CS 540 HW1 Problem 1 Solution – ID3 Decision Tree

At each step, we want to select the feature with the lowest remainder. Or in other words, the feature that breaks our examples into groups such that the average entropy of the groups, weighted by the number of examples in the groups, is as small as possible. We add leaf nodes when we encounter an ID3 base case.

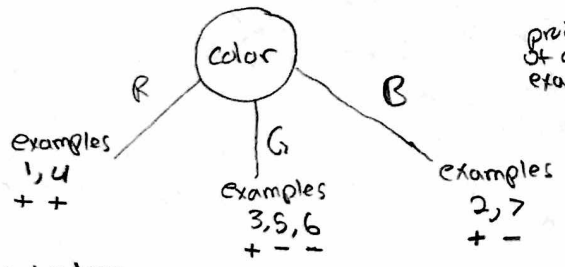
① Current examples (color, age, weight, category)

1. R, O, H, +
2. B, Y, L, +
3. G, Y, L, +
4. R, Y, H, +
5. G, O, L, −
6. G, Y, L, −
7. B, O, H, −

Current features: color, age, weight

Remainder for Color

3 possible values – R G B – break our examples into 3 groups



Remainder

the weighted average of the entropies of each group

$$R(color) = (2/7)E(red) + (3/7)E(green) + (2/7)E(blue)$$
$$= 2/7(0) + 3/7(0.92) + 2/7(1)$$
$$= 0.68$$

the weight is the # of examples in the group over total number of current examples

probability of a pos. example in group → $P(+) = 2/2$ $P(-) = 0/2$ ← probability of a negative example in group

entropy for R group

$$E(red) = -(2/2)\lg(2/2) - (0/2)\lg(0/2) = 0$$

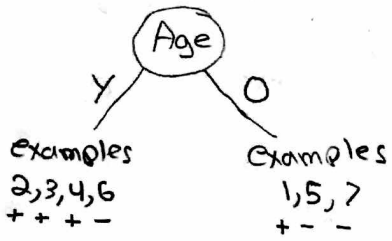
entropy for G group

$$P(+) = 1/3 \quad P(-) = 2/3$$
$$E(green) = -(1/3)\lg(1/3) - (2/3)\lg(2/3) = 0.92$$

entropy for B group

$$P(+) = 1/2 \quad P(-) = 1/2$$
$$E(blue) = -(1/2)\lg(1/2) - (1/2)\lg(1/2) = 1$$

Remainder for Age



entropy for Y group

$P(+) = 3/4 \quad P(-) = 1/4$

$E(Y) = -(3/4)\lg(3/4) - (1/4)\lg(1/4) = 0.81$

entropy for O group

$P(+) = 1/3 \quad P(-) = 2/3$

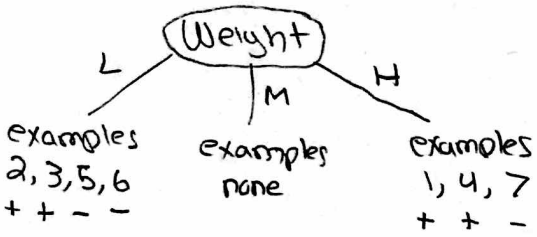
$E(O) = -(1/3)\lg(1/3) - (2/3)\lg(2/3) = 0.92$

Remainder

$R(age) = 4/7(0.81) + 3/7(0.92)$

$= 0.85$

Remainder for Weight



entropy for L group

$P(+) = 2/4 \quad P(-) = 2/4$

$E(L) = -(2/4)\lg(2/4) - (2/4)\lg(2/4) = 1$

entropy for H group

$E(H) = -(2/3)\lg(2/3) - (1/3)\lg(1/3) = 0.92$

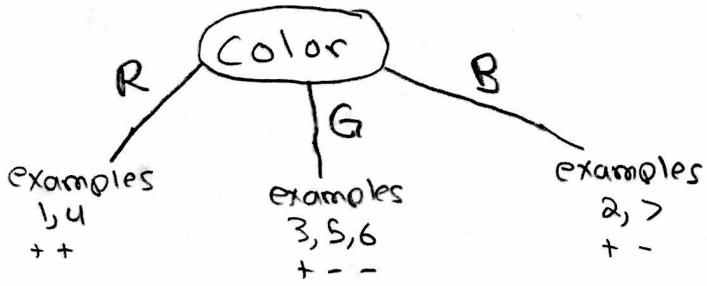
Remainder

$R(weight) = 4/7(1) + 3/7(0.92)$

$= 0.97$

Color has the smallest remainder of the currently available features, therefore we select it as the first node.

Current Decision Tree



Now we recurse on each branch.

(2) the R branch

current examples

R, O, H, +
R, Y, H, +

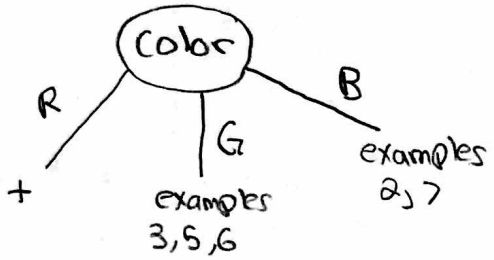
current features

age, weight

note: color not available since it was already selected for a parent node

All of the current examples are +. This is an ID3 base case. Add a leaf labeled with +.

Current Decision Tree



(3) The G branch

current examples

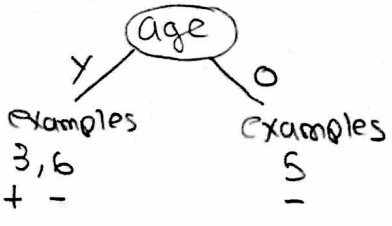
3 G, Y, L, +
5 G, O, L, -
6 G, Y, L -

current features

age, weight

We will once again select the feature with the lowest remainder.

Remainder for Age



entropy for Y group

$P(+) = 1/2 \quad P(-) = 1/2$

$E(Y) = -(1/2)\lg(1/2) - (1/2)\lg(1/2) = 1$

entropy for O group

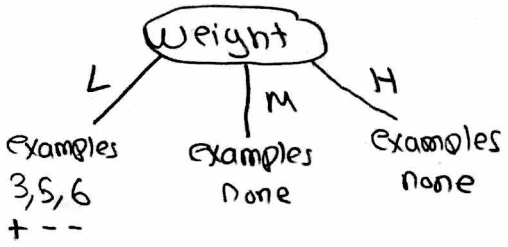
$P(+) = 0/1 \quad P(-) = 1/1$

$E(O) = -(0/1)\lg(0/1) - (1/1)\lg(1/1) = 0$

Remainder

$R(age) = 2/3(1) + 1/3(0) = 0.66$

Remainder for Weight



Entropy for L group

$P(+) = 1/3 \quad P(-) = 2/3$

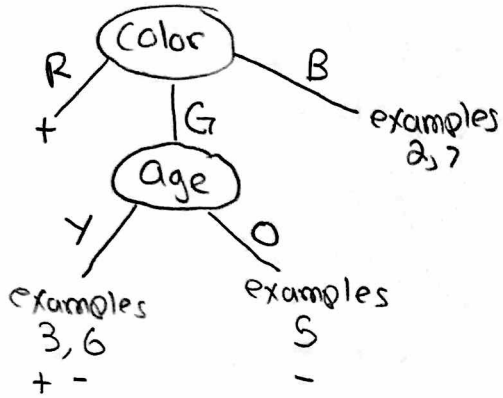
$E(L) = -(1/3)\lg(1/3) - (2/3)\lg(2/3) = 0.92$

Remainder

$R(\text{weight}) = E(L) = 0.92$

We select the feature with the lowest remainder, age.

Current Decision Tree



(4) the Y branch

Current examples

3 G, Y, L, +

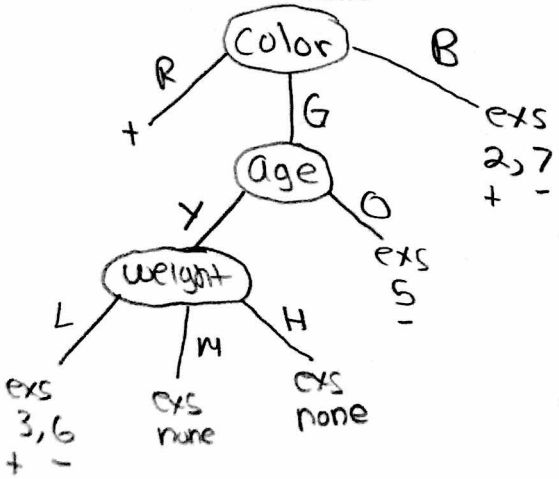
6. G, Y, L, -

Current features

weight

Since there is only one feature remaining, we do not need to compute remainder: just add feature to tree

Current tree



(5) the L branch

Current examples	current features
3 G, Y, L, +	none
6 G, Y, L, -	

There are no features remaining, this is an ID3 base case. Add a leaf with majority val of remaining examples. Since majority val is a tie, use label negative.

(6) the M branch

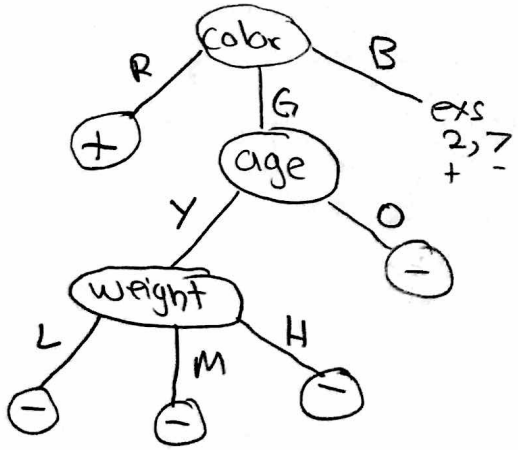
examples	features
none	none

Base case: no examples. Add leaf w/ majority label of parent. This is a tie, add negative.

(7) the H branch

Same as (6)

Current Tree



(8) the O branch

examples	features
5 G, O, L, -	none

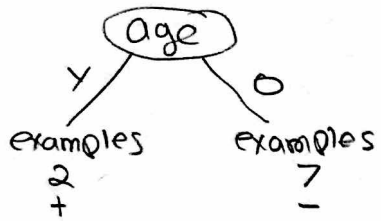
Base case: all examples have same label. Add leaf w/ this label.

⑨ the B branch

Current examples	current features
2 B, Y, L, +	age, weight
7 B, O, H, -	

Select feature w/ lowest remainder

Remainder Age



entropy Y group

$P(+) = 1/1 \quad P(-) = 0/1$

$-(1/1)\lg(1/1) - (0/1)\lg(0/1) = 0$

entropy O group

$P(+) = 0/1 \quad P(-) = 1/1$

$-(0/1)\lg(0/1) - (1/1)\lg(1/1) = 0$

$R(age) = 1/2(0) + 1/2(0) = 0$

Remainder Weight

entropy L group

$P(+) = 1/1 \quad P(-) = 0/1$

$E(L) = 0$

entropy O group

$P(+) = 0/1 \quad P(-) = 1/1$

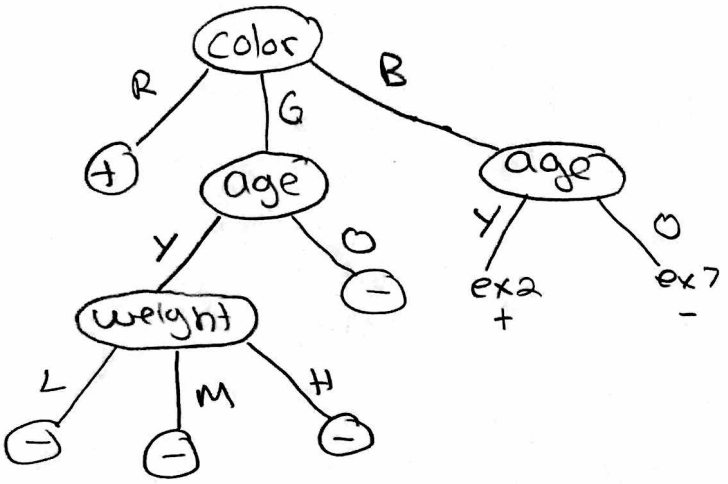
$E(O) = 0$

$R(weight) = 1/2(0) + 1/2(0)$

$= 0$

The remainders of age and weight are both 0. Select age because it comes first in alphabetical order.

Current Decision Tree



(10) the Y branch

examples
2 B, Y, L, +

features
weight

Since all examples have same label, add leaf w/ that label to tree.

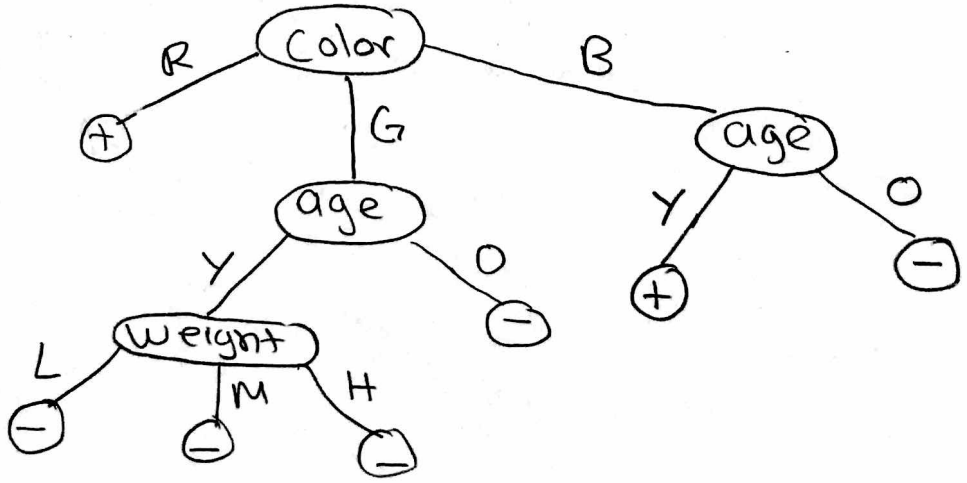
(11) the O branch

examples
7. B, O, H, -

features
weight

All examples have same label, add leaf w/ that label to the tree

Final Tree



Testing our tree with the test set

ex	color	age	weight	predicted	actual	correct?
8	B	Y	H	+	+	✓
9	G	Y	L	−	+	×
10	B	O	L	−	−	✓
11	R	Y	M	+	−	×
12	R	O	L	+	−	×

The decision tree correctly predicted the label for 2/5 of the testing examples. This is an accuracy of 40%..