Final Examination

CS 525 - Spring 2004

Thursday, May 13, 2004, 12:25pm-2:25pm

Write out your final solution to each problem clearly and unambiguously.
There are FIVE questions, each worth 20 points.
Each problem that involves tableaus can be solved in three pivots or fewer.

1. Solve the following linear program.

$$\begin{array}{llrcrcrcl} \text{min} & & 2x_1 & + & x_2 & + & 2x_3 & & \\ \text{subject to} & & 4x_1 & - & x_2 & + & 2x_3 & \geq & 4, \\ & & 2x_1 & + & 2x_2 & - & x_3 & \geq & 3, \\ & & x_1 & , & x_2 & , & x_3 & \geq & 0. \end{array}$$

2. Given the following linear program, define a Phase I problem (by adding artificial variables and defining an appropriate objective). Give a basic feasible starting point for the Phase I problem. (Do not try to solve it.)

$$\begin{array}{llrcrcrcrcl} \text{min} & & 3x_1 & + & 4x_2 & + & 4x_3 & & & & \\ \text{subject to} & & 2x_1 & - & 2x_2 & + & x_3 & = & 8, & & \\ & & x_1 & + & 6x_2 & + & 3x_3 & = & 12, & & \\ & & & & & & -5 & \leq & x_1, & & \\ & & & & & & 0 & \leq & x_2 & \leq & 2, \\ & & & & & & & & x_3 & \leq & 3. \end{array}$$

3. Solve the following problem for all t in the range $(-\infty, \infty)$. Tabulate your results, showing each range of t values together with the corresponding solution and the corresponding values of $z(t)$. Draw a graph of the optimal objective $z(t)$ vs t.

$$\begin{array}{lrcl} \text{minimize } z(t) = & x_1 + 2x_2 + t(2x_1 - x_2) & & \\ & x_1 - x_2 & \geq & 0 \\ \text{subject to} & -x_1 + 4x_2 & \geq & -3 \\ & x_1, x_2 & \geq & 0. \end{array}$$

4. Consider the following system of algebraic relationships:

$$\begin{array}{rcl} 3x_1 + 2x_2 & \geq & 5, \\ x_1 - x_2 & = & 2. \end{array}$$

Formulate (but do not solve!) a linear program that minimizes the violation of these relationships in the ℓ_1 sense, subject to the constraints $x_1 \geq 0$ and $x_2 \geq 0$.

(Write out your linear program clearly, in "general LP" form.)

5. Given a matrix A and a vector b, show that there exists a vector z such that $b'z > 0$, $A'z \leq 0$, and $z \geq 0$ if and only if there is *no* vector x such that $Ax \geq b$ and $x \geq 0$. Explain your reasoning clearly.