SPRING 2008 CS 525 MIDTERM – SOLUTIONS

1. Put into tableau

	x_1	x_2	x_3	x_4	1
$y_1 =$	0	1	4	7	1
$y_2 =$	−1	1	2	1	−2
$y_3 =$	1	2	10	20	5

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Pivot on (2,2)

	x_1	y_2	x_3	x_4	1
$y_1 =$	1	1	2	6	3
$x_2 =$	1	1	−2	−1	2
$y_3 =$	3	2	6	18	9

Pivot on (1,1)

	y_1	y_2	x_3	x_4	1
$x_1 =$	1	−1	−2	−6	−3
$x_2 =$	1	0	−4	−7	−1
$y_3 =$	3	−1	0	0	0

11 Cannot pivot y_3 to top of tableau.
However tableau is feasible if we set $y_1 = y_2 = y_3 = 0$,
so system has solutions

4 Dependence of rows of A: $A_{3\cdot} = 3A_{1\cdot} - A_{2\cdot}$

General form of solutions obtained by setting

$x_3 = \alpha$

$x_4 = \beta$

and obtain

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3 - 2\alpha - 6\beta \\ -1 - 4\alpha - 7\beta \\ \alpha \\ \beta \end{bmatrix}$$

(6)

$$= \begin{bmatrix} -3 \\ -1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ -4 \\ 1 \\ 0 \end{bmatrix} \alpha + \begin{bmatrix} -6 \\ -7 \\ 0 \\ 1 \end{bmatrix} \beta.$$

for all $\alpha \in \mathbb{R}$, $\beta \in \mathbb{R}$.

2. Standard form:

$$\min \quad 2x_1 + x_2 - 2x_3$$

$$\text{s.t.} \quad -2x_1 - \tfrac{5}{2}x_2 - x_3 \geq -6$$

$$-x_1 + x_2 - 2x_3 \geq -4$$

$$x_1, x_2, x_3 \geq 0$$

	x_1	x_2	x_3	1
$x_4 =$	-2	$-\frac{5}{2}$	-1	6
$x_5 =$	-1	1	-2	4
$z =$	2	1	-2	0

no need for phase I.

pivot on (2, 3).

	x_1	x_2	x_5	1
$x_4 =$	$-\frac{3}{2}$	-3	$\frac{1}{2}$	4
$x_3 =$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	2
$z =$	3	0	1	-4

OPTIMAL: solution is $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

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Solution is not unique - can pivot on (1,2) to obtain another vertex solution.

	x_1	x_4	x_5	1
$x_2 =$	$-\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{6}$	$\frac{4}{3}$
$x_3 =$	$-\frac{3}{4}$	$-\frac{1}{6}$	$-\frac{5}{12}$	$\frac{8}{3}$
$z =$	3	0	1	-4

giving solution $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 4/3 \\ 8/3 \end{pmatrix}$

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Actually all points of the form

$$\alpha \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + (1-\alpha) \begin{pmatrix} 0 \\ 4/3 \\ 8/3 \end{pmatrix}, \quad \alpha \in [0,1]$$

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are also solutions.

3. Scheme II

free

	x_1	x_2	x_3	1
equality → $x_4 =$	2	-1	1	-5
$x_5 =$	1	1	0	-10
$z =$	2	-1	2	0

pivot on (1,3).

	x_1	x_2	x_4	1
move to bottom → $x_3 =$	-2	1	1	5
$x_5 =$	1	1	0	-10
$z =$	-2	1	2	10

↑ delete

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	x_1	x_2	1
$x_5 =$	1	1	-10
$z =$	-2	1	10
$x_3 =$	-2	1	5

Tableau not feasible – apply Phase I.

	x_1	x_2	x_0	1
$x_5 =$	1	1	1	-10
$z =$	-2	1	0	10
$x_3 =$	-2	1	0	5
$z_0 =$	0	0	1	0

special pivot: (1, 3):

	x_1	x_2	x_5	1
→ $x_0 =$	−1	−1	1	10
$z =$	−2	1	0	10
$x_3 =$	−2	1	0	5
$z_0 =$	−1	−1	1	10

↑

pivot on (1,1):

	x_0	x_2	x_5	1
$x_1 =$	−1	−1	1	10
$z =$	2	3	−2	−10
$x_3 =$	2	3	−2	−15
$z_0 =$	1	0	0	0

(8) done with phase 1! delete column x_0, row z_0:

	x_2	x_5	1
$x_1 =$	−1	1	10
$z =$	3	−2	−10
$x_3 =$	3	−2	−15

Tableau indicates unboundedness!

by letting $x_5 \uparrow \infty$, we get $x_1 \uparrow \infty$, $x_2 = 0$

(9) while $z \downarrow -\infty$.

Specifically, if $x_5 = \alpha$, get

$$x_1 = 10 + \alpha$$
$$x_2 = 0$$
$$x_3 = -15 - 2\alpha$$
$$z = -10 - 2\alpha.$$

So direction of unboundedness is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 0 \\ -15 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \alpha, \quad \alpha \uparrow \infty.$$

4 (a) max $6u_1 + 7u_2$

st $u_1 + 3u_2 \le 2$

$-u_1 + u_2 \le -1$

$3u_1 + 2u_2 \le 5$

$u_1, u_2 \ge 0$

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(b) $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} 5/4 \\ 1/4 \end{pmatrix}$ is feasible for the

5 dual. (Many other possibilities)

(c) By weak duality, a lower bound for the primal is $6u_1 + 7u_2$,

5 where $\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ is any feasible dual point. Using the feasible point from (b), we obtain lower bound of

$$\frac{30}{4} + \frac{7}{4} = \frac{37}{4}$$