FINAL – CS525 – SPRING 2008.

SOLUTIONS

1. Tableau form:

	x_1	x_2	1
$x_3 =$	1	1	2
$x_4 =$	2	-1	-1
$x_5 =$	-1	0	4
$z =$	3	-2	0

x_4 = slack var for equality constraint

x_2 = free var.

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Scheme II pivot: (2,2) yields..

	x_1	x_4	1
$x_3 =$	3	-1	1
$x_2 =$	2	-1	-1
$x_5 =$	-1	0	4
$z =$	-1	2	2

delete x_4 column, move x_2 to bottom.

	x_1	1
$x_3 =$	3	1
$x_5 =$	-1	4
$z =$	-1	2
$x_2 =$	2	-1

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Tableau is primal feasible, proceed with "Phase II": pivot on (2,1)

	x_5	1
$x_3 =$	-3	13
$x_1 =$	-1	4
$z =$	1	-2
$x_2 =$	-2	7

(5)

(3) optimal! solution is $x^* = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$.

2. PA in tableau:

	x_1	x_2	x_3	1
$x_4 =$	3	1	-1	1
$x_5 =$	1	-1	2	-3
$x_6 =$	1	2	1	-4
$z =$	1	3	4	0

(6)

Dual feasible, so use dual simplex.
Select pivot row 3, ratio test gives col 1.

	x_6	x_2	x_3	1
$x_4 =$	3	-5	-4	13
$x_5 =$	1	-3	1	1
$x_1 =$	1	-2	-1	4
$z =$	1	1	3	4

(11)

Optimal! with solution

3 $x^* = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix}$.

3. Tableau form:

	x_1	x_2	1
$x_3 =$	1	-1	0
$x_4 =$	-1	2	4
$z =$	1	1	0
$z_0 =$	-1	0	0

Tableau is optimal for $t=0$.
In fact it is optimal with solution

$$x(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad z(t) = 0$$

whenever t satisfies

$$1-t \geq 0 \qquad 1 \geq 0$$

that is,

$$t \in (-\infty, 1].$$

5 As t crosses 1, first reduced cost becomes negative, so choose pivot column 1.
Ratio test gives row 2.

	x_4	x_2	1
$x_3 =$	-1	1	4
$x_1 =$	-1	2	4
$z =$	-1	3	4
$z_\delta :$	-1	-2	-4

optimal if $\quad -1 + t \geq 0, \quad 3 - 2t \geq 0$
which is true for $t \in [1, 3/2]$.

solution for this range is $x(t) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, $z(t) = 4 - 4t$

(5)

as t crosses $3/2$, second reduced cost goes negative. But there's no suitable pivot row, so we have unboundedness for $t > 3/2$

(4)

Summary:

$t \in (-\infty, 1]$: $x(t) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $z(t) = 0$

$t \in [1, 3/2]$: $x(t) = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$, $z(t) = 4 - 4t$.

(3) $t \in (3/2, \infty)$ unbounded.

At $t = 3/2$, have solution set

$$x(3/2) = \begin{pmatrix} 4 + 2\alpha \\ \alpha \end{pmatrix} \text{ for all } \alpha \geq 0,$$

with $z(3/2) = 4 - 4(3/2) = -2$.

(3)

4. replace $|x_2|$ by x_3, with constraints

$$x_3 \geq x_2$$
$$x_3 \geq -x_2$$

Thus objective becomes

$$\max(x_1 + x_2 + 4, -x_1 + 3x_3 - 2)$$

now use x_4 to represent objective, where

$$x_4 \geq x_1 + x_2 + 4$$
$$x_4 \geq -x_1 + 3x_3 - 2$$

Thus the LP formulation is.

$$\min \quad x_4$$

$$\text{s.t.} \quad \begin{aligned} x_4 &\geq x_1 + x_2 + 4 \\ x_4 &\geq -x_1 + 3x_3 - 2 \\ x_3 &\geq x_2 \\ x_3 &\geq -x_2 \\ 3x_1 + x_2 &\geq -5 \\ x_1 &\geq 0 \end{aligned}$$

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5. (i) Dual is

$$\max\ b'u \quad \text{s.t.}\quad A'u \le c$$
$$B'u = d$$
$$u \ge 0.$$

(6)

(ii) Since all elements of A are positive, and $u \ge 0$, we have $A'u \ge 0$, with $A'u = 0$ only if all components of u are zero. Since $c \le 0$, the constraint $A'u \le c$ can in fact be satisfied only if all components of u are zero. Hence $u = 0$ is the only possible feasible point for the dual.

(6)

Since the primal has a solution, the dual must also have a solution (strong duality) and it must be $u^* = 0$.

(3)

(iii) by strong duality, optimal primal objective = optimal dual objective $= b'u^* = 0$.

(5)