Final Examination

CS 525 - Spring 2002

Monday, May 13, 2002

Write out your final solution to each problem clearly and unambiguously.

Instructions (Please read carefully): If a problem has no solutions, many solutions, or is infeasible or unbounded, you must clearly state this to be the case and justify your statement. When the problem is unbounded, give a direction of unboundedness.

There are FIVE (5) questions. All questions are worth the same number of points.

Problems 1 and 5 each can be solved in three pivots or fewer.

1. Solve the following linear program.

$$\begin{array}{lrcrcrcr}
\min & x_1 & + & 3x_2 & + & 4x_3 & & \\
\text{subject to} & 3x_1 & + & x_2 & - & x_3 & \geq & -1, \\
 & x_1 & - & x_2 & + & 2x_3 & \geq & 3, \\
 & x_1 & + & 2x_2 & + & x_3 & \geq & 2, \\
 & x_1 & , & x_2 & , & x_2 & \geq & 0.
\end{array}$$

2. Suppose that at iteration k in the infeasible path-following interior-point method (Algorithm IPF), the algorithm takes a step length of α_k. Express the values of the following two vectors:

$$\mathcal{A}x^{k+1} - b$$
$$\mathcal{A}'y^{k+1} + s^{k+1} - c$$

in terms of α_k, the previous iterate (x^k, y^k, s^k), and the problem data $\mathcal{A}$, b, and c.

(Note that c and p in the class notes are used interchangeably; they both denote the cost vector for the primal linear program.)

3. Consider the linear program

$$\min_{x,y} \; c'x + d'y \;\; \text{subject to} \;\; Ax + By \geq b, \;\; x \geq 0, y \geq 0,$$

where A is a square matrix that satisfies $A \geq I$ (where I is the identity matrix), and $c \leq 0$. Suppose that this problem has a solution.

   (i) Write down the dual for this problem.

   (ii) By using the properties of A and c, find the solution of the dual. (Explain your reasoning.)

   (iii) What is the optimal objective value of the original (primal) problem? (Give reasons for your answer.)

4. Given the following linear program, define a Phase I problem (by adding artificial variables and defining an appropriate objective). Give a basic feasible starting point for the Phase I problem. (Do not try to solve it.)

$$\begin{array}{lrcrcrcr}
\min & -3x_1 & + & 4x_2 & + & 7x_3 & & \\
\\
\text{subject to} & x_1 & + & 3x_2 & + & 4x_3 & = & 10, \\
 & -x_1 & + & 2x_2 & + & x_3 & = & -5, \\
 & & & & & -1 \leq & x_1 & \leq \; 4, \\
 & & & & & 0 \leq & x_2 & \leq \; 1, \\
 & & & & & -1 \leq & x_3. &
\end{array}$$

5. Solve the following quadratic program by Lemke's method.

$$\begin{array}{ll}
\min & 2x_1^2 + x_2^2 + 2x_1x_2 - x_1 - x_2 \\
\\
\text{subject to} & x_1 + 2x_2 \geq 2, \\
 & x_1 \geq 0, x_2 \geq 0
\end{array}$$