Midterm Examination II

CS 525 - Spring 2005

Wednesday, March 16, 2005, 2:25-3:25p

Each question is worth 20 points. Each problem that involves tableaus can be solved in three pivots or fewer.

If a problem has no solution or multiple solutions, you must state so clearly and justify your claim.

No calculators allowed, You may bring one standard-size sheet of paper, handwritten on both sides, into the test.

3. Solve the following linear program.

$$
\begin{array}{lrcl}
\min & 2x_1 + x_2 + x_3 & & \\
\text{subject to} & 3x_1 + 2x_2 - x_3 & \geq & -2, \\
& x_1 + x_2 - x_3 & = & 1, \\
& -3x_1 - x_2 + x_3 & \geq & 3, \\
& x_1, x_2 & \geq & 0, \\
& x_3 & & \text{free}
\end{array}
$$

4. Consider the following linear program:

$$
\begin{array}{lrcl}
\min & x_1 + x_2 + 3x_3 & & \\
\text{subject to} & x_1 + 2x_2 & \geq & 1, \\
& x_1 - x_2 + x_3 & \geq & 2, \\
& x_1 & \leq & 1, \\
& x_1, x_2, x_3 & \geq & 0.
\end{array}
$$

   (a) Write down the dual of this problem.

   (b) By inspection, find a feasible point for the dual.

(c) Find a lower bound on the optimal value of the primal problem, without constructing any tableaus.

5. Consider the linear program in standard form:

$$\min p'x \text{ subject to } Ax \geq b, \quad x \geq 0.$$

Suppose that all components of the vector b are nonnegative, and that this problem and its dual are both feasible.

Show that the problem attains a solution x^*, and that the optimal objective satisfies $p'x^* \geq 0$. Cite any theorems that you need to justify this claim.