

# CS 577: Introduction to Algorithms

## Sample Quiz 1

### Ground Rules

- This quiz consists of two questions and two pages. Please answer all questions.
- The quiz is closed book and closed notes. You are allowed to use one letter-size (one-sided) “cheat-sheet”.
- You may use any result that we showed in class or in the homework, or any result from the textbook by Kleinberg and Tardos. Clearly state the results that you use. Substantiate all other claims that you make.

### Questions

1. The diameter of a graph is the maximum distance between any two vertices in the graph, i.e., the maximum over all vertices  $u$  and  $v$  of the length of the shortest path between  $u$  and  $v$ .

Give a **linear time** algorithm for finding the diameter of an undirected **tree** in which each edge has unit length. Prove the correctness of your algorithm.

2. Given an array  $A$  of  $n$  (positive or negative) numbers, the *maximum range-sum* problem is to find the maximum over pairs of indices  $i$  and  $j$ ,  $i < j$ , of the sum  $A[i] + A[i + 1] + \cdots + A[j]$ . For example, if the array is

$$2, \quad -10, \quad 30, \quad -8, \quad 3, \quad 4, \quad -12, \quad 17, \quad -6, \quad -3, \quad 5$$

then, the maximum range-sum is  $30 - 8 + 3 + 4 - 12 + 17 = 34$ , attained by setting  $i = 3$  and  $j = 8$ .

Give an  $O(n \log n)$  time algorithm for finding the maximum range-sum of an array of length  $n$ . Your algorithm only needs to return the sum and not the indices  $i$  and  $j$  for which the sum is attained. Prove the correctness as well as running time of your algorithm.