MATH 521 L1 22/09/08

Real number: Decimal expansion, Line segment (geometric)

Decimal expansion → Good for arithmetic → $(\times, +)$ → special role 10

Line segment (geometric) → No special 10 → arithmetic hard → $+$: Concat line → $\times$: area

Definition Real numbers are a complete ordered field

Notational Preliminaries

A set is a collection of objects, called elements

e.g. $\mathbb{Z}$ $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$ set of integers

$\mathbb{Q}$ set of rational numbers

$\{US\ States\}$ $|\{US\ States\}| = 50$

$\mathbb{R}$ set of real numbers

We say A is a subset of B $(A \subset B)$ if every element of A is an element of B (some people would write $A \subseteq B$)

Note that $A \subset A$

If $A \subset B$ but B is not equal to A, we say A is a proper subset of B

Another way to write $A \subset B$ is $\forall x \in A,\ x \in B$

$\exists x \in A,\ x \in B$ Alternatively, $A \cap B \neq \emptyset$

$\exists x \quad x \in A \cap B$

If A, B are sets the Cartesian product $A \times B$ is the set of ordered pairs (a, b) $a \in A$, $b \in B$

$A \times A = \{(a_1, a_2)\}$ $a_1, a_2 \in A$

$A \times A$ often called A^2 might be different elements of $A \times A$

What is $A \times A^2$?

An element of $A \times A^2$ is ordered pair (a, p) $\quad a \in A$, $p \in A^2$

i.e. the elements are of the form $p = (a', a'')$

$(a, (a', a''))$

What about $A^3 = \{(a, a', a'')\}$?

Functions

Def A, B sets. A Function F from A to B is a subset $F \subset A \times B$ s.t.

- If $(a,b) \in F$ and $(a,b') \in F$ then $b = b'$
- $\forall a \; \exists b \quad (a,b) \in F$

When $(a,b) \in F$, we write $F(a) = b$

We write $F: A \to B$ $\quad A \xrightarrow{F} B$

(one-to-one)

Def We say $f: A \to B$ is injective if

$f(a) = f(a') \Rightarrow a = a'$

is surjective if

$\forall b \in B, \; \exists a \in A, \; F(a) = b$

List of functions

$f: \mathbb{Q} \to \mathbb{Q}$	$f(x) = x$	$id_{\mathbb{Q}}$	I, S
$f: \mathbb{Q} \to \mathbb{Q}$	$f(x) = x^2$		
$f: \mathbb{Q}_{>0} \to \mathbb{Q}$	$f(x) = x^2$		I
$f: \mathbb{Q} \to \mathbb{Q}$	$f(x) = 2x$		I, S

$f: \mathbb{Q} \to \{0,1\}$ $\quad f(x) = \begin{cases} 1 & \text{if the denominator of } x \text{ in lowest terms is even} \\ 0 & \text{(S) is odd} \end{cases}$

C: US States $\to$ Cities $\quad C(\text{State}) = \text{its capital}$ $\quad$ I

S: Cities $\to$ states $\quad S(\text{city}) = \text{state it is in}$ $\quad$ S