Final Examination

CS 525 - Fall 2012

Monday, December 17, 2010, 12:25p-2:25p.

Each question is worth the same number of points.

No electronic devices, notes, or books allowed, except that you may bring one standard-size sheet of paper, handwritten on both sides, into the test. **Give reasoning and justify all your answers**, quoting any theorems you use.

1. Consider the following linear program.

$$\begin{aligned} \min_{x_1,x_2} \; & -2x_1 + 3x_2 \\ \text{subject to} \quad & x_1 + x_2 \geq -2 \\ & -x_1 + 2x_2 = 1 \\ & 0 \leq x_1 \leq 5, \quad (x_2 \text{ free}). \end{aligned}$$

   (a) Solve this problem. (Hint: Use Scheme II.)

   (b) Write down the KKT conditions for this problem.

   (c) Write down the dual, in any suitable form.

   (b) Find the solution of the dual, and check that the optimal objectives are the same for primal and dual.

2. Show that exactly one of the following two systems has a solution:

$$\begin{aligned} (I): \quad & Ax = b, \;\; 0 \leq x \leq e, \\ (II): \quad & A^T u - v \leq 0, \;\; v \geq 0, \;\; b^T u - e^T v = 1, \end{aligned}$$

   where e is the vector of 1s, with the same dimension as x.

3. Consider the following problem, which has variables (x_1, x_2), an absolute value term in the objective, and a parameter t (which appears in both objective and constraints):

$$\min_{x_1, x_2} |x_1 + t| \text{ subject to } 2x_1 + 3x_2 \geq -2 + 3t, \quad (x_1, x_2) \geq 0.$$

   (a) By introducing an auxiliary variable x_3, express this problem equivalently as a linear program, in which the parameter t appears only in the constraints.

   (b) Find solutions of the linear program for all values of the parameter t. Tabulate the values of (x_1, x_2) and the optimal objective as a function of t.

4. Consider the following problem in a single variable x_1:

$$\min_{x_1} x_1^2 - 2x_1 + \max(0, 5 - x_1) \text{ subject to } x_1 \geq 0.$$

   (a) By introducing an auxiliary variable x_2, reformulate this problem as an equivalent quadratic program.

   (b) Write down the KKT conditions for this quadratic program.

   (c) Use Lemke's method to find the optimal value of x_1.

   (d) Is the solution you found a *global* solution of the quadratic programming formulation? Explain why or why not.