- Assignment 2 out!
- Course Project
  - Form groups?
  - Project list by Monday (9/28)
  - Submit project bids by Thursday (10/1)
  - Assigned project by Friday (10/2)
**SETTING: FAIR SHARING**

**Equal Share**

- 100%
- 50%
- 0%

**Max-Min Share**

- Maximize the allocation for most poorly treated users
- Maximize the minimum
MOTIVATION: MULTI RESOURCES
Users have a demand vector
<2, 3, 1> means user’s task needs 2 R1, 3 R2, 1 R3

Resources given in multiples of demand vector
i.e., users might get <4, 6, 2>
PROPERTIES

Sharing Incentive

Pareto Efficiency

Strategy Proof

Envy free
PROPERTIES

Sharing Incentive
User is no worse off than a cluster with \( \frac{1}{n} \) resources

Strategy Proof
User should not benefit by lying about demands

Pareto Efficiency
Not possible to increase one user without decreasing another

Envy free
User should not desire the allocation of another user
### DRF: APPROACH

<table>
<thead>
<tr>
<th>Dominant Resource</th>
<th>Dominant Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resource user has the <strong>biggest</strong> share of</td>
<td>Fraction of the dominant resource user is allocated</td>
</tr>
<tr>
<td>Total: <code>&lt;10 CPU, 4 GB&gt;</code></td>
<td>E.g., for User 1 this is <strong>25% or 1/4</strong></td>
</tr>
<tr>
<td>User 1: <code>&lt;1 CPU, 1 GB&gt;</code></td>
<td></td>
</tr>
<tr>
<td>Dominant resource is <strong>memory</strong></td>
<td></td>
</tr>
</tbody>
</table>
### DRF: Approach

Equalize the dominant share of users

<table>
<thead>
<tr>
<th>User</th>
<th>Allocation</th>
<th>Dominant Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>User1</td>
<td>&lt;0 CPU, 0 GB&gt;</td>
<td>0</td>
</tr>
<tr>
<td>User2</td>
<td>&lt;0 CPU, 0 GB&gt;</td>
<td>0</td>
</tr>
</tbody>
</table>

**Total:** <9 CPU, 18 GB>

**User1:** <1 CPU, 4 GB>
- dom res: mem

**User2:** <3 CPU, 1 GB>
- dom res: CPU
DRF: APPROACH

Total: <9 CPU, 18 GB>

User1: <1 CPU, 4 GB> per task
  <3 CPU, 12 GB> for 3 tasks
  dom res: mem
  dom share: 12/18 = 2/3

User2: <3 CPU, 1 GB>
  <6 GPU, 2 GB> for 2 tasks
  dom res: CPU
  dom share: 6/9 = 2/3
Whenever there are available resources:
Schedule a task to the user with **smallest dominant share**
Algorithm 1 DRF pseudo-code

\[ R = \langle r_1, \cdots, r_m \rangle \quad \triangleright \text{total resource capacities} \]
\[ C = \langle c_1, \cdots, c_m \rangle \quad \triangleright \text{consumed resources, initially 0} \]
\[ s_i \ (i = 1..n) \quad \triangleright \text{user } i\text{'s dominant shares, initially 0} \]
\[ U_i = \langle u_{i,1}, \cdots, u_{i,m} \rangle \ (i = 1..n) \quad \triangleright \text{resources given to user } i, \text{initially 0} \]

**pick** user \( i \) with lowest dominant share \( s_i \)

\( D_i \leftarrow \text{demand of user } i\text{'s next task} \)

**if** \( C + D_i \leq R \) **then**

\[ C = C + D_i \quad \triangleright \text{update consumed vector} \]
\[ U_i = U_i + D_i \quad \triangleright \text{update } i\text{'s allocation vector} \]
\[ s_i = \max_{j=1}^m \{ u_{i,j} / r_j \} \]

**else**

**return** \( \quad \triangleright \text{the cluster is full} \)

**end if**
COMPARISON: ASSET FAIRNESS

Asset Fairness: Equalize each user’s sum of resource shares

Violates Sharing Incentive

Consider total of 70 CPUs, 70 GB RAM
U1 needs <2 CPU, 2 GB RAM> per task
U2 needs <1 CPU, 2 GB RAM> per task

Asset Fair Allocation:
U1:
U2:
COMPARISON: ASSET FAIRNESS

Asset Fairness: Equalize each user’s sum of resource shares

Violates Sharing Incentive

Consider total of 70 CPUs, 70 GB RAM
U1 needs <2 CPU, 2 GB RAM> per task
U2 needs <1 CPU, 2 GB RAM> per task

Asset Fair Allocation:
U1: 15 tasks: 30 CPU, 30 GB (Sum = 60)
U2: 20 tasks: 20 CPU, 40 GB (Sum = 60)
**COMPARISON: CEEI**

CEEI: Competitive Equilibrium from Equal Incomes

- Each user receives initially $1/n$ of every resource,
- Subsequently, each user can trade resources with other users in a perfectly competitive market
- Computed by maximizing **product of utilities** across users
COMPARISON: CEEI

Total: <9 CPU, 18 GB>  
User1: <1 CPU, 4 GB>  
User2: <3 CPU, 1 GB>

\[
\begin{align*}
\max (x \cdot y) \\
\text{subject to} \\
x + 3y & \leq 9 \\
4x + y & \leq 18
\end{align*}
\]
Total: <9 CPU, 18 GB>

User2 Before:
CEEI: 55% CPU, 9% mem

Total: <9 CPU, 18 GB>
User1: <1 CPU, 4 GB>
User2: <3 CPU, 2 GB>
## COMPARISON

<table>
<thead>
<tr>
<th>Property</th>
<th>Allocation Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Asset</td>
</tr>
<tr>
<td>Sharing Incentive</td>
<td>✓</td>
</tr>
<tr>
<td>Strategy-proofness</td>
<td>✓</td>
</tr>
<tr>
<td>Envy-freeness</td>
<td>✓</td>
</tr>
<tr>
<td>Pareto efficiency</td>
<td>✓</td>
</tr>
<tr>
<td>Single Resource Fairness</td>
<td>✓</td>
</tr>
<tr>
<td>Bottleneck Fairness</td>
<td></td>
</tr>
<tr>
<td>Population Monotonicity</td>
<td>✓</td>
</tr>
<tr>
<td>Resource Monotonicity</td>
<td>✓</td>
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</tbody>
</table>

Table 2: Properties of Asset Fairness, CEEI and DRF.
SUMMARY

DRF: Dominant Resource Fairness
Allocation policy for scheduling
Provides multi-resource fairness
Ensures sharing incentive, strategy proofness
DISCUSSION

https://forms.gle/i7m7xXxKhtfvL9UD9
Consider a system with 100 units of CPU, 50 units of memory and 200 units of disk. Consider three users with the following requirements

Alice (4 CPU, 1 memory, 1 disk)
Bob (1 CPU, 4 memory and 4 disk)
Carol (1 CPU, 2 memory and 16 disk)

List the dominant resource as defined in DRF for Alice, Bob and Carol
What would be the final task allocation in the given cluster for Alice, Bob and Carol?
What could be one workload / cluster scenario where DRF implemented on Mesos will NOT be optimal?
Next Week: Machine Learning
Assignment 2 out!