Final Examination

CS 525 - Spring 2003

Monday, May 12, 2003, 10:05am-12:05pm

Each question is worth 20 points. Each problem that involves tableaus can be solved in two pivots or fewer.

1. Solve the following linear program: If it is unbounded, give a direction of unboundedness.

$$\begin{array}{rcl} \min\ \ 4x_1 + x_2 & & \text{subject to} \\ 2x_1 + 3x_2 & \geq & -2, \\ x_1 + x_2 & = & 5, \\ x_1 & \leq & 10, \\ x_1 & \geq & 0, \quad x_2 \text{ free.} \end{array}$$

   (Hint: Use Scheme II.)

2. Consider the following linear program in standard form:

$$\min_x p'x \ \text{ subject to } \ Ax \geq b, \ \ x \geq 0.$$

   Suppose we know the following two facts about this problem: (i) its solution exists, and the optimal objective value is zero; (ii) all components of the right hand side b are strictly positive.

   (a) Write down the dual of this linear program.

   (b) Show that the dual has solution $u = 0$. (Mention any theorems you use in deriving this result.)

   (c) Show that $p \geq 0$.

3. Solve the following linear program for all values of the parameter t in the interval $(-\infty, \infty)$:

$$
\begin{array}{rcl}
\min \;\; x_1 + t(-2x_1 - x_2) & & \text{subject to} \\
x_1 - x_2 & \geq & -2, \\
-2x_1 + x_2 & \geq & -4, \\
x_1, x_2 & \geq & 0.
\end{array}
$$

Indicate clearly each interval of t, together with the solution and optimal objective value for this interval.

4. Consider the following linear program with bounds:

$$
\begin{array}{llllllllll}
\min & -4x_1 & + & 5x_2 & + & x_3 & & & & \\
\text{subject to} & x_1 & + & 2x_2 & + & 4x_3 & = & 10, & & \\
& -x_1 & + & 3x_2 & + & 2x_3 & = & -5, & & \\
& & & & & -1 & \leq & x_1 & \leq & 2, \\
& & & & & 0 & \leq & x_2 & \leq & 2, \\
& & & & & -2 & \leq & x_3. & &
\end{array}
$$

(a) Define a Phase I problem, by adding artificial variables and defining an appropriate objective.

(b) Give a basic feasible starting point for the Phase I problem. (Do not try to solve it.)

5. Solve the following approximation problem for x:

$$
\min_x \; \|Ax - b\|_\infty, \;\; \text{subject to} \;\; x \geq 0,
$$

where

$$
A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \;\; b = \begin{bmatrix} 0 \\ 2 \end{bmatrix}.
$$

(a) Write down a linear programming formulation of this problem containing three variables. (Remember to include the constraints $x \geq 0$.)

(b) Solve this linear program using the dual simplex method, and write down the optimal values of x_1 and x_2.