfactory, the planner may adjust the beam orientations (among other variables) iteratively to improve the plan. The quality of the plan therefore depends to some degree on the experience of the planner and the number of iterations involved. However, it is difficult for the planner to determine the suitability or number of beam orientations for a particular clinical case *a priori*. The addition of beam orientation variables to the optimization problem increases its size and complexity, which may result in unacceptable increases in the solution time and may also exhaust available memory on computers. Therefore, limited research effort has been focused on incorporating full beam orientation optimization into the objective function for the entire inverse-planning process.

Experience in clinical situations indicates that there is value to careful selection of beam orientation in IMRT treatment planning. The ability to generate improved dose distributions by carefully selecting appropriate beam orientations has been demonstrated in lung (Das *et al* 2000), prostate (Pickett *et al* 1994, Rowbottom *et al* 1998) and parotid gland (Rowbottom *et al* 2001a, 2001b) tumour IMRT plans. It is generally believed that treatment quality improves with additional beams although the amount of improvement diminishes with increasing beam numbers. However, an increased number of beams increases the treatment time and may introduce potential errors in terms of patient movement and increased discomfort. Additionally, some attention has been focused on the possible impact of prolonged delivery times involved in IMRT treatments on cell survival (Morgan *et al* 2002). In particular, theoretical calculations have suggested a decrease in tumour control probability for treatment times exceeding 20–30 min for prostate tumours (Wang *et al* 2003).

Previous work on the optimization of beam orientation in radiation therapy has involved maximal geometric separation of treatment beams (Sailer *et al* 1993, Das *et al* 1997) applied to simple geometric shapes and later extended to clinical examples. Fourier transform and simulated annealing methods have also been used to select suitable beam portals (Soderstrom and Brahme 1992, 1995, Stein *et al* 1997, Pugachev *et al* 2000). The use of beam's eye-view methods in beam angle selection has been demonstrated (Pugachev and Xing 2001, 2002). In their approach the beam's eye-view projection of the target is divided into many beamlets and for a given beam angle, a score is calculated based on the maximum target dose deliverable from each beamlet without exceeding the OAR tolerances averaged over all voxels traversed by the beamlet.

In this work we describe an effective method for ranking beam orientations based on mean OAR dose (MOD) obtained from a single-beam unmodulated plan. First, we will demonstrate the viability of a single-beam index such as MOD in the selection of beam orientations.

Second we will show the insensitivity of DVHs to the number of beams (for $\geqslant 5$ beams), assuming similar average MODs for the OARs. Third, we will illustrate the implementation of an IP model based on MOD data for beam orientation selection for a complex case (as represented by a case with multiple OARs). We then show how these rankings may be used within a beam orientation optimization framework using integer programming (IP) and the branch-and-cut algorithm (see Nemhauser and Wolsey (1988)). Our approach is similar in concept to the beam's eye-view methods proposed by Pugachev and Xing (Pugachev and Xing 2001, 2002); however we employ a forward calculation of flat beams (unmodulated) which are then input into a IP model that is solved to global optimality subject to user defined constraints. IP has been previously used by us and other investigators in radiation therapy. It was effectively used in the optimization of permanent prostate implants (Lee *et al* 1999, D'Souza *et al* 2001) and has found application in radiosurgery treatment planning (Lee *et al* 2000). Since the branch-and-cut algorithm searches the solution space in a systematic (not probabilistic) manner, the time associated with finding the solution may be unacceptable in a full IMRT optimization problem where the variables are beam orientations, tumour and OAR doses and beamlet weights. However, since we only consider beam orientations in this work, the solution time involved is $\leqslant 5$ s on a 750 MHz CPU.

2. Method

The pelvic and abdominal sections of an anthropomorphic phantom (Alderson Research Laboratories, Inc., Stamford, CT) were CT scanned (Picker 5000, Acqsim, Cleveland, OH). A PTV was simulated in each phantom along with one OAR in the pelvic phantom and four OARs in the abdominal phantom. The PTV was somewhat centrally simulated in the pelvic part of the anatomy and one OAR was simulated in close proximity to the PTV (single-OAR case) in order to mimic a prostate-like case. In the abdominal phantom (multi-OAR case), the PTV was located towards the right of the phantom and four OARs were simulated such that each beam (in 10° increments) passed through at least one OAR in order to mimic a pancreas-like case (where the potential OARs are liver, kidneys, spinal cord and stomach). Figure 1 shows the transverse section of the pelvic and abdominal phantoms with the relative positions of the PTV and OARs.

2.1. Single-beam plan indices

All calculations described in this work were performed in the Corvus, v.4.0 (NOMOS, PA) planning system. The beam orientation space was discretized in increments of 10° from 0° to 350°. An initial treatment plan consisting of an unmodulated single beam was generated for each of the 36 orientations. In each case, the beam was shaped to the beam's eye-view projection of the PTV. A dose of 1.8 Gy was prescribed to the PTV and the plan was normalized so that the mean PTV dose was 2 Gy. This was done because mean dose is proportional to the integral dose delivered to an organ. Therefore, normalizing each single-beam plan to a mean PTV dose of 2 Gy ensures the same energy deposition in the PTV mass. A fractional dose of 1.8 Gy was prescribed since this is the most commonly used dose prescription in our clinic. We do not expect the results presented here to be altered by using a different fractional prescription dose (or normalized mean dose). No intensity modulation was used in the generation of the initial single-beam plan. For each single-beam plan, the minimum, mean and maximum OAR doses were obtained. In addition, the minimum and maximum PTV doses were also determined.

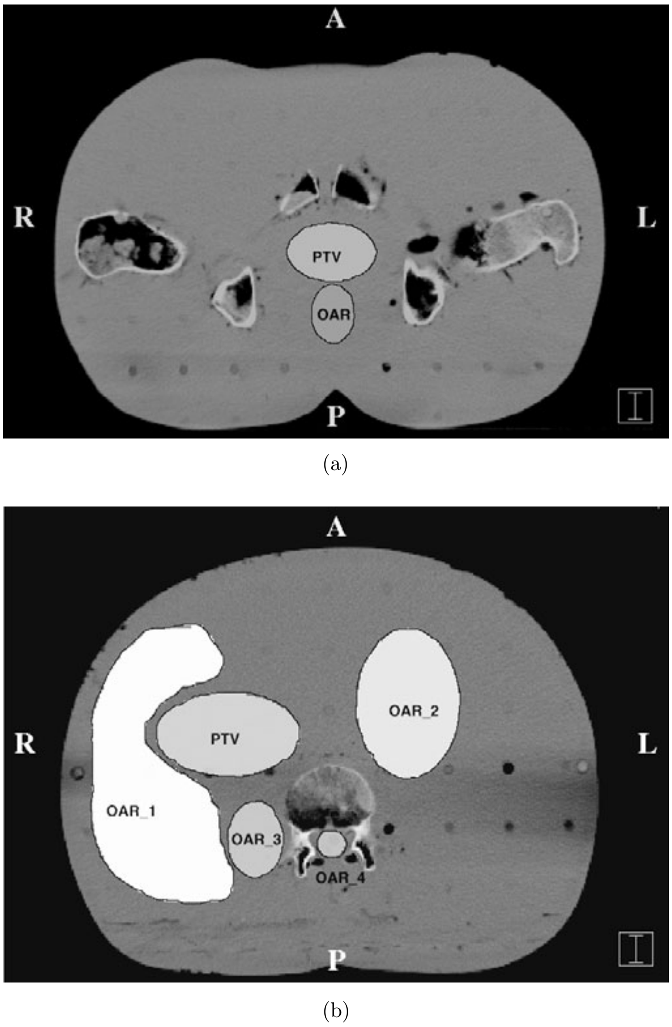


Figure 1. Transverse CT images of (a) the pelvic anthropomorphic phantom with one OAR and (b) the abdominal anthropomorphic phantom with four OARs.

Figure 2 shows the minimum, mean and maximum OAR doses for the single-beam plan as a function of beam orientation for the single-OAR case. Figures 3(a) and (b) show the minimum, mean and maximum OAR doses for OAR1 and OAR3 in the multi-OAR case. We only show the graphs for two OARs for purposes of simplicity.

We propose and will show that MOD derived from the single-beam plan is a reliable predictor of 'good' beam orientations (in terms of improved OAR sparing), i.e. beam orientations with lower MODs, produce better OAR sparing. Henceforth, we will use MOD to denote the mean organ-at-risk dose obtained from a single-beam plan in which the PTV mean dose is normalized to 2 Gy. While we have shown the minimum and maximum OAR doses as a function of beam orientation for single-beam plans, we do not use them in the selections of beams. It is observed that in general, the shape of maximum and minimum OAR dose curves as a function of beam angle somewhat resemble the shape of the mean OAR dose curve. From

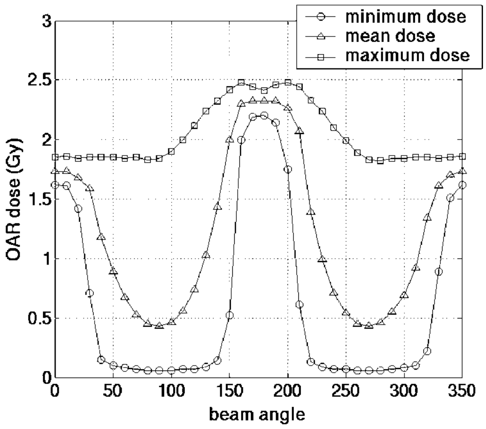
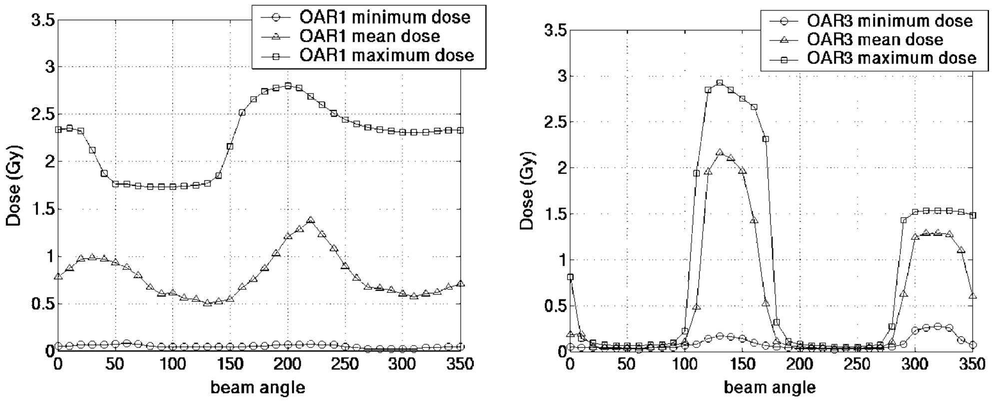


Figure 2. Single-OAR case: minimum, mean and maximum OAR doses resulting from single-beam plans in which the mean PTV dose is normalized to 2 Gy.



(a) (b)

Figure 3. Multi-OAR case: minimum, mean and maximum doses for (a) OAR1 and (b) OAR3 resulting from single-beam plans in which the mean PTV dose was normalized to 2 Gy.

figure 2, MOD for the single OAR case is relatively low for beam orientations 45°–130° and 230°–315°. Similar ranges can be obtained for the respective OARs in the multi-OAR case. We note the relatively suitable beam orientations in figures 3(a) and (b) for OAR1 (70°–170°, 260°–0°) and OAR3 (0°–100°, 190°–280°) in the multi-OAR case. Beam orientation resulting in relatively lower MOD can likewise be determined from the appropriate figures for OAR2 and OAR4. It is clear from figures 3(a) and (b) that ‘good’ beam orientations for one OAR may not be desirable for another OAR as is often the case in many clinical situations. We present an IP formulation to deal with such situations.

2.2. *Treatment plan metric—MOD*

Treatment plans using five, six, seven and nine equi-spaced intensity-modulated beams were generated using Corvus, v.4.0. The same constraints and optimization algorithm (simulated annealing) were used in all plans (using Corvus, v.4.0) so that beam orientation was the only variable between plans. Doses of 75.6 Gy and 63 Gy were prescribed for the single- and multi-OAR cases respectively. All plans were normalized so that 97% of the PTV received the prescribed dose.

The validity of MOD as a reliable indicator of desirable beam orientations was tested by generating plans with the same beam number but different average MOD. The average MOD was defined as the mean of the MODs for the beam orientations in the plan. For example, in the single-OAR case, consider three six-beam plans with beam orientations of 30, 80, 135, 225, 280 and 330 (plan 1), 40, 80, 125, 235, 280 and 320 (plan 2) and 50, 80, 120, 240, 280 and 310 (plan 3). The average MOD for the OAR was 1.088 Gy, 0.862 Gy and 0.695 Gy for plan 1, plan 2 and plan 3, respectively. In the multi-OAR case, two six-beam plans were generated with beam orientations of 30, 70, 180, 220, 260 and 350 (mplan 1) and 0, 80, 120, 160, 280 and 320 (mplan 2). For purposes of brevity, we describe the differences in these plans in terms of OAR1 and OAR3. The average MOD for OAR1 was 0.918 Gy and 0.653 Gy from mplan 1 and mplan 2, respectively. The average MOD for OAR3 was 0.803 Gy and 0.932 Gy from mplan 1 and mplan 2, respectively. The quality of these plans from a DVH perspective will be considered in detail below and contrasted with plans generated via an optimization model.

The impact of the number of beams is also of concern in this research and was tested by manually generating plans with approximately the same average MOD but differing number of beams. The beams were selected such that there was a minimum spacing constraint: 40° for five and six beams, 30° for seven beams and 20° for nine beams. (The same spacing constraints are used in the IP model below.) The choices for the minimum beam spacing were based on clinical experience. Five-, six-, seven- and nine-beam plans with average MODs of 0.680 Gy, 0.693 Gy, 0.677 Gy and 0.673 Gy, respectively were devised for the single-OAR case. The average MODs in this case were within 3% of one another. For the multi-OAR case the impact of beam number was illustrated by generating 5–9 beam plans with average MODs within 3% of one another for OAR1 and OAR3. (We do not consider the MODs for OAR2 and OAR4 for purposes of simplicity and because the purpose of this exercise is to demonstrate the impact of number of beams on the respective OARs in a multi-OAR setting.) For example, the average MOD for OAR1 was 0.704 Gy, 0.697 Gy, 0.706 Gy and 0.694 Gy for the five-, six-, seven- and nine-beam plans, respectively. The average MOD for OAR3 was 0.472 Gy, 0.475 Gy, 0.478 Gy and 0.476 Gy for the five-, six-, seven- and nine-beam plans, respectively with the same 5–9 beams.

We also investigated if a higher number of beams, or a lower average MOD, results in 'better' plans by comparing plans with differing number of beams in which the plan with a lower beam number had a lower average MOD. Five-beam and nine-beam plans (beam orientations were manually selected) were generated for the single-OAR case. The beam orientations used in the five-beam plan were 45, 110, 240, 270 and 310 resulting in an average MOD of 0.732 Gy. The nine-beam (equi-spaced) plan consisted of beams from 0, 40, 80, 120, 160, 200, 240, 280 and 320 with an average MOD of 1.241 Gy. We propose that MOD is a more reliable predictor of OAR sparing than number of beams (i.e. plans with beam orientations resulting in a lower average MOD produce better OAR sparing independent of beam number).

2.3. Selection of beam orientations via integer programming

Using the average MOD from the single-beam plans (which, as will be shown, is a reliable indicator for beam orientation quality), we can automate the process of beam selection by formulating the problem as a IP program, including in the problem beam-angle spacing constraints and additional constraints that are appropriate for the treatment planning system for which the beam angles will serve as input (our clinical experience has shown that directly opposed beams yield undesirable hot spots in the Corvus planning system). The number of beams to be used in the plan is also input as one of the constraints. Since the IP model is solved rapidly, several different plans with varying numbers of beams are easily generated. The objective function is the weighted average MOD over all the beams. While the beam orientation space was discretized in increments of 10° to generate single-beam plans within a reasonable amount of time, MOD was estimated at finer intervals of 5° by interpolating the MODs obtained at 10° increments. The mathematical representation of the problem with these 5° increments is as follows:

$$\text{minimize} \sum_{\text{OAR}} \alpha_{\text{OAR}} \left(\sum_{\theta} w_\theta \text{MOD}_{\theta,\text{OAR}} \right) \tag{1}$$

subject to

$$\sum_{\theta} w_\theta = n \tag{2}$$

$$w_\theta + w_{\theta+\delta} + w_{\theta+2\delta} + w_{\theta+(m-1)\delta} \leqslant 1 \qquad \text{for} \quad \theta = 0, 5, 10, \ldots, 355 \tag{3}$$

$$w_\theta + w_{\theta+k} \leqslant 1 \qquad \text{for} \quad k = 180 - \delta, 180, 180 + \delta \qquad \text{and for} \quad \theta = 0, 5, 10, \ldots, 355 \tag{4}$$

$$w_\theta = 0, 1 \tag{5}$$

where α_{OAR} is the weight associated with an OAR in the multi-objective cost function, θ is the beam orientation index, w_θ is the binary selection variable for a beam at angle θ, $\text{MOD}_{\theta,\text{OAR}}$ is the MOD for an OAR from a single beam at angle θ, n is the number of beams selected, δ is the spacing between adjacent beams (in this case, 5°) and $m\delta$ is the minimum geometric spacing allowed between beams. The inequality constraints specify the minimum spacing between adjacent beams (equation (3)) and the exclusion of opposed (or nearly opposed) beams (equation (4)). The minimum spacing constraint was imposed to prevent clustering of beams. Directly opposed (or nearly opposed) beams, i.e. beams that are 175–185° apart were excluded, since, in our experience, such beams lead to undesirable plans in the Corvus planning system. This integer programming problem can be solved using the branch-and-cut algorithm (BC) as implemented in the CPLEX (ILOG, Inc., CA) software.

Details of the BC algorithm can be found elsewhere (Nemhauser and Wolsey 1988) but a brief description of its application here is provided. The BC algorithm recursively partitions the feasible solution set. It makes use of the relaxed optimality condition (ROC) and the divide-and-conquer principle. The ROC considers an original problem

$$\begin{aligned} &\min\ f(x) \\ &\text{subject to } x \in S \end{aligned}$$

and a relaxed problem with feasible set T containing S:

$$\begin{aligned} &\min\ f(x) \\ &\text{subject to } x \in T. \end{aligned}$$

In an IP program the relaxation corresponds to allowing binary variables to assume real continuous values in [0, 1]. If x solves the relaxed problem and x is in S (i.e. it is as integer), then x solves the original problem. Otherwise, branching is required and the algorithm fixes a relaxed variable to 0 on the left branch, and 1 on the right branch, i.e. the solution path considers two routes. The relaxed problem arising from multiple such branches is

$$\begin{aligned} &\min\ f(x) \\ &\text{subject to } x \text{ is in the union of the sets } T_1, T_2, \ldots, T_k \end{aligned}$$

where the union of the sets $T_1, T_2, \ldots, T_k$ contains S. Each such relaxed problem corresponds to a set of linear programs each of which is solved by the simplex method. Certain branches (subproblems) can be eliminated from further exploration if the relaxed solution is worse than the solution already obtained from a different branch.

For the beam orientation problem discussed here, the weight assigned to each beam orientation (in 5° increments) is a binary variable. Branching begins when the algorithm fixes a beam orientation variable at 0 for the left branch and 1 for the right branch. The objective function for the LP relaxation at each branch is optimized, and this provides the lower bound for the optimal value of the IP problem at that branch. This process is continued in a systematic manner until all the beam orientation weight variables have binary values at the leaf node with the best objective value.

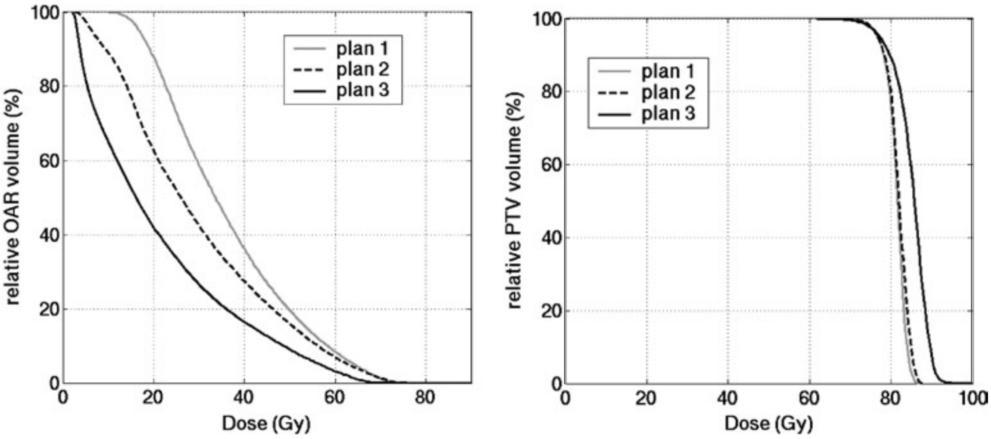
The integer model was tested on the multi-OAR case, to investigate the significance of weighted average MODs with different weights for different OARs in contrast to the single-OAR case where the objective function is the average MOD for one OAR. It was considered redundant to test the IP model on the single-OAR case since we have shown that the degree of OAR sparing is directly related to the average MOD of the beam orientations used. OAR weights (α_{OAR}) of 20, 3, 3 and 3 were assigned to OAR1, OAR2, OAR3 and OAR4, respectively. These weights were assigned arbitrarily but can be adjusted depending on the relative importance of the critical structures. We generated six- and seven-beam plans by using the IP model described above selecting the required spacing of beams as 40° for six beams and 30° for seven beams. The beam orientations resulting from the IP formulation were 50°, 80°, 110°, 280°, 310° and 350° for the six-beam plan and 50°, 80°, 110°, 250°, 280°, 310° and 350° for the seven-beam plan. To compare the IP model with manually generated plans, we generated six- and seven-beam plans by examining the beam's eye-view projection of the PTV and its positional relation to the projection of the OARs. The beam orientations arrived at by manual intervention were 35°, 70°, 105°, 145°, 310° and 350° for the six-beam plan. Seven equi-spaced beam orientations (0°, 51°, 103°, . . . , 308°) were used for the seven-beam plan.

These beam orientations were input into Corvus, with the prescription dose to the PTV set at 63 Gy, and the dose distribution was then optimized via the simulated annealing algorithm (available in Corvus). The constraints on the OARs were specified as follows: no more than 20% (volume) of OAR1 was allowed to exceed 25 Gy, 10% of OAR2 was allowed to exceed 25 Gy, 10% of OAR3 was allowed to exceed 20 Gy and 20% of OAR4 was allowed to exceed 30 Gy. Minimum doses of 5 Gy for all OARs and maximum doses of 55 Gy, 25 Gy, 25 Gy and 30 Gy were specified for OAR1, OAR2, OAR3 and OAR4, respectively. The plans were normalized so that at least 97% of the PTV received the prescribed dose. It should be noted that beam selection was conducted using the IP but the dose optimization (once beam orientations were selected) was performed in Corvus.

3. Results

We describe three results: (1) the utility of MODs from single-beam plans in the selection of beam orientations, (2) the insensitivity of DVHs to the number of beams (for $\geqslant 5$ beams), assuming similar average MOD for the OARs, and (3) the implementation of an IP model based on MOD data for beam orientation selection for a complex case (as represented by the multi-OAR case).

We describe in some detail the significance of average MOD for single-OAR case in which six-beam orientations were used. Figure 4(a) shows the cumulative OAR DVH for the three six-beam plans. The degree of sparing of the OAR was directly related to these



(a) (b)

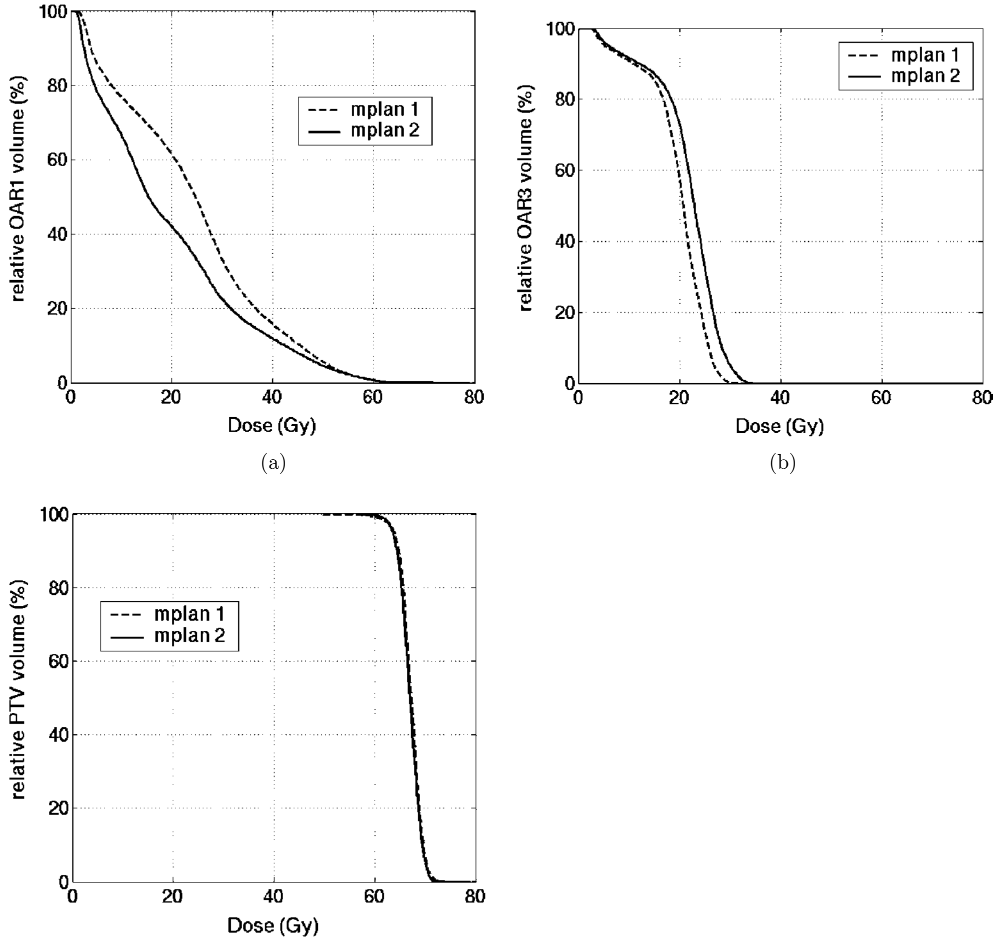
Figure 4. Single-OAR case: dose–volume histograms for the (a) OAR and the (b) PTV resulting from three plans each with six beams and with average MODs of 1.088 Gy (plan 1), 0.862 Gy (plan 2) and 0.695 Gy (plan 3).

average MODs since the use of beam orientations with lower OAR mean doses in the single-beam plans led to superior DVHs. The OAR volumes receiving at least 45 Gy were 26.9%, 21.3% and 12.6% for the three plans in which the average MODs for the beam orientations used were 1.088 Gy, 0.862 Gy and 0.695 Gy. Similar reduction in OAR volume at other doses were observed. The PTV DVHs provide the same target coverage (97% of PTV received the precribed dose) since that is how the plans were normalized. These DVHs show differences in the dose homogeneity with plan 3 resulting in the greatest inhomogeneity (see figure 4(b)).

Similar results were obtained in the multi-OAR case. We only highlight the results as applied to OAR1 and OAR3 for purposes of brevity. Figure 5 shows the DVHs resulting from two six-beam manual plans (mplan 1 and mplan 2) for which the average MODs for both OAR1 and OAR3 differed significantly. The volume of OAR1 receiving at least 35 Gy was 22.7% and 16.0% for mplan 1 and mplan 2, respectively. The corresponding average MODs were 0.918 Gy and 0.653 Gy, respectively. The volume of OAR3 receiving 25 Gy was 16.3% and 33.8%, respectively. The corresponding average MODs were 0.803 Gy and 0.932 Gy, respectively. Unlike the single OAR case, there is almost no difference in the PTV DVHs for mplan 1 and mplan 2.

Figure 6(a) shows the insensitivity of the OAR DVH to beam number for beam orientations that result in similar (within 2%) average MODs for the single OAR case. The average MODs for selected sets of five, six, seven and nine beams were 0.680 Gy, 0.695 Gy, 0.677 Gy and 0.673 Gy, respectively. The DVHs for six or more beams seem to converge. For example, the volume of the OAR receiving 45 Gy was 14.2%, 12.6%, 13.2% and 11.7% from the five-, six-, seven- and nine-beam plans, respectively. The difference in the PTV DVHs is not significant with a difference of <2 Gy between the maximum doses from each of the plans (see figure 6(b)).

A similar result is observed in the multi-OAR case. Again, we only highlight the results for OAR1 and OAR3. Figure 7 shows the DVHs for OAR1 and OAR3 as a function of beam number for beam orientations with average MODs within 2% of one another. The volume of OAR1 receiving 35 Gy is 16.3%, 16.1%, 14.1% and 15.8% for the five-, six-, seven-



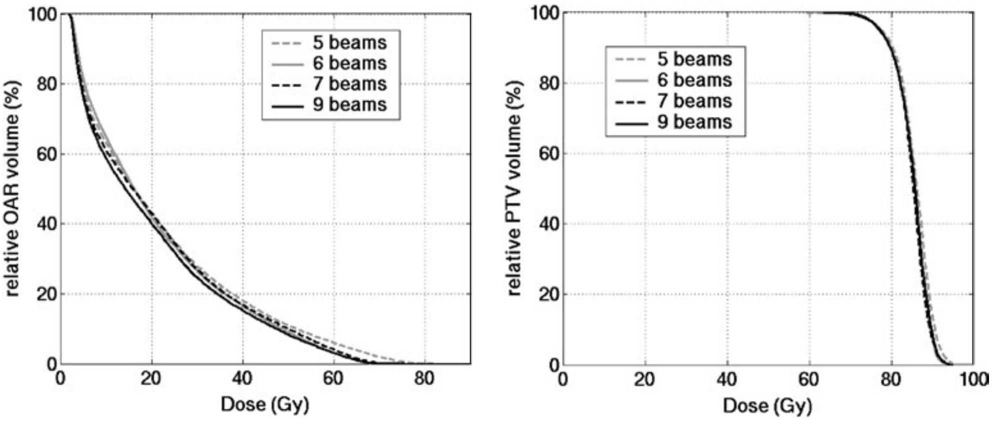
(a) (b)

(c)

Figure 5. Multi-OAR case: dose–volume histograms for (a) OAR1 and (b) OAR3 and (c) PTV for two six-beam plans with average MODs of 0.918 Gy (mplan 1) and 0.653 Gy (mplan 2), respectively for OAR1 and 0.803 Gy (mplan 1) and 0.932 Gy (mplan 2), respectively for OAR3.

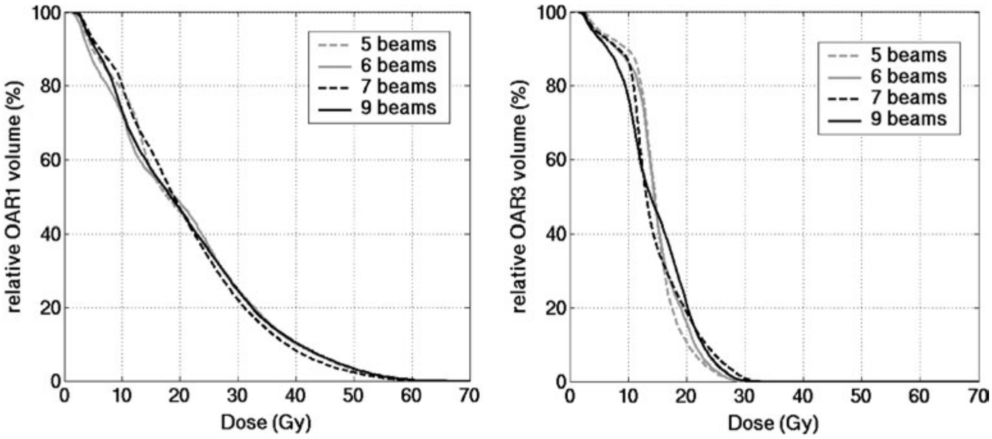
and nine-beam plans, respectively. The volume of OAR3 receiving 25 Gy is 2.2%, 2.8%, 6.8% and 4.9% for the corresponding plans. The average MODs for OAR1 were 0.704 Gy, 0.697 Gy, 0.706 Gy and 0.694 Gy and for OAR3 were 0.472 Gy, 0.475 Gy, 0.478 Gy and 0.476 Gy from the five-, six-, seven- and nine-beam plans, respectively.

Figure 8(a) shows the comparison of the OAR DVH for 5 (45°, 110°, 240°, 270° and 310° resulting in an average MOD of 0.732 Gy) and nine-beam (0°, 40°, 80°, 120°, 160°, 200°, 240°, 280° and 320°) plans described (in section 2) in which the five beams have an average MOD of 0.680 Gy and the nine beams have an average MOD of 1.241 Gy. The DVHs show the clear superiority of the five-beam plan compared with the nine-beam plan. For example, the volume of the OAR exceeding 45 Gy is 14.2% and 25.8% for the five- and nine-beam plans, respectively. While both plans were normalized so that $\geqslant$97% of the PTV received the prescribed dose of 75.6 Gy, the five-beam plan results in a higher maximum dose (93.9 Gy) than the nine-beam plan (88.3 Gy).



(a) (b)

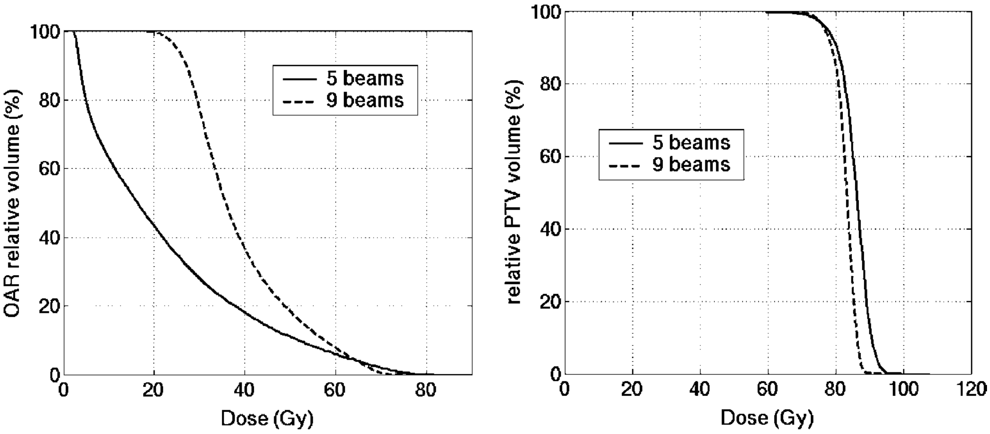
Figure 6. Single-OAR case: dose–volume histograms for (a) OAR and (b) PTV as a function of beam number. The beam orientations for these plans were such that the average MODs were within 3% of one another. The average MODs for the five-, six-, seven- and nine-beam plans were 0.680 Gy, 0.693 Gy, 0.677 Gy and 0.673 Gy, respectively.



(a) (b)

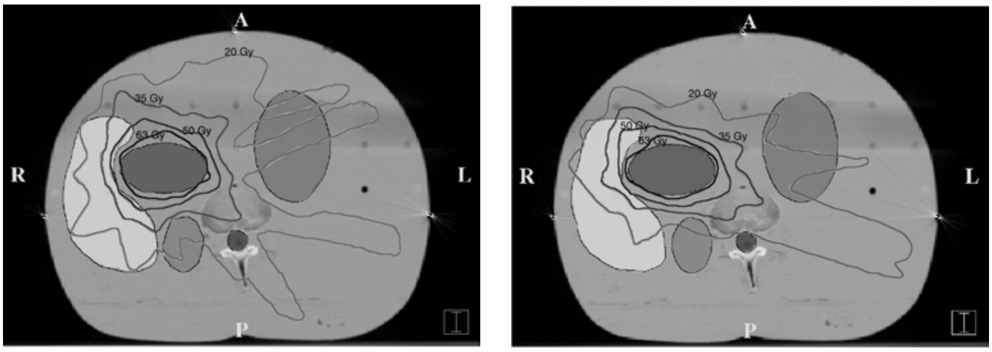
Figure 7. Multi-OAR case: dose–volume histograms for (a) OAR1 and (b) OAR3 as a function of beam number. The beam orientations for these plans were such that the average MODs were within 3% of one another. The average MODs from the five-, six-, seven- and nine-beam plans were 0.704 Gy, 0.697 Gy, 0.706 Gy and 0.694 Gy, respectively for OAR1 and 0.472 Gy, 0.475 Gy, 0.478 Gy and 0.476 Gy, respectively for OAR3.

Finally, we compare the results from the implementation of the IP model for beam selection using the branch-and-cut algorithm with manually generated plans. For the six-field plans, the manually selected (by an experienced treatment planner) beam orientations resulted in average MODs of 0.695 Gy, 0.452 Gy, 0.853 Gy and 0.715 Gy for OAR1, OAR2, OAR3 and OAR4, respectively. Correspondingly, the average MODs for the beam orientations using the IP approach were 0.683 Gy, 0.583 Gy, 0.463 Gy and 0.428 Gy for the respective OARs. Our weighting scheme produced average MODs that were lower than those of the manual plan for all OARs except OAR2. The average MOD for OAR2 may be lowered by increasing the



(a) (b)

Figure 8. Comparison of a five- and nine-beam plan in the single-OAR case showing that beam orientations with lower average MODs result in better OAR sparing independent of beam number. The five- and nine-beam plans had average MODs of 0.680 Gy and 1.241 Gy, respectively.

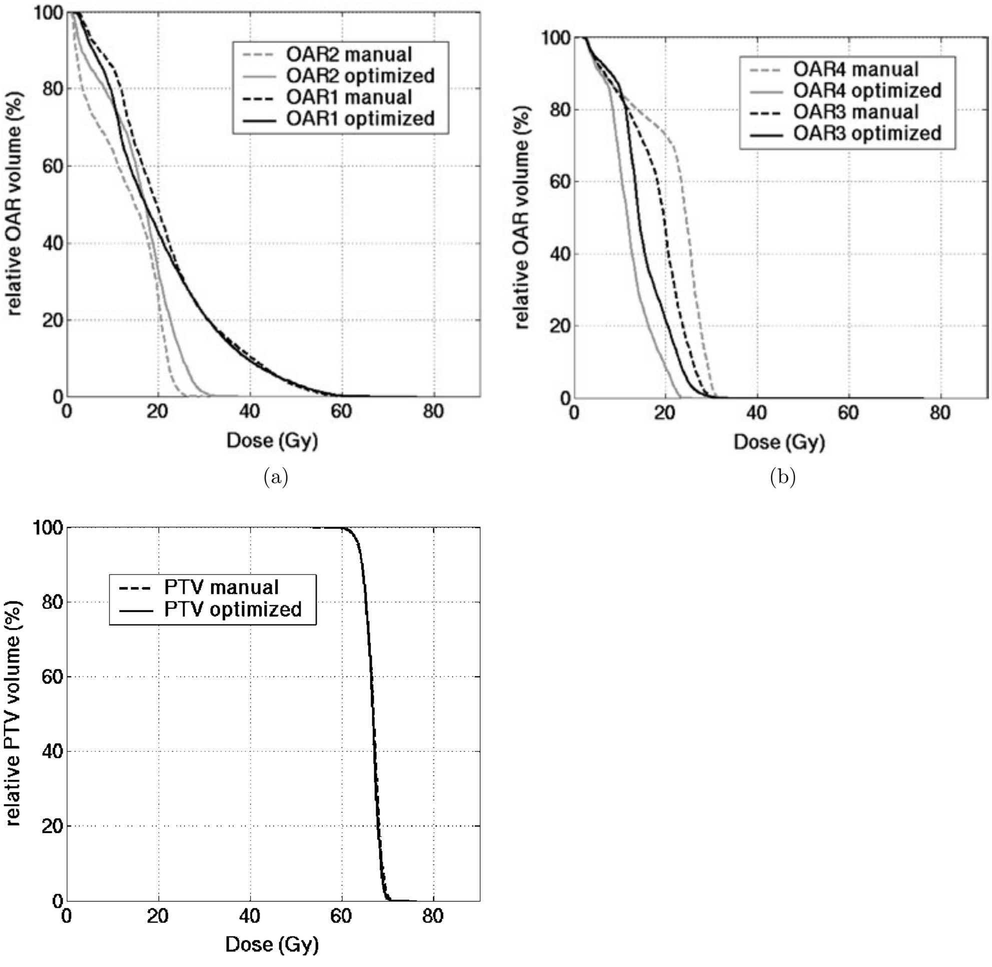


(a) (b)

Figure 9. Isodose comparison between treatment plans resulting from beam orientations (a) selected manually and (b) with the IP approach. In each case six beams were used. Shown are the 63 Gy, 50 Gy, 35 Gy and 20 Gy isodose lines.

weight assigned to it in the IP model in which case a different set of beam orientations would be generated. However, we decided against this since the lowering of the average MOD for OAR2 would occur at the expense of the average MOD for the other OARs.

A comparison of the isodose distribution between the six-beam orientations using the manual and IP approach is shown in figure 9. The 63 Gy, 50 Gy, 35 Gy and 20 Gy lines are shown. Figure 10 shows the DVH comparison for the OARs and PTV between the optimized plans in which the six-beam orientations were selected manually and versus using our approach. The volume of OAR1 exceeding 25 Gy was 31.2% and 30.8% with the manual approach and the IP approach, respectively. The lack of difference is not surprising due to the proximity of the average MOD for the two-beam orientation sets. On the other hand, the volume of OAR3 exceeding 20 Gy was 45.8% and 21.2% via the two methods with the IP approach of beam selection producing more than a 20% improvement in sparing (see figure 10(b)). The PTV DVHs for both beam orientation sets is almost identical (see figure 10(c)).

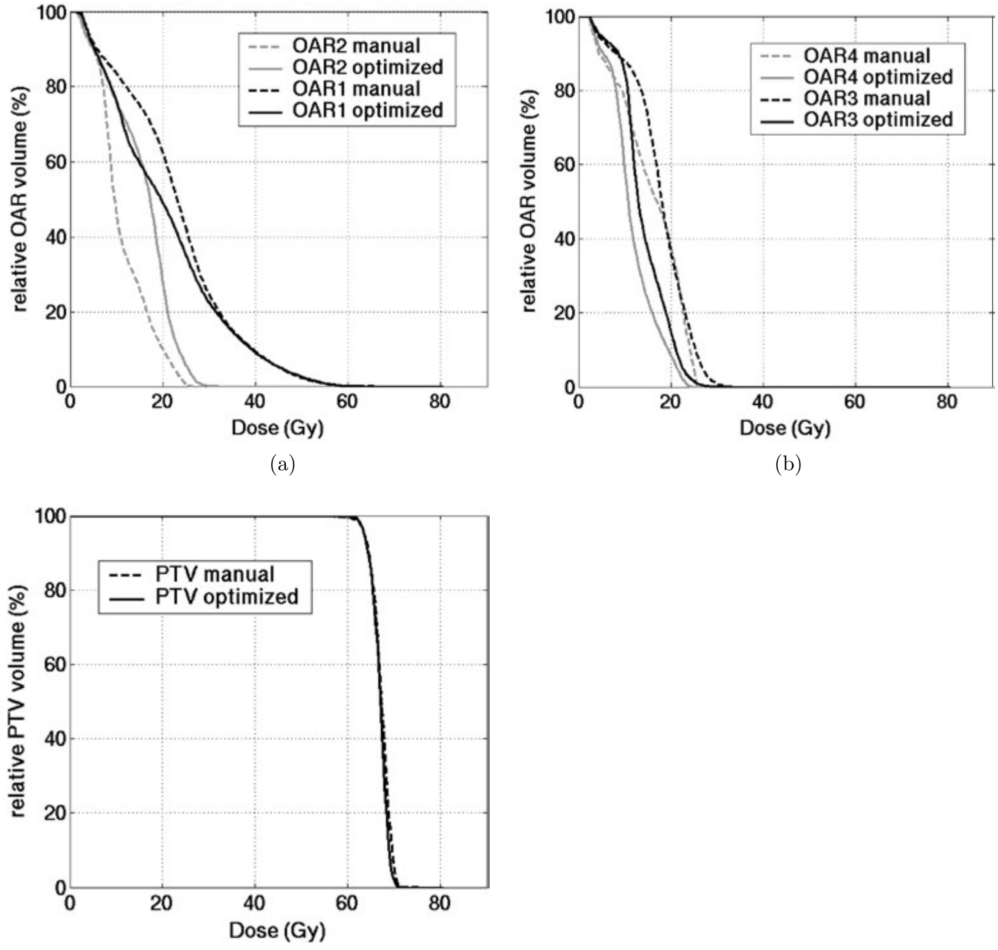


(a) (b)

(c)

Figure 10. A DVH comparison between the optimized six-field treatment plans generated using beams arrived at manually and using the IP model for (a) OAR1 and OAR2, (b) OAR3 and OAR4 and (c) PTV. The volume of OAR1 exceeding 25 Gy was 31.2% and 30.8% with the manual approach and the IP approach, respectively. The volume of OAR3 exceeding 20 Gy was 45.8% and 21.2%, respectively with manual beam selection and the IP approach.

For the seven-field optimized plans, the manual beam orientation set resulted in average MODs of 0.790 Gy, 0.359 Gy, 0.816 Gy and 0.504 Gy for OAR1, 0AR2, OAR3 and OAR4, respectively. The corresponding average MODs for the seven beams selected using the IP approach were 0.713 Gy, 0.630 Gy, 0.404 Gy and 0.374 Gy. Figure 11 shows a comparison of the OAR and PTV DVHs for the seven-field plans using the two different beam orientation sets. The volume of OAR1 exceeding 25 Gy was 42.2% and 35.4% for the manually selected beams and IP model beams, respectively. The volume of OAR3 exceeding 20 Gy was 37.1% and 15.0% for the two beam orientation sets with the IP approach again generating beam orientations resulting in much better OAR sparing (for all OARs except OAR2). The PTV DVHs for the two beam orientation sets are very similar (see figure 11(c)).



(a) (b)

(c)

Figure 11. A comparison between the optimized seven-field treatment plans generated using beams arrived at manually and using the IP model for (a) OAR1 and OAR2, (b) OAR3 and OAR4 and (c) PTV. The volume of OAR1 exceeding 25 Gy was 42.2% and 35.4% for the manually selected beams and IP model beams, respectively. The volume of OAR3 exceeding 20 Gy was 37.1% and 15.0% for the manually selected beams and IP model beams, respectively.

4. Discussion

We have presented a simple but effective strategy for selecting beam orientations in IMRT treatment planning based on a single-beam dosimetric index (mean organ dose to organs-at-risk for single-beam plans normalized to deliver 2 Gy to the PTV). Our strategy indicates that there is a correlation between the ranking of unmodulated single beams and the utility of those beam orientations within a set of beams in a full IMRT optimization process. It was found that beam orientations that produced a lower MOD for a particular OAR also resulted in better sparing for the same OAR in an IMRT plan. The degree of OAR sparing as a whole in an IMRT plan was directly related to the average MOD for the beam orientation set selected for delivery. The predictive capability of average MOD was relatively independent of the number of beams for 5 or more beams. Other investigators have demonstrated similar results with no improvement in OAR sparing beyond seven–nine beams (Bortfeld and Schlegel 1993, Soderstrom and Brahme 1995, Crooks *et al* 2002).

In clinical situations we are often confronted with cases in which several OARs need to be simultaneously spared. We have provided an example of a multi-OAR case in this paper in which we have designated weights to each OAR in the IP model to derive a composite dosimetric index that takes into account all OARs. The IP model takes into account this composite objective as well as constraints on beam spacing and opposing beams. Thus, the objective function is a compromise between all OARs. The beam orientations generated by the IP are not necessarily the 'best' beam orientations for any particular OAR. They simply reflect the 'best' beam orientations given the particular weighting scheme. By increasing the relative weight assigned to an OAR, one can further improve the sparing for that OAR. To find the most suitable beam orientations for an OAR, one can assign a zero weight to the other OARs in the objective function of the IP model. In our example of the multi-OAR case, the DVHs for OAR1, OAR3 and OAR4 were superior for the optimized approach in comparison with the manual beam selection set. However, the manual beam selection set produced a superior DVH for OAR2. The results were in agreement with the average MOD for each beam orientation set since the IP approach produced lower average MODs for OAR1, OAR3 and OAR4 and the manual beam orientation set resulted in a lower average MOD for OAR2. If desired, we could produce a better DVH for OAR2 by simply increasing the weight assigned to it in the IP model. This improvement would come at the expense of some deterioration in the sparing of the rest of the OARs. The overall process demonstrates the flexibility of the IP model.

The limitation in the approach presented here is that it finds beam orientations which as a set yield the lowest weighted average MOD subject to beam spacing and beam opposition constraints. The average MOD for an OAR is proportional to the area under the DVH curve, i.e. the integral OAR dose (D'Souza and Rosen 2003). Hence our approach seeks to reduce the area under the DVH curve for an OAR of interest. The area under the DVH curve for an OAR may be minimized by selecting beam orientations that result in the lowest average MOD for that OAR. However, while our approach considers the general nature of the DVH curve, it does not consider specific dose–volume constraints. For example, if a clinician requires that $<20\%$ of an OAR receive more less than a certain dose, our current approach cannot handle this constraint from a beam selection perspective. Our research at present is focusing on incorporating specific dose–volume constraints into the IP model for beam selection. Further, we are investigating the (in)variance of a particular OAR integral dose (area under the DVH curve) for the 'best' beam orientations (for that OAR) under the influence of different dose–volume constraints.

Our approach can be extended to include non-coplanar fields as well as different collimator angles. Currently, the beam orientation space is discretized in 10° increments with an interpolation every 5° which results in 72 beam orientations. If we assume that the couch can rotate 180° and if the couch rotation space is discretized in 5° increments, the search space would include a total of 2592 beam orientations. Further if we include collimator rotations (also in 5° increments), it would result in 36 collimator positions for every beam orientation (only 180° of collimator rotation need to be considered since the remaining 180° are redundant). This translates into a total of 93 312 possible selections. It would be prohibitive to employ such a large number of selections in a full IMRT optimization process. However, incorporating these selections in our approach outside of the full IMRT dose optimization may be feasible.

5. Conclusion

In summary, we have presented a simple but effective approach to beam-orientation selection in IMRT treatment planning based on unmodulated single-beam characteristics (i.e. optimizing beam intensity profiles). This approach can be used by a planner with minimal experience to generate treatment plans of good quality. It eliminates the iterative process that is necessary in most treatment plans in which beam orientations are subsequently adjusted. This method can easily be implemented as an add-on to an existing planning system. The IP model is flexible so that the user may specify the number of beams as well as the weighting assigned to each OAR in a multi-OAR setting and other constraints such as beam spacing and opposition constraints. The user may also employ the average MOD as an crude index when evaluating the sparing of a particular OAR between rival beam orientation sets (this requires only a few seconds of computation) without carrying out a full IMRT optimization (requiring several minutes). It also encourages the use of a few well-chosen portals which are deemed suitable for the case in question rather than a large number of equi-spaced beams (typically used in many clinical scenarios). The selection of few good beam orientations (5–7) will decrease the total delivery time, which may reduce patient discomfort and decrease patient motion errors. We are currently implementing this approach on clinical treatment plans to evaluate its efficacy.

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