CS 577: Introduction to Algorithms	Homework 8
Out: 04/20/16	**Due: 04/27/16**

Ground Rules

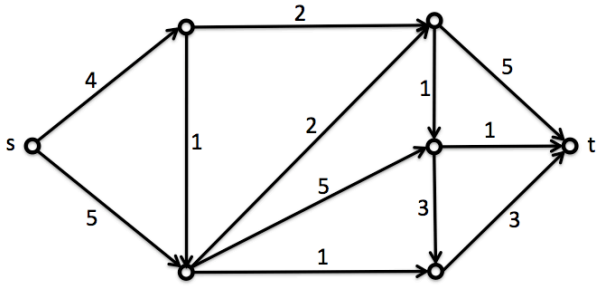
- See HW1.
- All problems will be graded.

Problems

1. The manager at a local toy store, Algos-R-Us, is in need of some algorithmic expertise. A few days back he received a shipment of Russian nesting dolls that was damaged in transit. All of the sets were disassembled, that is, none of the dolls were nested inside another. There are n dolls in all and k boxes to pack them into. The manager needs to figure out how to assemble the n dolls into k or fewer nested sets, if at all possible. For any two dolls, it is possible to tell if one can be nested inside the other, but there is no other way of telling whether two dolls belong to the same set. For some pairs of dolls, neither can be nested inside the other (e.g. one may be taller than the other and the other wider than the first). Design an efficient algorithm to find a partition of the n dolls into k or fewer nested sets, if such a partition exists.

2. An edge in a network is called *upper-binding* if increasing its capacity by one unit increases the maximum flow in the network. An edge is called *lower-binding* if reducing its capacity by one unit decreases the maximum flow in the network.

   (a) For the network G below determine the max s-t flow, f^*, the residual network G_{f^*}, and a minimum s-t cut. Also identify all of the upper-binding and all of the lower-binding edges in the graph.



   (b) Develop an algorithm for finding all the upper-binding edges in a network G when given a maximum flow f^* in G. Your algorithm should run in linear time.

   (c) Develop an algorithm for finding all the lower-binding edges in a network G when given a maximum flow f^* in G. Your algorithm should run in time $O(mn)$.