Final Exam

CS 525, Semester I, 1999-2000

Friday December 17, 1999
2 hours (starting 12:25)

All questions carry equal credit. No calculators allowed. Be sure to quote any results you use accurately.

Use a different book for questions 3 and 4.

1. Use the simplex method to solve the following problem:

$$\begin{array}{lccccccc}
\min & & & 3x_1 + 2x_2 - 5 & & & & \\
 & 5x_1 & + & 5x_2 & + & x_3 & \geq & 23 \\
\text{subject to} & x_1 & + & \frac{1}{2}x_2 & + & \frac{1}{2}x_3 & \geq & 4 \\
 & x_1 & + & x_2 & & & \geq & 5 \\
 & x_1 \leq 3 & & & x_2 \geq 1 & & & x_3 \geq 0
\end{array}$$

   Is the solution unique?

2. Let $z(t)$ be the solution of

$$\begin{array}{ll}
\min & 2 + x_1 + 3x_2 \\
\text{subject to} & x_1 + x_2 \geq 4 - t \\
 & x_1 - x_2 \geq 1 - 2t \\
 & x_1, x_2 \geq 0
\end{array}$$

   Find $z(t)$ for all values of t. For $-3 \leq t \leq 4$, what is the dual solution? Is this dual solution unique?

3. Solve

$$\begin{array}{ll}
\min & \frac{1}{2}x_1^2 + x_1x_2 + \frac{3}{2}x_2^2 - x_1 - x_2 + 4 \\
\text{subject to} & x_1 + x_2 \geq 3 \\
 & x_1 - x_2 \geq 1 \\
 & x_1, x_2 \geq 0
\end{array}$$

   What are the optimal multipliers on the constraint equations? Make sure you quote any theorems that you use accurately.

4. Show that $Ax = b$, $x \geq l$ has no solution if and only if there exists a vector (u, v) with $\|(u, v)\|_\infty = 1$, $A^T u + v = 0$, $v \geq 0$ and $b^T u + l^T v > 0$.