Final Exam

CS 525, Semester II, 1996-97

Thursday May 15, 1997
2 hours (starting 5:00)

All questions carry equal credit. Be sure to quote any results you use accurately. Use a different book for questions 4 and 5.

1. Use the simplex method to solve the following problem:

$$\begin{array}{llcccccc} \max & & & 2x_1 + x_2 - x_3 & & & & \\ \text{subject to} & 5x_1 & & & + & x_3 & \geq & 1 \\ & -2x_1 & & & + & x_3 & \leq & 22 \\ & -4x_1 & + & x_2 & - & x_3 & \leq & -6 \\ & x_1 \leq 0 & & x_2 \geq 3 & & & & x_3 \geq 0 \end{array}$$

2. Let $z(t)$ be the solution of

$$\begin{array}{ll} \max & -x_1 - x_2 - x_3 \\ \text{subject to} & 1 + 3t \leq x_1 + x_2 - x_3 \\ & -x_1 + x_2 \leq -2 - t \\ & x_1, x_2 \geq 0 \end{array}$$

   Find $z(t)$ for all values of t. What properties does this function of t have?

3. Solve

$$\begin{array}{ll} \max & x_1 - x_2 - x_1^2 - x_2^2 - 2x_1x_2 \\ \text{subject to} & x_1 + x_2 \geq 1 \\ & x_1, x_2 \geq 0 \end{array}$$

   Make sure you quote any theorems that you use accurately.

4. Show that $Ax \geq b$, $x \geq 0$ has no solution if and only if there exists a point z with $\|z\|_1 = 1$, $A^T z \leq 0$, $z \geq 0$ and $b^T z > 0$.

5. Let the n–vector x represent a production schedule in a certain company. As a manager, you want to find a "good" production schedule. After months of investigation you find that there are $m(m > n)$ constraints on x:

$$\sum_{j=1}^{n} A_{ij} x_j = b_j \quad i = 1, 2, \ldots, m$$

These constraints are so numerous that it is unlikely that any production schedule will satisfy them all. Fortunately, however, these constraints are "loose"in the sense that they need not be satisfied if you are willing to pay a penalty for their violation. More specifically, you are required to pay c_i yen for each unit of violation (either excess or shortage) of the i–th constraint ($c_i > 0$). Formulate, as a linear program, the problem of finding x which minimizes the total penalty incurred.