Problem 1

i) The full joint table should sum to 1, so to find the missing probability $P(A, \neg B, C, D)$ we can do:

1 - sum of given probabilities

$= 1 - (0.002 + 0.018 + 0.013 + \ldots) = \boxed{0.14}$

ii) $P(\neg B, \neg C, D) = 0.018 + 0.019 = \boxed{0.037}$

iii) $P(A \vee \neg B) = P(A) + P(\neg B) - P(A, \neg B)$

$= 0.031 + 0.019 + 0.150 + 0.140 + 0.080 + 0.070 + 0.060 + 0.090$
$+ 0.002 + 0.018 + 0.013 + 0.127 + 0.031 + 0.019 + 0.150 + 0.140$
$- (0.031 + 0.019 + 0.150 + 0.140)$

$= \boxed{0.800}$

iv) $P(A \mid \neg B, \neg C, D) = \dfrac{P(A, \neg B, \neg C, D)}{P(\neg B, \neg C, D)} = \dfrac{0.019}{\underbrace{0.037}_{\text{from Part ii)}}} = \boxed{0.514}$

v) $P(\neg B, \neg C, D \mid A) = \dfrac{P(A, \neg B, \neg C, D)}{P(A)} = \dfrac{0.019}{0.640} = \boxed{0.0296}$

$P(A) = 0.031 + 0.019 + 0.150 + 0.140 + 0.080 + 0.070 + 0.060 + 0.090$
$= \boxed{0.640}$

Problem 2

i) $P(A,B,C,D) = P(A)\cdot P(B|A)\cdot P(C|A,B)\cdot P(D|B,C)$

$= 0.7\cdot 0.8\cdot 0.6\cdot 0.2 = \boxed{0.0672}$

ii) $P(\neg B, \neg C, \neg D) = P(A,\neg B,\neg C,\neg D) + P(\neg A,\neg B,\neg C,\neg D)$

$P(A,\neg B,\neg C,\neg D) = P(A)\cdot P(\neg B|A)\cdot P(\neg C|A,\neg B)\cdot P(\neg D|\neg B,\neg C)$

$= 0.7\cdot 0.2\cdot 0.5\cdot 0.1$

$= \boxed{0.007}$

$P(\neg A,\neg B,\neg C,\neg D) = P(\neg A)\cdot P(\neg B|\neg A)\cdot P(\neg C|\neg A,\neg B)\cdot P(\neg D|\neg B,\neg C)$

$= 0.3\cdot 0.9\cdot 0.7\cdot 0.1$

$= \boxed{0.0189}$

$P(\neg B,\neg C,\neg D) = 0.007 + 0.0189 = \boxed{0.0259}$

iii) $P(A|B,C,\neg D) = \dfrac{P(A,B,C,\neg D)}{P(B,C,\neg D)}$

$P(A,B,C,\neg D) = P(A)\cdot P(B|A)\cdot P(C|A,B)\cdot P(\neg D|B,C)$

$= 0.7\cdot 0.8\cdot 0.6\cdot 0.8$

$= \boxed{0.2688}$

$P(B,C,\neg D) = P(A,B,C,\neg D) + P(\neg A,B,C,\neg D)$

$P(\neg A,B,C,\neg D) = P(\neg A)\cdot P(B|\neg A)\cdot P(C|\neg A,B)\cdot P(\neg D|B,C)$

$= 0.3\cdot 0.1\cdot 0.4\cdot 0.8$

$= \boxed{0.0096}$

$P(B,C,\neg D) = 0.2688 + 0.0096 = \boxed{0.2784}$

$P(A|B,C,\neg D) = \dfrac{0.2688}{0.2784} = \boxed{0.9655}$

Problem 2 Continued

iii) You could have also observed that due to the Markov Blanket Property,

$P(A \mid B, C, \neg D) = P(A \mid B, C)$, which would have expanded into

$$P(A \mid B, C) = \frac{P(A, B, C)}{P(B, C)}$$

$$P(A, B, C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A, B)$$

$$= 0.7 \cdot 0.8 \cdot 0.6$$

$$= \boxed{0.336}$$

$$P(B, C) = P(A, B, C) + P(\neg A, B, C)$$

$$P(\neg A, B, C) = 0.3 \cdot 0.1 \cdot 0.4 = \boxed{0.012}$$

$$P(B, C) = 0.336 + 0.012 = \boxed{0.348}$$

$$P(A \mid B, C) = \frac{0.336}{0.348} = \boxed{0.9655}$$

iv) $P(\neg D \mid A, B, C) = \dfrac{P(A, B, C, \neg D)}{P(A, B, C)}$ (without Markov Blanket)

$P(A, B, C, \neg D) = \boxed{0.2688}$ computed on previous page

$P(A, B, C) = \boxed{0.336}$ computed above

$$P(\neg D \mid A, B, C) = \frac{0.2688}{0.336} = \boxed{0.8}$$

Problem 2 Continued

Can also see this by examining probability table (with Markov Blanket)

iv continued) $P(\neg D \mid B,C) = \dfrac{P(B,C,\neg D)}{P(B,C)}$

$P(B,C,\neg D) = \boxed{0.2784}$ (computed on page 2)

$P(B,C) = \boxed{0.348}$ (computed on page 3)

$P(\neg D \mid B,C) = \dfrac{0.2784}{0.348} = \boxed{0.8} = P(\neg D \mid A,B,C)$

v) $P((A,D) \vee (B,C)) = P(A,D) + P(B,C) - P(A,B,C,D)$

$P(A,D) = P(A,B,C,D) + P(A,\neg B,C,D)$
$\quad + P(A,B,\neg C,D) + P(A,\neg B,\neg C,D)$

$P(A,B,C,D) = \boxed{0.0672}$ page 2

$P(A,\neg B,C,D) = P(A)P(\neg B \mid A)P(C \mid A,\neg B)P(D \mid \neg B,C)$
$= 0.7 \cdot 0.2 \cdot 0.5 \cdot 0.3$
$= \boxed{0.021}$

$P(A,B,\neg C,D) = 0.7 \cdot 0.8 \cdot 0.4 \cdot 0.4 = \boxed{0.0896}$

$P(A,\neg B,\neg C,D) = 0.7 \cdot 0.2 \cdot 0.5 \cdot 0.9 = \boxed{0.063}$

$P(A,D) = 0.0672 + 0.021 + 0.0896 + 0.063 = \boxed{0.2408}$

$P(B,C) = \boxed{0.348}$ page 3

$P((A,D) \vee (B,C)) = 0.2408 + 0.348 - 0.0672 = \boxed{0.5216}$

Problem 3

$$P(A,B,\neg C,\neg D) = P(A)P(B|A)P(\neg C|A,B)P(\neg D|B,\neg C)$$
$$= 0.7 \cdot 0.8 \cdot 0.4 \cdot 0.6$$
$$= \boxed{0.1344}$$

$P(A,B,\neg C,D) = \boxed{0.0896}$ page 4

$P(A,B,C,\neg D) = \boxed{0.2688}$ page 2

$P(A,B,C,D) = \boxed{0.0672}$ page 2

The main point of this question is to highlight that a Bayes Net can fill all the cells in a full joint table.

Problem 4

See next page for alternative solution

i) $$P(flu \mid \neg shot) = \frac{P(\neg shot \mid flu)\,P(flu)}{P(\neg shot)}$$ using Bayes' Rule

We know $P(flu)$ and $P(\neg shot)$, so let's focus on $P(\neg shot \mid flu)$.

$$P(\neg shot \mid flu) = 1 - P(shot \mid flu)$$

$$P(shot \mid flu) = \frac{P(flu \mid shot)\,P(shot)}{P(flu)} = \frac{(0.1)(0.7)}{(0.2)} = \boxed{0.35}$$

$$P(\neg shot \mid flu) = 1 - 0.35 = \boxed{0.65}$$

Now solving the original question,

$$P(flu \mid \neg shot) = \frac{P(\neg shot \mid flu)\,P(flu)}{P(\neg shot)} = \frac{(0.65)(0.2)}{1-0.7} = \boxed{0.433}$$

ii) $$P(shot \mid \neg flu) = \frac{P(\neg flu \mid shot)\,P(shot)}{P(\neg flu)}$$ using Bayes' Rule

$$P(\neg flu \mid shot) = 1 - P(flu \mid shot) = 1 - 0.1 = \boxed{0.9}$$

$$P(shot \mid \neg flu) = \frac{(0.9)(0.7)}{0.8} = \boxed{0.788}$$

iii) $$\left.\begin{aligned} P(DE) &= \frac{57}{1000000} \\ P(T4 \mid DE) &= 0.95 \\ P(T4 \mid \neg DE) &= 0.05 \end{aligned}\right\}$$ given in problem description

$$P(DE \mid T4) = \frac{P(T4 \mid DE)\,P(DE)}{P(T4)}$$

$$= \frac{(0.95)\left(\frac{57}{1000000}\right)}{0.05} = \boxed{0.00108}$$

$$P(T4) = P(T4, DE) + P(T4, \neg DE)$$

$$P(T4, DE) = P(T4 \mid DE)\,P(DE) = (0.95)\left(\frac{57}{1000000}\right) = \boxed{5.415 \times 10^{-5}}$$

$$P(T4, \neg DE) = P(T4 \mid \neg DE)\,P(\neg DE) = (0.05)\left(\frac{999943}{1000000}\right) = \boxed{0.04999}$$

$$P(T4) = 0.00005415 + 0.04999 = \boxed{0.05}$$

Problem 4

i) Alternative Solution

$P(\text{flu} \mid \neg\text{shot}) = ?$

Using conditioning, we observe the following.

$$P(\text{flu}) = P(\text{flu} \mid \text{shot}) \cdot P(\text{shot}) + P(\text{flu} \mid \neg\text{shot}) \cdot P(\neg\text{shot})$$

$$0.2 = 0.1 \cdot 0.7 + P(\text{flu} \mid \neg\text{shot}) \cdot (1 - 0.7)$$

$$\frac{0.2 - 0.07}{0.3} = P(\text{flu} \mid \neg\text{shot})$$

$$P(\text{flu} \mid \neg\text{shot}) = \boxed{0.433}$$