CS 577: Introduction to Algorithms (Section 1) — Homework 3

Out: 10/08/14 — **Due: 10/20/14**

Ground Rules

See homework 1. Please note: **this homework is due on a Monday and not on a Wednesday**.

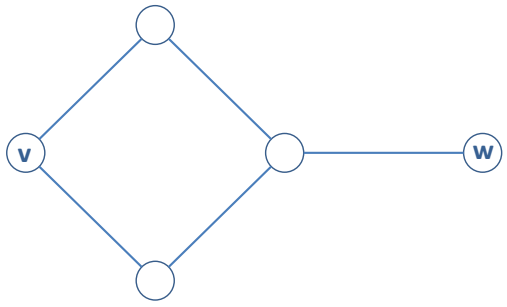
Ungraded problems

1. Problem 3.12 in the textbook (Pg. 112).
2. In class we saw that to prove the correctness of Dijkstra's algorithm we need to assert that all edges weights in the graph are nonnegative. In this problem we examine this assumption closely.
   (a) Consider the following algorithm for finding shortest paths in graphs with negative edge weights: add a large constant to each edge weight so that all the weights become positive; then run Dijkstra's algorithm. Is this algorithm correct? Prove your answer.
   (b) Suppose that all of the negative weight edges are those that leave the source node s. Can Dijkstra's algorithm, started at s, fail on such a graph? Prove your answer.
3. Problem 4.20 in the textbook (Pg. 199–200).

Graded problems

4. **(5 points)** Often there are multiple shortest paths between two nodes of a graph. Give a linear-time algorithm for the following task: given an undirected graph $G = (V, E)$ with unit edge lengths and nodes v and w, output the number of distinct shortest paths from v to w. For example, for the graph below, on input v and w your algorithm should output 2. (See problem 3.10 in the book for the "story" behind this question.)

   Hint: Think about modifying BFS.



5. **(5 points)** Problem 4.21 in the textbook (Pg. 200).
6. **(5 points)** Problem 4.5 in the textbook (Pg. 190).

   Hint: Use a greedy approach that "stays ahead".