Final Examination

CS 525 - Fall 2010

Thursday, December 23, 2010, 10:05a-12:05p.

Each question is worth the same number of points.

No electronic devices, notes, or books allowed, except that you may bring one standard-size sheet of paper, handwritten on both sides, into the test. **Give reasoning and justify all your answers**, quoting any theorems you use.

1. (a) Solve the following linear program: If it is unbounded, give a direction of unboundedness.

$$\begin{aligned} \min\ & -2x_1 + 3x_2 + x_3 \\ \text{subject to}\ & x_1 + x_3 \geq 2, \\ & 4x_1 - x_2 = -1 \\ & (x_1, x_2, x_3) \geq 0. \end{aligned}$$

   (b) Formulate a linear program that finds the Chebyshev approximate solution of the system $Ax = b$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ 4 & 7 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 2 \\ 10 \end{bmatrix}.$$

   Write the problem in tableau form. *Do not solve* it, but indicate which element of the tableau would be your first pivot.

2. Solve the following linear program for all values of the parameter t in the interval $(-\infty, \infty)$. For each piece of the solution indicate clearly:

parameter range, solution $x(t)$, and optimal objective value $z(t)$.

$$\begin{aligned} \min\ & x_1 + 2x_2 \\ \text{subject to}\ & -x_1 \geq 4 + t, \\ & x_1 + x_2 \geq -2t, \\ & x_1, x_2 \geq 0. \end{aligned}$$

3. Consider the following quadratic program:

$$\begin{aligned} \min\ & x_1^2 - 2x_1x_2 + 4x_2^2 + 2x_1 \\ \text{subject to}\ & 2x_1 + x_2 \geq 4, \\ & x_1, x_2 \text{ both free.} \end{aligned}$$

   (a) Write down the KKT conditions for this problem.

   (b) Solve the problem using Lemke's method.

   (c) Does the solution change if we change the constraint to an equality constraint: $2x_1 + x_2 = 4$? Explain.

4. Suppose that in a two-player zero-sum game, the loss matrix for Player 1 is as follows:

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

   (a) Write down the expected loss for Player 1, when Player 1 plays a randomized strategy $x = (x_1, x_2, x_3)$ and Player 2 plays a randomized strategy $y = (y_1, y_2, y_3)$.

   (b) Show that Player 1 can achieve a *negative* expected loss (i.e. an expected gain) if if Player 2 plays any strategy *other than* $(y_1, y_2, y_3) = (1/3, 1/3, 1/3)$.

   (c) Show that $\bar{x} = (1/3, 1/3, 1/3)$ and $\bar{y} = (1/3, 1/3, 1/3)$ form a Nash equilibrium pair.

   (d) Let $\bar{x} = (1/3, 1/3, 1/3)$ as in part (c). Is it possible for $(\bar{x}, \hat{y})$ to be a Nash equilibrium pair, for some strategy vector $\hat{y}$ *not equal to* $(1/3, 1/3, 1/3)$? Explain.