Final Examination

CS 525 - Spring 2014

Monday, May 12, 2014, 5:05p-7:05p.

No electronic devices, notes, or books allowed, except that you may bring one standard-size sheet of paper, handwritten on both sides, into the test. **Give reasoning and justify all your answers**, quoting any theorems you use.

1. Consider the following linear program in which the objective is parametrized by a scalar t:

$$\begin{aligned} & \min_{x_1,x_2} 2x_1 + (3+t)x_2 \\ \text{subject to} \quad & \frac{1}{2}x_1 + x_2 \geq \frac{1}{2}, \\ & x_1 - 3x_2 \geq -3, \\ & (x_1, x_2) \geq 0. \end{aligned}$$

   Solve this problem for all values $t \in (-\infty, \infty)$. Tabulate your solutions $x(t)$ and the corresponding objective function values $z(t)$ on each interval of t values.

2. Given a matrix B, show that exactly one of these two systems has a solution:

$$\text{(I)} \quad 0 \neq Bx \geq 0,$$

$$\text{(II)} \quad B^T y = 0, \quad y > 0.$$

   (Hint: Construct a primal-dual pair of linear programs and use a duality theorem.)

3. Consider the following linear program:

$$\begin{aligned} \min \quad & 4x_1 - 5x_2 - x_3 \\ \text{subject to} \quad & 2x_1 + x_3 \geq 2, \\ & 3x_2 + x_3 = 1, \\ & x_1 \text{ free}, \ x_2, x_3 \geq 0. \end{aligned}$$

   (a) Solve this problem. (If the problem is unbounded, give a direction of unboundedness.)

   (b) Write down the dual of this problem.

   (c) Write down the KKT conditions for this problem.

   (d) Without doing any further pivots, find the solution of the dual.

   (e) Will the solution of the original (primal) problem change if we change the coefficient of x_2 in the objective from -5 to -8? Explain.

4. A husband and wife are deciding how to spend their evening. The wife prefers to go to a baseball game, while the husband would prefer to visit an art gallery, but in either case, they would like to go out together. In formulating their decision as a bimatrix game, the loss matrix entry for each person is 4 if they go to separate events, 1 if they go to their preferred event together, and 2 if they go together to their less favored event together.

   (a) Write down the loss matrix A for the wife and B for the husband. Let strategy 1 be "attend baseball" and strategy 2 be "visit art gallery".

   (b) Is this a zero-sum game?

   (c) If the randomized strategy vectors for the wife and husband are denoted by x and y, respectively, show that $\bar{x} = (1, 0)'$ and $\bar{y} = (1, 0)'$ is a Nash equilibrium pair.

   (d) Show that $\bar{x} = (0.6, 0.4)'$ and $\bar{y} = (0.4, 0.6)'$ is also a Nash equilibrium pair.

   (e) By inspection, can you identify any other Nash equilibria?