

*Good morning!*

# CS 744: DRF

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Fall 2021

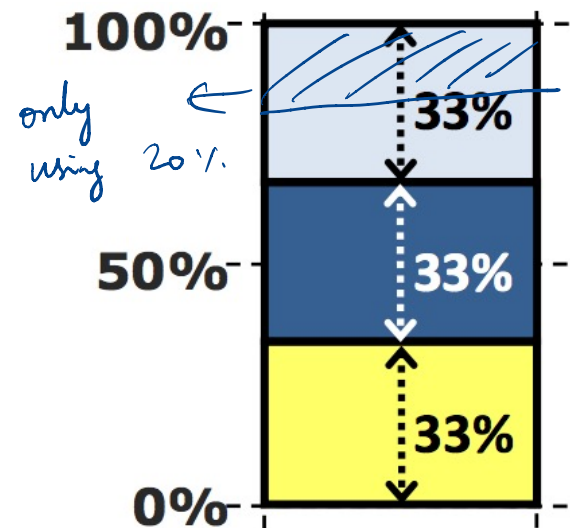
# ADMINISTRIVIA

- Assignment 2 out! *→ PyTorch ~2 weeks*
- Course Project
  - Project list by Oct 1
  - Form groups and submit project bids by Oct 8
  - Assigned project by Oct 14
  - Introductions due Oct 25

# SETTING: FAIR SHARING → Background

not work  
conserving

## Equal Share



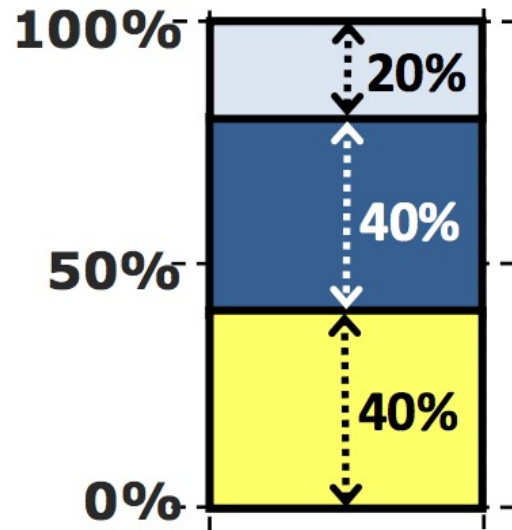
## Max-Min Share

Maximize the allocation  
for most poorly treated  
users

max-min fairness

Maximize the minimum

CPU scheduling : Lottery  
network sched

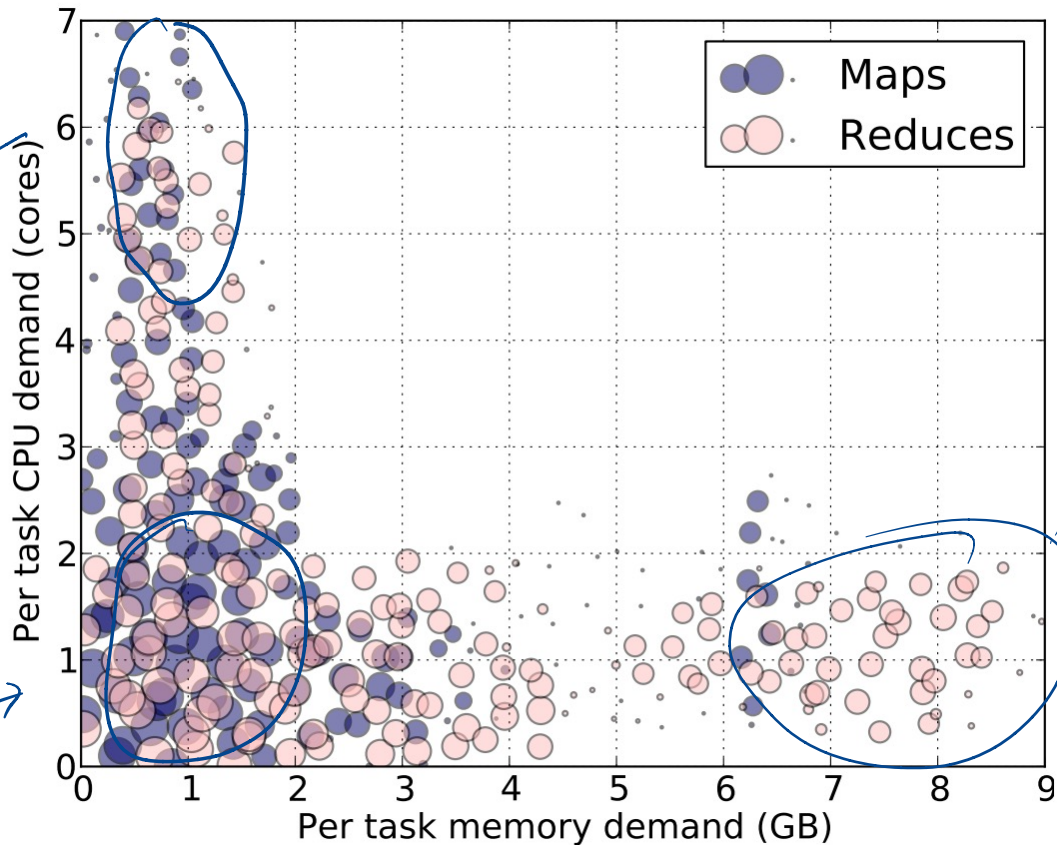


# MOTIVATION: MULTI RESOURCES

lot of CPU  
very little memory

mix of  
Maps/  
Reduces

lots of  
small  
tasks



likely to be  
Reduce

lot of memory  
very little  
CPU

# DRF: MODEL

Users have a **demand vector**

$\langle 2, 3, 1 \rangle$  means user's task needs 2 R1, 3 R2, 1 R3

*Multi resource*

*2 CPUs, 3GB, 1 disk space*

Resources given in multiples of demand vector

i.e., users might get  $\langle 4, 6, 2 \rangle$

*↳ can run multiple tasks of  $\langle 2, 3, 1 \rangle$*

# PROPERTIES

## Sharing Incentive

Users should be not  
work off when  
sharing the cluster

## Strategy Proof

Users cannot get  
more resources by  
lying

## Pareto Efficiency

cannot give resources  
to one user without  
taking away from  
another

## Envy free

User does not wish  
for another users  
allocation

# PROPERTIES

## Sharing Incentive

User is no worse off than a cluster with  
 $1/n$  resources

## Strategy Proof

User should not benefit by  
lying about demands

## Pareto Efficiency

Not possible to increase  
one user without  
decreasing another

## Envy free

User should not desire the  
allocation of another user

# DRF: APPROACH

## Dominant Resource

Resource user has the **biggest** share of

Total: <10 CPU, 4 GB> CPU   Mem

User 1: <1 CPU, 1 GB> =  $\frac{1}{10}$  <  $\frac{1}{4}$

Dominant resource is memory

## Dominant Share

Fraction of the dominant resource user is allocated

E.g., for User 1 this is **25% or 1/4**

Allocate 2 tasks with  
<1 CPU, 1 GB> =  $\frac{2}{4}$   
Dominant share



# DRF: APPROACH

If User 1 :  $\langle 2 \text{ CPU}, 4 \text{ GB} \rangle$

Equalize the dominant share of users

Total:  $\langle 9 \text{ CPU}, 18 \text{ GB} \rangle$

User1:  $\langle 1 \text{ CPU}, 4 \text{ GB} \rangle$

dom res: **mem**

User2:  $\langle 3 \text{ CPU}, 1 \text{ GB} \rangle$

dom res: **CPU**

Total Used: 9 CPU,  
14 GB

User	Allocation	Dominant Share
User1	$\langle 0 \text{ CPU}, 0 \text{ GB} \rangle$ $\langle 1 \text{ CPU}, 4 \text{ GB} \rangle$ $\langle 2 \text{ CPU}, 8 \text{ GB} \rangle$ $\langle 3 \text{ CPU}, 12 \text{ GB} \rangle$	0 $4/18 = 2/9$ $8/18 = 4/9$ $12/18 = 6/9$
User2	$\langle 0 \text{ CPU}, 0 \text{ GB} \rangle$ $\langle 3 \text{ CPU}, 1 \text{ GB} \rangle$ $\langle 6 \text{ CPU}, 2 \text{ GB} \rangle$	0 $3/9 = 1/3$ $6/9$

# DRF: APPROACH

Total: <9 CPU, 18 GB>

User1: <1 CPU, 4 GB> per task  
<3 CPU, 12 GB> for 3 tasks

dom res: mem

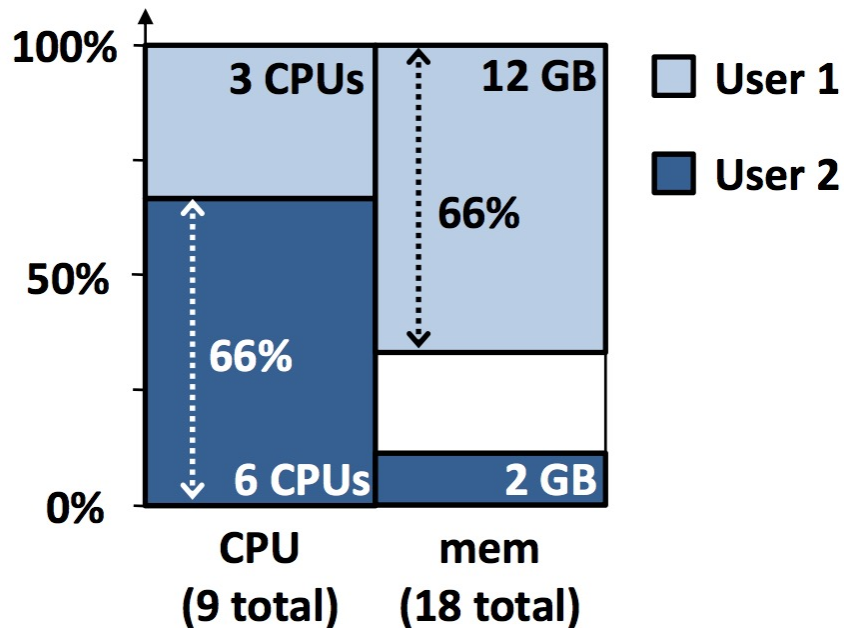
dom share:  $12/18 = 2/3$

User2: <3 CPU, 1 GB>

<6 CPU, 2 GB> for 2 tasks

dom res: CPU

dom share:  $6/9 = 2/3$



# DRF ALGORITHM

Whenever there are available resources:

Schedule a task to the user with **smallest dominant share**

# DRF ALGORITHM

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## Algorithm 1 DRF pseudo-code

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$R = \langle r_1, \dots, r_m \rangle$   $\triangleright$  total resource capacities

$C = \langle c_1, \dots, c_m \rangle$   $\triangleright$  consumed resources, initially 0

$s_i$  ( $i = 1..n$ )  $\triangleright$  user  $i$ 's dominant shares, initially 0

$U_i = \langle u_{i,1}, \dots, u_{i,m} \rangle$  ( $i = 1..n$ )  $\triangleright$  resources given to user  $i$ , initially 0

**pick** user  $i$  with lowest dominant share  $s_i$

$D_i \leftarrow$  demand of user  $i$ 's next task

**if**  $C + D_i \leq R$  **then**  $\rightarrow$  fits within capacity

$C = C + D_i$   $\triangleright$  update consumed vector

$U_i = U_i + D_i$   $\triangleright$  update  $i$ 's allocation vector

$s_i = \max_{j=1}^m \{u_{i,j}/r_j\}$   $\rightarrow$  update their dominant share

**else**

**return**

**end if**

$\rightarrow$  cluster capacity  
 $\rightarrow$  Resources used so far  
 $\rightarrow$  Dominant share updated as we make allocation

$\rightarrow$  a different user can fit in the available resources

$\rightarrow$  using available  $\triangleright$  the cluster is full

# COMPARISON: ASSET FAIRNESS

Asset Fairness: Equalize each user's sum of resource shares

Consider total of 70 CPUs, 70 GB RAM

U1 needs <2 CPU, 2 GB RAM> per task = 4

U2 needs <1 CPU, 2 GB RAM> per task = 3

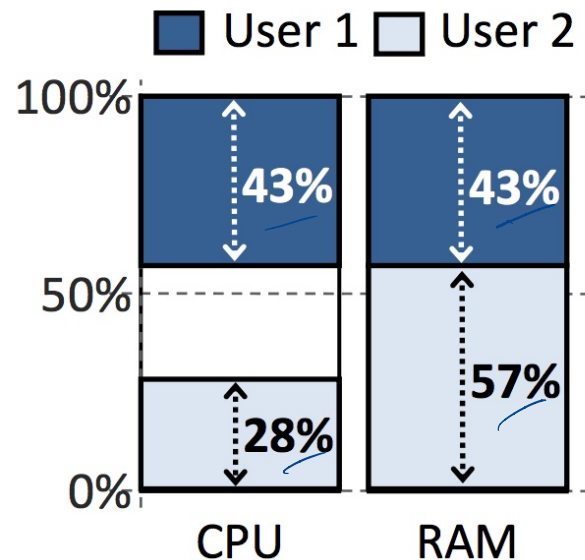
Asset Fair Allocation:

U1: 15 tasks: 30 CPU, 30 GB (Sum = 60)

U2: 20 tasks: 20 CPU, 40 GB (Sum = 60)

U2: 3, 6, ..., 60

U1: 4, 8, ..., 60



# COMPARISON: ASSET FAIRNESS

Asset Fairness: Equalize each user's sum of resource shares

Violates Sharing Incentive

Consider total of 70 CPUs, 70 GB RAM  
U1 needs <2 CPU, 2 GB RAM> per task  
U2 needs <1 CPU, 2 GB RAM> per task

*Prev slide*

*U1: 15 tasks  
U2: 20 tasks*

Sharing incentive?

Half of the cluster is 35 CPU, 35 GB RAM

U1: *17 tasks*

U2: *17 tasks*

*is better off with half of cluster*

# COMPARISON: CEEI

CEEI: Competitive Equilibrium from Equal Incomes

- Each user receives initially  $1/n$  of every resource,
- Subsequently, each user can trade resources with other users in a perfectly competitive market
- Computed by maximizing product of utilities across users

Utility: Number of  
tasks user gets

# COMPARISON: CEEI

Total: <9 CPU, 18 GB>

User1: <1 CPU, 4 GB>

User2: <3 CPU, 1 GB>

User 1

$\max(x \cdot y)$   
subject to

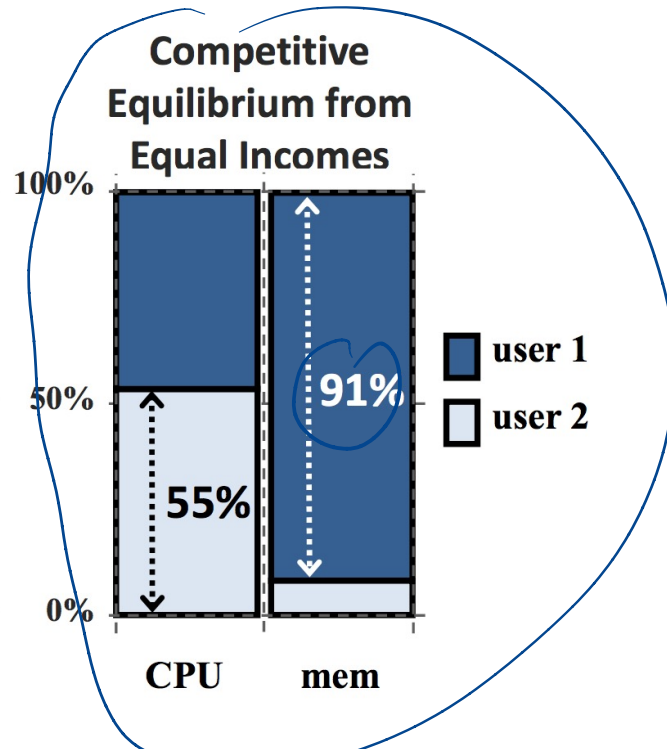
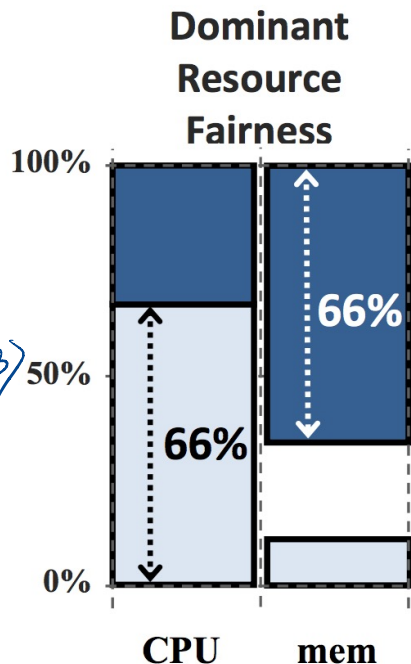
$$\begin{aligned} x + 3y &\leq 9 && \text{CPU} \\ 4x + y &\leq 18 && \text{Mem} \end{aligned}$$

User 1

$$x = 4.05$$

$$\langle 4.05, \sim 16 \text{ GB} \rangle$$

$$y = 1.62$$





# CEEI: STRATEGY PROOFNESS

Total: <9 CPU, 18 GB>

User2 Before:

CEEI: 55% CPU, 9% mem

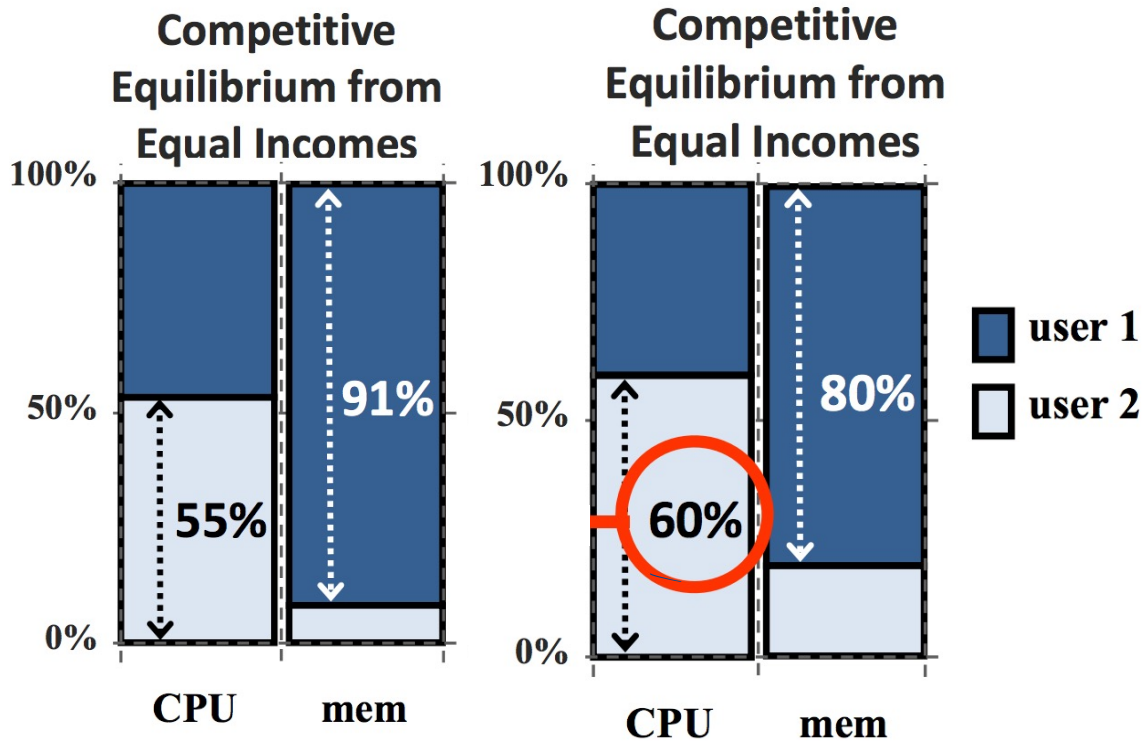
Total: <9 CPU, 18 GB>

User1: <1 CPU, 4 GB>

User2: <3 CPU, 2 GB>

$$\begin{aligned} \max \quad & x \cdot y \\ \text{s.t.} \quad & x + 3y \leq 9 \\ & 4x + 2y \leq 18 \end{aligned}$$

$$\begin{aligned} x &= 3.6 \\ y &= 1.8 \end{aligned}$$



# COMPARISON

Property	Allocation Policy		
	Asset	CEEI	DRF
Sharing Incentive		✓	✓
Strategy-proofness	✓		✓
Envy-freeness	✓	✓	✓
Pareto efficiency	✓	✓	✓
Single Resource Fairness	✓	✓	✓
Bottleneck Fairness		✓	✓
Population Monotonicity	✓		✓
Resource Monotonicity			

Table 2: Properties of Asset Fairness, CEEI and DRF.

# SUMMARY

DRF: Dominant Resource Fairness

Allocation policy for scheduling

Provides multi-resource fairness

Ensures sharing incentive, strategy proofness

# DISCUSSION

<https://forms.gle/wdN8bwqxEwjPEcAq7>

Consider a system with 40 units of CPU, 20 units of memory and 160 units of disk. Consider three users with the following requirements

Alice (4 CPU, 1 memory, 1 disk)

Bob (1 CPU, 4 memory and 4 disk)

Carol (1 CPU, 2 memory and 16 disk)

List the dominant resource as defined in DRF for Alice, Bob and Carol

$$\begin{array}{ll} \text{Alice} & \left\langle \left( \frac{1}{10} \right), \frac{1}{20}, \frac{1}{160} \right\rangle \quad \text{CPU} \\ \text{Bob} & \left\langle \frac{1}{40}, \frac{1}{5}, \frac{1}{40} \right\rangle \quad \text{Memory} \\ \text{Carol} & \left\langle \frac{1}{40}, \frac{1}{10}, \frac{1}{10} \right\rangle \quad \text{Memory / Disk} \end{array}$$

What would be the final task allocation in the given cluster for Alice, Bob and Carol ?

Alice	Dom. Share	Bob	Dom. Share	Carol	Dom. Share
4, 1, 1	$\frac{1}{10}$	1, 4, 4	$\frac{1}{5}$	1, 2, 16	$\frac{1}{10}$
8, 2, 2	$\frac{2}{10} = \frac{1}{5}$	2, 8, 8	$\frac{2}{5} = \frac{4}{10}$	2, 4, 32	$\frac{1}{5}$
12, 3, 3	$\frac{3}{10}$	<hr/> 1 task in static $\frac{1}{3}$ <sup>rd</sup> cluster		3, 6, 48	$\frac{3}{10}$
16, 4, 4	$\frac{4}{10}$			4, 8, 64	$\frac{4}{10}$
4 tasks		<hr/> Total: 40, 20, 160 $\frac{1}{3}$ <sup>rd</sup> = 13, 6.3, 53		3 tasks in static $\frac{1}{3}$ <sup>rd</sup> cluster	
3 tasks in static $\frac{1}{3}$ <sup>rd</sup> cluster					

What could be one workload / cluster scenario where DRF implemented on Mesos will NOT be optimal?

placement preferences or locality

# NEXT STEPS

Next Week: Machine Learning  
Assignment 2 out!