1. Search algorithms:

a) Using depth-first search, the algorithm will scan the states in the following order: A, B, C, H, I, J, E, N, S, T, Y, X, W, V, U, Q, L, **M**, where it reaches the goal state. The solution path is A, B, C, H, **M**.

b) Looking at the entire maze, we will find a solution at depth 5.
Tree 1: A. No goal state found.
Tree 2: A, B, F. No goal state found.
Tree 3: A, B, C, F. No goal state found.
Tree 4: A, B, C, H, F. No goal state found.
Tree 5: A, B, C, H, I, **M**, F. Goal state found!

c) If $h(n)$ is defined as the Manhattan distance, then it is admissable, as it is the optimistic guess of the distance to the goal, assuming that there are no obstacles in the way. There are, in fact, obstacles in the way, so $h(n)$ will predict a distance that is less than or equal to the actual distance to the goal ($h^*(n) \geq h(n)$).

d) Let's use A search using $h(n)$ defined above. The OPEN list is as follows (format $S^{\#} = (f(S) = g(n) + h(n))$):
A^1 $\mathbf{(4 = 0 + 4)}$
B^2 $\mathbf{(4 = 1 + 3)}$
F^2 $(4 = 1 + 3)$
C^3 $\mathbf{(4 = 2 + 2)}$
G^3 $(\infty = 2 + \infty)$
K^3 $(\infty = 2 + \infty)$
D^4 $(\infty = 3 + \infty)$
H^4 $\mathbf{(4 = 3 + 1)}$
I^5 $(5 = 4 + 1)$
M^5 $\mathbf{(4 = 4 + 0)}$
Dequeue M^5, found goal state!
The path in bold is the path that is followed to the goal state; reiterated, that path is A^1, B^2, C^3, H^4, M^5.

e) We're using a new heuristic function $h_2(n)$, where its values are based off of the values of $h(n)$ above:

$h(n)$	0	1	2	3	4	5	6	7	8	9	∞
$h_2(n)$	0	1	3	1	2	3	1	2	1	1	∞

With this new function, the OPEN list is as follows:
A^1 $\mathbf{(2 = 0 + 2)}$
B^2 $\mathbf{(2 = 1 + 1)}$
F^2 $(2 = 1 + 1)$
C^3 $\mathbf{(5 = 2 + 3)}$
G^3 $(\infty = 2 + \infty)$
K^3 $(\infty = 2 + \infty)$
D^4 $(\infty = 3 + \infty)$
H^4 $\mathbf{(4 = 3 + 1)}$
I^5 $(7 = 4 + 3)$
M^5 $\mathbf{(4 = 4 + 0)}$
Dequeue M^5, found goal state!

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The path is the same as d) above; A^1, B^2, C^3, H^4, M^5.

2. Probability, given the following table:

	Y		$\neg Y$	
	Z	$\neg Z$	Z	$\neg Z$
X	0.5	0.03	0.02	0.1
$\neg X$	0.2	0.05	0	0.1

(a) What is $P(\neg X)$?

$$\begin{aligned} P(\neg X) &= P(\neg X, Y, Z) + P(\neg X, Y, \neg Z) + P(\neg X, \neg Y, Z) + P(\neg X, \neg Y, \neg Z) \\ &= 0.2 + 0.05 + 0 + 0.1 \\ &= 0.35 \end{aligned}$$

(b) What is $P(Y|\neg X)$? We can reuse $P(\neg X)$ from the previous answer.

$$\begin{aligned} P(Y|\neg X) &= \frac{P(\neg X, Y)}{P(\neg X)} \\ &= \frac{P(\neg X, Y, Z) + P(\neg X, Y, \neg Z)}{P(\neg X)} \\ &= \frac{0.2 + 0.05}{0.35} \\ &= \frac{0.25}{0.35} = 0.71 \end{aligned}$$

(c) What is $P(\neg Z|X, Y)$? We can use the chain rule here, knowing that $P(X, Y, Z) = P(X)P(Y|X)P(Z|X, Y)$. We can also reuse the fact that we know $P(\neg X)$ already.

$$\begin{aligned} P(\neg Z|X, Y) &= \frac{P(X, Y, \neg Z)}{P(X)P(Y|X)} \\ &= \frac{P(X, Y, \neg Z)}{(1 - P(\neg X))\frac{P(X,Y)}{P(X)}} \\ &= \frac{P(X, Y, \neg Z)}{(1 - P(\neg X))\frac{P(X,Y,Z)+P(X,Y,\neg Z)}{1-P(\neg X)}} \\ &= \frac{0.03}{(1 - 0.35)\frac{0.5+0.03}{1-0.35}} = \frac{0.03}{0.65\frac{0.53}{0.65}} \\ &= \frac{0.03}{0.65 \cdot 0.82} = \frac{0.03}{0.53} \\ &= 0.05 \end{aligned}$$

(d) To find $P(\neg Z|X, \neg Y, Z)$, we can again apply the chain rule.

$$\begin{aligned} P(\neg Z, X, \neg Y, Z) &= P(X)P(\neg Y|X)P(Z|X, \neg Y)P(\neg Z|X, \neg Y, Z) \\ P(\neg Z|X, \neg Y, Z) &= \frac{P(\neg Z, X, \neg Y, Z)}{P(X)P(\neg Y|X)P(Z|X, \neg Y)} \\ &= \frac{0}{P(X)P(\neg Y|X)P(Z|X, \neg Y)} \\ &= 0 \end{aligned}$$

Since the numerator is zero, $P(\neg Z|X, \neg Y, Z)$ is zero. We can verify this conceptually by realizing that it is impossible to observe Z if a $\neg Z$ has already been observed.

(e) Two random variables are independent iff $P(A,B) = P(A)P(B)$. When we apply this to X and Z (using previous answers):

$$\begin{aligned}
P(X,Z) &= P(X)P(Z) \\
P(X,Y,Z) + P(X,\neg Y,Z) &= [1 - P(\neg X)]\,[P(X,Y,Z) + P(\neg X,Y,Z) + P(X,\neg Y,Z) + P(\neg X,\neg Y,Z)] \\
0.5 + 0.02 &= (1 - 0.35)(0.5 + 0.2 + 0.02 + 0) \\
0.52 &= (0.65)(0.72) \\
0.52 &\neq 0.468
\end{aligned}$$

Since the equality described above does not hold, X **and** Z **are not independent**.

3. Answering questions using a Bayesian Network

(a) $P(B)$ can be computed by the joint probability of all possible values of its parents, C and S. We can eliminate W and A from the joint probability through variable elimination (they don't contribute to B).

$$\begin{aligned}
P(B) &= P(B,C,S) + P(B,C,\neg S) + P(B,\neg C,S) + P(B,\neg C,\neg S) \\
&= P(B|C,S)P(C)P(S) + P(B|C,\neg S)P(C)P(\neg S) + P(B|\neg C,S)P(\neg C)P(S) + P(B|\neg C,\neg S)P(\neg C)P(\neg S) \\
&= (0.9 \cdot 0.8 \cdot 0.02) + (0.2 \cdot 0.8 \cdot 0.98) + (0.9 \cdot 0.2 \cdot 0.02) + (0.01 \cdot 0.2 \cdot 0.98) \\
&= 0.0144 + 0.1568 + 0.0036 + 0.00196 \\
P(B) &= 0.17676
\end{aligned}$$

$$\begin{aligned}
P(B) = {} & P(A,B,W,C,S) + P(A,B,W,C,\neg S) + P(A,B,W,\neg C,S) + P(A,B,W,\neg C,\neg S) \\
& P(A,B,\neg W,C,S) + P(A,B,\neg W,C,\neg S) + P(A,B,\neg W,\neg C,S) + P(A,B,\neg W,\neg C,\neg S) \\
& P(\neg A,B,W,C,S) + P(\neg A,B,W,C,\neg S) + P(\neg A,B,W,\neg C,S) + P(\neg A,B,W,\neg C,\neg S) \\
& P(\neg A,B,\neg W,C,S) + P(\neg A,B,\neg W,C,\neg S) + P(\neg A,B,\neg W,\neg C,S) + P(\neg A,B,\neg W,\neg C,\neg S)
\end{aligned}$$

(b) The probability that a co-worker complains of a backache is $P(W)$, computable by the joint probability of all possible values of its parent C.

$$\begin{aligned}
P(W) &= P(W,C) + P(W,\neg C) = P(W|C)P(C) + P(W|\neg C)P(\neg C) \\
&= (0.9 \cdot 0.8) + (0.01 \cdot 0.2) = 0.72 + 0.002 \\
P(W) &= 0.722
\end{aligned}$$

(c) The probability that a backache doesn't happen is $P(\neg A)$, calculated by joint probability. W doesn't contribute here.

$$\begin{aligned}
P(\neg A) = {} & P(\neg A,B,C,S) + P(\neg A,B,C,\neg S) + P(\neg A,B,\neg C,S) + P(\neg A,B\neg C,\neg S) + \\
& P(\neg A,\neg B,C,S) + P(\neg A,\neg B,C,\neg S) + P(\neg A,\neg B,\neg C,S) + P(\neg A,\neg B,\neg C,\neg S) \\
= {} & P(\neg A|B)P(B|C,S)P(C)P(S) + P(\neg A|B)P(B|C,\neg S)P(C)P(\neg S) + \\
& P(\neg A|B)P(B|\neg C,S)P(\neg C)P(S) + P(\neg A|B)P(B|\neg C,\neg S)P(\neg C)P(\neg S) + \\
& P(\neg A|\neg B)P(\neg B|C,S)P(C)P(S) + P(\neg A|\neg B)P(\neg B|C,\neg S)P(C)P(\neg S) + \\
& P(\neg A|\neg B)P(\neg B|\neg C,S)P(\neg C)P(S) + P(\neg A|\neg B)P(\neg B|\neg C,\neg S)P(\neg C)P(\neg S) \\
= {} & (0.3 \cdot 0.9 \cdot 0.8 \cdot 0.02) + (0.3 \cdot 0.2 \cdot 0.8 \cdot 0.98) + (0.3 \cdot 0.9 \cdot 0.2 \cdot 0.02) + (0.3 \cdot 0.01 \cdot 0.2 \cdot 0.98) \\
& + (0.9 \cdot 0.1 \cdot 0.8 \cdot 0.02) + (0.9 \cdot 0.8 \cdot 0.8 \cdot 0.98) + (0.9 \cdot 0.9 \cdot 0.2 \cdot 0.02) + (0.9 \cdot 0.99 \cdot 0.2 \cdot 0.98) \\
= {} & 0.00432 + 0.04704 + 0.00108 + 0.000588 + 0.00144 + 0.56448 + 0.00324 + 0.174636 \\
P(\neg A) = {} & 0.796824
\end{aligned}$$

(d) If a coworker has a backache, the probability that a person has a back injury and a back ache is $P(A,B|W)$. We can compute this by knowing $P(A,B|W)$ is equivalent to $P(A,B,W)/P(W)$. We have already computed P(W) in (b), we will compute $P(A,B,W)$ below.

$$\begin{aligned}
P(A,B,W) &= P(A,B,W,C,S) + P(A,B,W,C,\neg S) + P(A,B,W,\neg C,S) + P(A,B,W,\neg C,\neg S)\\
&= P(A|B)P(B|C,S)P(W|C)P(C)P(S) + P(A|B)P(B|C,\neg S)P(W|C)P(C)P(\neg S) +\\
&\quad P(A|B)P(B|\neg C,S)P(W|\neg C)P(\neg C)P(S) + P(A|B)P(B|\neg C,\neg S)P(W|\neg C)P(\neg C)P(\neg S)\\
&= (0.7\cdot 0.9\cdot 0.9\cdot 0.8\cdot 0.02) + (0.7\cdot 0.2\cdot 0.9\cdot 0.8\cdot 0.98) + (0.7\cdot 0.9\cdot 0.01\cdot 0.2\cdot 0.02) +\\
&\quad (0.7\cdot 0.01\cdot 0.01\cdot 0.2\cdot 0.98)\\
&= 0.009072 + 0.098784 + 0.0000252 + 0.00001372\\
P(A,B,W) &= 0.10789492
\end{aligned}$$

Given we know know $P(A,B,W) = 0.108$ and $P(W) = 0.722$ from (b), $P(A,B|W) = 0.108/0.722 = 0.149$. There is a 14.9% chance that a person has a back ache and injury if their co-worker is complaining.

(e) The probability that the chairs are causing co-workers' discomfort is described by $P(C|W)$. Again, this is equivalent to $P(C,W)/P(W)$. $P(W)$ has been calculated in (b).

$$\begin{aligned}
P(C,W) &= P(C)P(W|C) = 0.8\cdot 0.9 = 0.72\\
P(C|W) &= \frac{P(C,W)}{P(W)} = \frac{0.72}{0.722} = 0.997
\end{aligned}$$

There is a 99.7% chance of the chairs being new and uncomfortable if a co-worker has a backache.

(f) The probability that the wrong sport was played if one has a backache is outlined by $P(S|A)$. It is computed much like the above; $P(A,S)/P(A)$. $P(\neg A)$ has been computed in part (c). W does not contribute to this computation and is left out.

$$\begin{aligned}
P(A,S) &= P(A,B,C,S) + P(A,B,\neg C,S) + P(A,\neg B,C,S) + P(A,\neg B,\neg C,S)\\
&= P(A|B)P(B|C,S)P(C)P(S) + P(A|B)P(B|\neg C,S)P(\neg C)P(S) +\\
&\quad P(A|\neg B)P(\neg B|C,S)P(C)P(S) + P(A|\neg B)P(\neg B|\neg C,S)P(\neg C)P(S)\\
&= (0.7\cdot 0.9\cdot 0.8\cdot 0.02) + (0.7\cdot 0.9\cdot 0.2\cdot 0.02) + (0.7\cdot 0.1\cdot 0.8\cdot 0.02) + (0.7\cdot 0.1\cdot 0.2\cdot 0.02)\\
&= 0.01008 + 0.00252 + 0.00112 + 0.00028 = 0.014\\
P(S|A) &= \frac{P(A,S)}{P(A)} = \frac{P(A,S)}{1-P(\neg A)} = \frac{0.014}{1-0.796824} = 0.0689
\end{aligned}$$

The probability the wrong sport was played if one has a backache is 6.89%.

(g) The probability that one has a back injury if they have a backache is $P(B|A) = P(B,A)/P(A)$.

$$\begin{aligned}
P(A,B) &= P(A,B,C,S) + P(A,B,C,\neg S) + P(A,B,\neg C,S) + P(A,B,\neg C,\neg S)\\
&= P(A|B)P(B|C,S)P(C)P(S) + P(A|B)P(B|C,\neg S)P(C)P(\neg S)\\
&\quad P(A|B)P(B|\neg C,S)P(\neg C)P(S) + P(A|B)P(B|\neg C,\neg S)P(\neg C)P(\neg S)\\
&= (0.7\cdot 0.9\cdot 0.9\cdot 0.8\cdot 0.02) + (0.7\cdot 0.2\cdot 0.9\cdot 0.8\cdot 0.98) + (0.7\cdot 0.9\cdot 0.01\cdot 0.2\cdot 0.02)\\
&= 0.00432 + 0.04704 + 0.00108 + 0.000588 = 0.053028\\
P(B|A) &= \frac{P(B,A)}{P(A)} = \frac{P(B,A)}{1-P(\neg A)} = \frac{0.053028}{1-0.796824} = 0.2610
\end{aligned}$$

There is a 26.1% probability that one has a back injury if one has a backache.

(h) The probability of having a backache if one plays a wrong sport and sits on new, uncomfortable chairs can be represented by $P(A|C,S)$. This value can be represented by $P(A,C,S)/P(C,S)$. Since C and S are independent, $P(C,S) = P(C)P(S)$. W does not factor in this equation and is ignored.

$$\begin{aligned}
P(A,C,S) &= P(A,B,C,S) + P(A,\neg B,C,S) \\
&= P(A|B)P(B|C,S)P(C)P(S) + P(A|\neg B)P(\neg B|C,S)P(C)P(S) \\
&= (0.7 \cdot 0.9 \cdot 0.8 \cdot 0.02) + (0.1 \cdot 0.1 \cdot 0.8 \cdot 0.02) \\
&= 0.01008 + 0.00016 = 0.01024 \\
P(A|C,S) &= \frac{P(A,C,S)}{P(C,S)} = \frac{P(A,C,S)}{P(C)P(S)} = \frac{0.01024}{0.8 \cdot 0.02} = \frac{0.01024}{0.016} = 0.64
\end{aligned}$$

If one plays a wrong sport and sits on new, uncomfortable chairs, the chance of backache is 64%.

(i) The probability of a co-worker having a backache given one plays a wrong sport and sits on new, uncomfortable chairs is given by $P(W|C,S)$, which can be computed from $P(W,C,S)/P(C,S)$.

$$\begin{aligned}
P(W,C,S) &= P(A,B,C,S,W) + P(A,\neg B,C,S,W) + P(\neg A,B,C,S,W) + P(\neg A,\neg B,C,S,W) \\
&= P(A|B)P(B|C,S)P(W|C)P(C)P(S) + P(A|\neg B)P(\neg B|C,S)P(W|C)P(C)P(S) + \\
&\quad P(\neg A|B)P(B|C,S)P(W|C)P(C)P(S) + P(\neg A|\neg B)P(\neg B|C,S)P(W|C)P(C)P(S) \\
&= (0.9 \cdot 0.8 \cdot 0.02)\,[(0.7 \cdot 0.9) + (0.1 \cdot 0.1) + (0.3 \cdot 0.9) + (0.9 \cdot 0.1)] \\
&= (0.0144)\,[0.63 + 0.01 + 0.27 + 0.09] = 0.0144 \cdot 1.09 \\
P(W,C,S) &= 0.015696 \\
P(C,S) &= P(A,B,C,S,W) + P(A,B,C,S,\neg W) + P(A,\neg B,C,S,W) + P(A,\neg B,C,S,\neg W) + \\
&\quad P(\neg A,B,C,S,W) + P(\neg A,B,C,S,\neg W) + P(\neg A,\neg B,C,S,W) + P(\neg A,\neg B,C,S,\neg W) \\
\frac{P(C,S)}{P(C)P(S)} &= P(A|B)P(B|C,S)P(W|C) + P(A|B)P(B|C,S)P(\neg W|C) + P(A|\neg B)P(\neg B|C,S)P(W|C) + \\
&\quad P(A|\neg B)P(\neg B|C,S)P(\neg W|C) + P(\neg A|B)P(B|C,S)P(W|C) + P(\neg A|B)P(B|C,S)P(\neg W|C) + \\
&\quad P(\neg A|\neg B)P(\neg B|C,S)P(W|C) + P(\neg A|\neg B)P(\neg B|C,S)P(\neg W|C) \\
\frac{P(C,S)}{(0.8 \cdot 0.2)} &= (0.7 \cdot 0.9 \cdot 0.9) + (0.7 \cdot 0.9 \cdot 0.1) + (0.1 \cdot 0.1 \cdot 0.9) + (0.1 \cdot 0.1 \cdot 0.1) \\
&\quad (0.1 \cdot 0.1 \cdot 0.1) + (0.3 \cdot 0.9 \cdot 0.9) + (0.3 \cdot 0.9 \cdot 0.1) + (0.9 \cdot 0.1 \cdot 0.1) \\
P(C,S) &= (0.16)(0.567 + 0.063 + 0.009 + 0.001 + 0.001 + 0.729 + 0.027 + 0.009) \\
&= (0.16)(1.406) = 0.22496 \\
P(W|C,S) &= \frac{P(W,C,S)}{P(C,S)} = \frac{0.015696}{0.22496} = 0.069772
\end{aligned}$$

The probability of a co-worker having a backache given one plays a wrong sport and sits on new, uncomfortable chairs is 6.98%.

(j) The probability that one sits on new, uncomfortable chairs and plays the wrong sport if one also has a back injury is given by $P(C,S|B)$. This can be computed from $P(B,C,S)/P(B)$. $P(B)$ was calculated in part (a).

$$\begin{aligned}
P(B,C,S) &= P(A,B,C,S,W) + P(A,B,C,S,\neg W) + P(\neg A,B,C,S,W) + P(\neg A,B,C,S,\neg W) \\
\frac{P(B,C,S)}{P(B|C,S)P(C)P(S)} &= P(A|B)P(W|C) + P(A|B)P(\neg W|C) + P(\neg A|B)P(W|C) + P(\neg A|B)P(\neg W|C) \\
\frac{P(B,C,S)}{0.9 \cdot 0.8 \cdot 0.02} &= (0.7 \cdot 0.9) + (0.7 \cdot 0.1) + (0.3 \cdot 0.9) + (0.3 \cdot 0.1) \\
P(B,C,S) &= (0.0144)(0.63 + 0.07 + 0.27 + 0.03) = 0.0144 \cdot 1.9 = 0.02736 \\
P(C,S|B) &= \frac{P(B,C,S)}{P(B)} = \frac{0.02736}{0.17676} = 0.15479
\end{aligned}$$

The probability that one sits on new, uncomfortable chairs and plays the wrong sport if one also has a back injury is 15.5%.

4. Implement a Naive Bayes classifier: see program turned in via the *handin* tool.

   (4) The Naive Bayes classifier doesn't care about the order that the characters are in, only the counts of each individual character normalized by the number of total characters. My classifier will still work. If order matters, a multi-order Markov model may be a better classifier.