

Final Examination

CS 525 - Spring 2005

Friday, May 13, 2005, 5:05pm-7:05pm

Write out your final solution to each problem clearly and unambiguously. There are FIVE questions, each worth 20 points. Each problem that involves tableaus can be solved in three pivots or fewer.

1. Solve the following linear program:

$$\begin{array}{ll} \min & 3x_1 + 2x_2 \\ & x_1 + x_2 \geq -2, \\ & 2x_1 - x_2 = 2, \\ \text{subject to} & x_1 \leq 5, \\ & x_1 \geq 0, \\ & x_2 \text{ free.} \end{array}$$

2. Solve the following quadratic program:

$$\begin{array}{ll} \min_x & 2x_1^2 + 2x_1x_2 + x_2^2 - x_1 - 2x_2 \\ & 2x_1 + 2x_2 \geq 1, \\ \text{subject to} & x_1 - x_2 \geq -2, \\ & x_1 \geq 0, x_2 \geq 0. \end{array}$$

3. Find a solution to the following ℓ_1 approximation problem:

$$\min_{x \geq 0} \|Ax - b\|_1,$$

where

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 3 \end{bmatrix}.$$

(Be sure to enforce the nonnegativity constraints $x \geq 0$.)

4. Solve the following linear program for all values of the parameter t in the interval $(-\infty, \infty)$. For each piece of the solution indicate clearly: parameter range, solution $x(t)$, and optimal objective value $z(t)$.

$$\begin{array}{ll} \min & -x_1 + t(3x_1 + x_2) \\ & -x_1 + x_2 \geq -1, \\ \text{subject to} & -x_2 \geq -3, \\ & x_1 \geq 0, x_2 \geq 0. \end{array}$$

5. Suppose that A is an $n \times n$ matrix with the property $A' = -A$, that c is a vector with n elements, and that the following linear program has a solution:

$$\min_x c'x \text{ subject to } Ax \geq -c, \quad x \geq 0.$$

What is the optimal objective value of this problem?