Midterm Examination

CS 525 - Spring 2008

Tuesday, March 11, 2008, 7:15-9:15pm

Each question is worth the same number of points.

No electronic devices, notes, or books allowed, except that you may bring one standard-size sheet of paper, handwritten on both sides, into the test. You need to give reasoning and justify all your answers.

1. Solve the system of equations $Ax = b$, where

$$A = \begin{bmatrix} 0 & 1 & 4 & 7 \\ -1 & 1 & 2 & 1 \\ 1 & 2 & 10 & 20 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 2 \\ -5 \end{bmatrix}.$$

   If there are multiple solutions, describe the full solution set. If there are linear dependence relations between the rows of the coefficient matrix, state them.

2. Solve the following linear program. If it infeasible, say so. If it is unbounded, give a direction of unboundedness. If there are multiple solutions, describe the full set of solutions.

$$\begin{array}{lrcl} \max & -2x_1 - x_2 + 2x_3 & & \\ & 2x_1 + \frac{5}{2}x_2 + x_3 & \leq & 6, \\ \text{subject to} & x_1 - x_2 + 2x_3 & \leq & 4, \\ & x_1, x_2, x_3 & \geq & 0. \end{array}$$

3. Solve the following linear program. (Hint: Use Scheme II.) If it infeasible, say so. If it is unbounded, give a direction of unboundedness. If

there are multiple solutions, describe the full set of solutions.

$$\begin{array}{lrcl} \min & 2x_1 - x_2 + 2x_3 & & \\ & 2x_1 - x_2 + x_3 & = & 5, \\ \text{subject to} & x_1 + x_2 & \geq & 10, \\ & x_1, x_2 & \geq & 0, \quad x_3 \text{ free.} \end{array}$$

4. Consider the following linear program:

$$\begin{array}{lrcl} \min & 2x_1 - x_2 + 5x_3 & & \\ & x_1 - x_2 + 3x_3 & \geq & 6, \\ \text{subject to} & 3x_1 + x_2 + 2x_3 & \geq & 7, \\ & x_1, x_2, x_3 & \geq & 0. \end{array}$$

   (a) Write down the dual of this problem.

   (b) By inspection, find a feasible point for the dual.

   (c) Without performing any simplex iterations, find a lower bound on the optimal objective for the given linear program.