Ground Rules

- **(Grading)** You will be graded on the correctness as well as clarity of your solutions. Please state and prove any assumptions or claims that you make.

- **(Collaboration)** You are allowed to discuss questions with other people in the class. However, **you must solve and write your answers yourself without any help**. You must also give explicit citations to any sources besides the textbook and class notes, including discussions with classmates.

- **(Lateness)** Late submissions do not get any credit.

- Start working on your homework early. Plan your work in such a way that you have the opportunity to put some problems on the back burner for a while and revisit them later. Good luck!

Problems

1. **(10 pts)** Let $\textbf{FACT}_7 = \{n | n \text{ has a prime factor that is } \equiv 7 \, (\text{mod } 10)\}$. Prove that if $\textbf{FACT}_7$ is NP-complete, then NP = Co-NP. Note: this problem is not known to be in P.

   (Hint: Prove that $\textbf{FACT}_7$ is in NP∩Co-NP.)

2. **(10 pts)** Problem 7.26 in the book (pg. 297).

3. **(12 pts)** Consider the following problem: Given a Boolean circuit C, find the smallest circuit C' that computes the same Boolean function as C, where "smallest" refers to a lexicographic ordering of the description of the circuit.

   Show that this problem can be solved in polynomial time if and only if P = NP.

4. **(8 pts)** Problem 8.11 in the book (pg. 330).

5. **(10 pts)** Give a deterministic algorithm that runs in logarithmic space to determine whether a given undirected graph has a cycle.