CS 577: Introduction to Algorithms (Section 1)	Homework 5
Out: 11/12/14	Due: 11/26/14

Ground Rules

See homework 1.

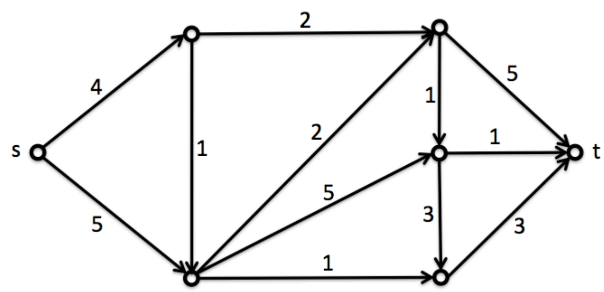
Ungraded problems

1. Problem 7.20 in the textbook (Pg. 426-27).
2. Problem 7.44 in the textbook (Pg. 444).
3. Problem 7.46 in the textbook (Pg. 444).

Graded problems

4. **(5 points)** An edge in a network is called *upper-binding* if increasing its capacity by one unit increases the maximum flow in the network. An edge is called *lower-binding* if reducing its capacity by one unit decreases the maximum flow in the network.

   (a) For the network G below determine the max s-t flow, f^*, the residual networkG_{f^*}, and a minimum s-t cut. Also identify all of the upper-binding and all of the lower-binding edges in the graph.



   (b) Develop an algorithm for finding all the upper-binding edges in a network G when given a maximum flow f^* in G. Your algorithm should run in linear time.

   (c) Develop an algorithm for finding all the lower-binding edges in a network G when given a maximum flow f^* in G. Your algorithm should run in time $O(mn)$.

5. **(5 points)** Consider the BINARY MAGIC SQUARE problem: Given a list of n integers $\vec{r} = (r_1, \cdots, r_n)$ and a list of m integers $\vec{c} = (c_1, \cdots, c_m)$, we ask whether there is an $n \times m$ grid of 0s and 1s such that row i sums to r_i and column j sums to c_j. We assume $\sum_i r_i = \sum_j c_j$.

   Examples: $n = m = 3$ with $\vec{c} = (1, 2, 0)$ and $\vec{r} = (1, 1, 1)$ (answer = yes) or $\vec{r} = (3, 0, 0)$ (answer = no):

Yes

1	0	0	1
0	1	0	1
0	1	0	1
1	2	0	

No

1	1	1	3
			0
			0
1	2	0	

Use Network Flow to design a polynomial-time algorithm to decide whether it is possible to construct a 0/1 grid that obeys the given $\vec{r}$ and $\vec{c}$ sums.

6. **(5 points)** Problem 7.27 in the textbook (Pg. 431).

7. **(Extra Credit)** Problem 7.51 in the textbook (Pg. 448).