CS 577: Introduction to Algorithms	Homework 4b
Out: 11/09/06	**Due: 11/16/06**

Ground Rules

- Grading. You will be graded on the correctness as well as clarity of your solutions. You are required to prove any claims that you make. In particular, when you are asked to design an algorithm, you must argue its correctness and running time.
- Collaboration. You are allowed to discuss questions with other people in the class. However, **you must solve and write your answers yourself without any help**. You must also give explicit citations to any sources besides the textbook and class notes, including discussions with classmates. Solutions taken from external sources such as the WWW, even if cited, will receive no credit unless there is significant "value added". In cases of doubt, you may be asked to explain your answer to the instructor and this will determine your grade.
- Lateness. Please see the class webpage for details on the lateness policy.
- This homework is due in *one week*. Start working early. Plan your work in such a way that you have the opportunity to put some problems on the back burner for a while and revisit them later. Good luck!

Problems

1. Problem 7.31 in the textbook (p. 434–435).
2. Problem 7.50 in the textbook (p. 447–448).
3. **(Fun with coins.)** A coin is said to have bias p if upon tossing it the probability that it comes up heads is p and the probability that it comes up tails is $1 - p$. In algorithm design, we are often interested in coins of bias $1/2$, but natural sources of randomness (e.g. whether the number of solar flares in a given year is even or odd) are mostly quite skewed.
   (a) Given a coin C of bias p, give an algorithm for generating a coin toss with bias $1/2$, using as few tosses of C (in expectation) as possible. State the expected number of tosses you require.
   Extra credit: Design an algorithm for the above that **does not** require knowledge of the bias p.
   (b) Sometimes, to the contrary, we have available a coin of bias $1/2$, but require a coin of bias p for some value $p < 1/2$. (As an example of when we might require a coin of small bias, recall the contention resolution problem discussed in class.)
   Give an algorithm for generating a coin toss of bias p using a coin C with bias $1/2$ using as few tosses of C as possible. State the expected number of tosses you make.
   (c) Given a deck of n cards, you want to shuffle the cards so that any permutation of the cards is equally likely. Show how to do this using a coin of bias $1/2$. How many coin tosses do you require?
   Extra credit: Prove that your answer is the best possible in terms of the number of coin tosses.