CS 540 (Shavlik) HW 5 – Logic-Based Representation and Reasoning

Assigned: 11/29/16
Due: 12/13/16, 11:55pm (can turn in late until 11:55pm Dec 15, 2016)
Points: 100

Problem 1- Inference in Propositional Logic (15 points)

Using the inference rules in Section 7.5 of the textbook plus any additional inference rules presented in lecture, show that S can be inferred if the following logical sentences are believed. Use the notation for derivations used in class to show your work. That is, make a table with columns "Number," "WFF," and "Justification." (Do the same for Problem 7.)

1. P

2. R

3. ¬ R ∨ K

4. S ∨ ¬ (¬ Q)

5. ¬ P → Q

6. K → (S ∨ ¬P)

7. (K ∧ ¬ Q) → S

8. (¬ K ∧ R) → S

Problem 2 – Conjunctive Normal (aka Clausal) Form (10 points)

Put the following WFF into Conjunctive Normal Form (CNF).   Show your work.

$$(P \rightarrow Q) \rightarrow (R \leftrightarrow S)$$

Problem 3 – Satisfiability (15 points)

We wish to use <u>search</u> to find an interpretation that satisfies all the clauses below.

a) <u>Show</u> how this set of clauses can be simplified *before* starting the search. Justify your simplification(s).

$(P \lor \neg Q \lor R \lor \neg Z)$

$(\neg P)$

$(\neg R \lor S \lor \neg W)$

$(P \lor \neg Q \lor \neg R)$

$(R \lor \neg S \lor W)$

$(\neg Q \lor \neg R \lor \neg W)$

b) What is one <u>*model*</u> of these sentences?   Explain.

Problem 4 - Representation in FOPC (30 points)

Give one (1) predicate calculus representation for each of the following English sentences. If you feel a sentence is ambiguous, provide a more detailed sentence that better captures the version represented by your FOPC. Choose reasonable constants, predicates, and functions − the predicate *thereAreNoCatsThatLikeSpinach( )* is <u>not</u> an acceptable answer to the first question. (In addition to the lecture notes, Chapter 8, especially Sections 8.3 and 8.4, contains related examples.)

    i.   No cat likes spinach.

    ii.   Every friend of a friend of Sue likes pizza.

    iii.   Joe's neighbor has a red bike.

    iv.   Except for Al, everyone in CS 540 knows Pete.

    v.   If pigs fly, the Packers win the 2017 Super Bowl.

    vi.   There exists an owner of every car. [Does <u>not</u> mean <u>one</u> person owns every car.]

    vii.   Nannon games have a winner and a loser. [The same player cannot both win and lose.]

    viii.   Every student at UW-Madison took English in high school.

    ix.   Watching a Badger game does not impact one's height.
        [You must use situation calculus here.]

    x.   Selling the CS 540 textbook changes who owns it, but not who wrote it.
        [You must use situation calculus here.]

Problem 5 - Unification (15 points)

For each of the following pairs of wff's, state the most general unifier (mgu), or say that the mgu does not exist and explain why. Terms starting with lowercase letters are universally quantified variables; otherwise they are constants (or functions). Show your work (by walking through the unification algorithm; don't simply guess!).

    i.   P(y, x, y)   and   Q(G(z), B, w)

    ii.   P(x, y, x)   and   P(A, w, F(z))

    iii.   P(x, F(y), z, y)   and   P(F(9, a), b, 7, a)

    iv.   P(i, z, z, a)   and   P(i, e, Sausage, Pepperoni)

    v.   Q(F(x, y), z, G(x, z))   and   Q(a, b, G(c, H(2, 0)))

Problem 6 – Resolution Theorem Proving (15 points)

Given the following clauses, use the *resolution* inference rule to do a proof by contradiction to show that **∃z S(z)**.   Use the notation of Figure 9.12 of textbook (there are also examples in the lecture notes). Lowercase letters indicate universal variables. (Do you work initially on 'scratch paper' and then remove any 'dead ends' before reporting your solution.)

   P(1)

   Q(2)

   R(1, 2)

   R(3, 4)

   ¬ P(a) ∨ ¬ P(b) ∨ R(a, b)

   ¬ Q(d) ∨ ¬ R(d, d) ∨ S(d)

   ¬ P(f) ∨ ¬ Q(g) ∨ R(f, g)

   ¬ R(m, n) ∨ R(n, n)

   ¬ Q(w) ∨ ¬ S(w) ∨ P(w)

Problem 7 - Inference in FOPC (ZERO points, but we will provide a solution; you should not turn in a solution, but do try to solve this question)

[Note: below, the sentences in this 'Murder Mystery' are converted to FOPC, using constants A, B, C, and D as shorthand for the people in the story, based on the first letter in their names.]

Dana has been murdered. Alfonso, Bonnie, and Clyde are suspects. Only one is guilty and the other two are innocent. The innocent ones told the truth to the police, but the guilty one may have lied.

Alfonso said that Bonnie and Dana were friends and that Clyde did not like Dana. Bonnie said that she was not in town at the time of the murder, and moreover, she did not know Dana. Clyde

Your job is to prove that Bonnie is the murderer (i.e., *murderer(B)*). You should do this via a proof by contradiction. I.e., assume ¬ *murderer(B)* and show that this leads to a something of the form $P \land \neg P$, which is a contradiction since P cannot not be both true and false.

First convert all the wff's below of the form $P \rightarrow Q$ into the logically equivalent form $\neg P \lor Q$; this will make it easier to use the inference rules involving resolution. Then use the inference rules of Section 7.5, Section 9.1, and any others presented in class. Be sure to use the notation for proofs presented in class (See Problem 1).

If Alfonso is innocent, Bonnie and Dana were friends and Clyde did not like Dana.

      1a.   innocent(A) → friends(B, D)
      1b.   innocent (A) → ¬ likes(C, D)

If Bonnie is innocent, she was not in town, and she did not know Dana.

      2a.   innocent (B) → ¬ inTown(B)
      2b.   innocent (B) → ¬ knows(B, D)

If Clyde is innocent, Alfonso and Bonnie were both with Dana just before Dana was murdered.

      3a.   innocent (C) → with(A, D)
      3b.   innocent (C) → with(B, D)

Everyone who was with Dana was in town. (Some slightly contrived background knowledge.)

      4.   ∀ x   with(x,D) → inTown(x)

Everyone knows his/her friends. (Some common-sense, background knowledge.)

      5.   ∀ x ∀ y friends (x, y) → knows(x, y)

Everyone knows the people he/she likes. (Some common-sense, background knowledge.)

      6.   ∀ x ∀ y likes(x, y) → knows(x, y)

Among Alfonso, Bonnie, and Clyde, two are innocent.

      7a.   innocent (A) ∨ innocent (B)
      7b.   innocent (A) ∨ innocent (C)
      7c.   innocent (B) ∨ innocent (C)

If someone is not innocent, then he/she is the murderer.

      8.   ∀ x ¬ innocent (x) → murderer(x)