Final Examination

CS 525 - Spring 2008

Sunday, May 11, 2008, 2:45pm-4:45pm

Write out your final solution to each problem clearly and unambiguously, and explain your reasoning in each case.

There are FIVE questions, each worth 20 points.

Each problem that involves tableaus can be solved in three pivots or fewer.

1. Solve the following linear program:

$$\begin{array}{lrcl} \min & \multicolumn{3}{c}{3x_1 - 2x_2} \\ & x_1 + x_2 & \geq & -2, \\ & 2x_1 - x_2 & = & 1, \\ \text{subject to} & x_1 & \leq & 4, \\ & x_1 & \geq & 0, \\ & x_2 & & \text{free.} \end{array}$$

   (Hint: Use Scheme II.)

2. Solve the following linear program.

$$\begin{array}{lrcrcrcr} \min & x_1 & + & 3x_2 & + & 4x_3 & & \\ & & & & & & & \\ \text{subject to} & 3x_1 & + & x_2 & - & x_3 & \geq & -1, \\ & x_1 & - & x_2 & + & 2x_3 & \geq & 3, \\ & x_1 & + & 2x_2 & + & x_3 & \geq & 4, \\ & x_1 & , & x_2 & , & x_2 & \geq & 0. \end{array}$$

3. Solve the following linear program for all values of the parameter t in the interval $(-\infty, \infty)$. For each piece of the solution indicate clearly: parameter range, solution $x(t)$, and optimal objective value $z(t)$.

$$\begin{array}{ll} \min & (1-t)x_1 + x_2 \\ \text{subject to} & \begin{array}{rcl} x_1 - x_2 & \geq & 0, \\ -x_1 + 2x_2 & \geq & -4, \\ x_1 & \geq & 0, \\ x_2 & \geq & 0. \end{array} \end{array}$$

4. Formulate *but do not solve* the following problem as a linear program (in any convenient form):

$$\min \max(x_1+x_2+4, -x_1+3|x_2|-2) \text{ subject to } 3x_1+2x_2 \geq -5, \quad x_1 \geq 0.$$

5. Consider the following linear program in general form:

$$\min c'x + d'y \text{ subject to } Ax + By \geq b, \quad x \geq 0.$$

Suppose that all elements of the matrix A are strictly positive and that all elements of the vector c are nonpositive. Suppose that this problem has a solution.

(i) Write down the dual of this problem.

(ii) By using the given properties of A and c, find a solution for the dual.

(iii) What is the optimal objective for the primal problem stated above?

Be sure to explain your reasoning for parts (ii) and (iii).