# Final Examination

CS 525 - Fall 2009

Monday, December 21, 2009, 10:05a-12:05p.

Each question is worth the same number of points.

No electronic devices, notes, or books allowed, except that you may bring one standard-size sheet of paper, handwritten on both sides, into the test. **You need to give reasoning and justify all your answers**, quoting any theorems you use.

1. Solve the following linear program: If it is unbounded, give a direction of unboundedness.

$$\begin{aligned} \min\ & 4x_1 - 12x_2 - x_3 \\ \text{subject to}\ & 2x_1 + x_3 \geq 2, \\ & 3x_2 + x_3 = 1, \\ & x_1 \text{ free},\ x_2, x_3 \geq 0. \end{aligned}$$

2. Solve the following linear program for all values of the parameter $t$ in the interval $(-\infty, \infty)$. For each piece of the solution indicate clearly: parameter range, solution $x(t)$, and optimal objective value $z(t)$.

$$\begin{aligned} \min\ & -x_1 + t(x_1 + x_2) \\ \text{subject to}\ & -x_1 + x_2 \geq -1, \\ & -x_2 \geq -3, \\ & x_1, x_2 \geq 0. \end{aligned}$$

3. Consider the following quadratic program:

$$\begin{aligned} \min\ & x_1^2 + 2x_1x_2 + 2x_2^2 + x_1 \\ \text{subject to}\ & x_1 + 2x_2 \geq 3, \\ & x_1 - x_2 \geq -2, \\ & x_1, x_2 \geq 0. \end{aligned}$$

   (a) Write down KKT conditions for this problem.

   (b) Solve the problem using Lemke's method.

   (c) Does the solution change if we change the coefficient of $x_1$ in the objective from 1 to 4? Explain.

4. Given a matrix $A$, show that *exactly one* of the following two statements is true:

   I. There is a vector $x$ such that $Ax > 0$ (that is, all components of $Ax$ are strictly positive);

   II. There is a vector $u$ such that $A'u = 0$, $u \geq 0$, and $u \neq 0$.

5. A husband and wife are deciding how to spend their evening. The wife prefers to go to a baseball game, while the husband would prefer to visit an art gallery, but in either case, they would like to go out together. In formulating their decision as a bimatrix game, the loss matrix entry for each person is 4 if they go to separate events, 1 if they go to their preferred event together, and 2 if they go together to their less favored event together.

   (a) Write down the loss matrix $A$ for the wife and $B$ for the husband. Let strategy 1 be "attend baseball" and strategy 2 be "visit art gallery".

   (b) If the randomized strategy vectors for the husband and wife are denoted by $x$ and $y$, respectively, show that $\bar{x} = (1, 0)'$ and $\bar{y} = (1, 0)'$ is a Nash equilibrium pair.

   (c) Show that $\bar{x} = (0.6, 0.4)'$ and $\bar{y} = (0.4, 0.6)'$ is also a Nash equilibrium pair.