Midterm Examination

CS 525 - Spring 2009

Wednesday, March 11, 2009, 7:15-9:15pm

Each question is worth the same number of points.

No electronic devices, notes, or books allowed, except that you may bring one standard-size sheet of paper, handwritten on both sides, into the test. **You need to give reasoning and justify all your answers**, citing the appropriate theorems where necessary.

1. (a) For the following choice of A and b, solve the system of equations $Ax = b$. If there are multiple solutions, describe the full solution set. If there are linear dependence relations between the rows of the coefficient matrix, state them.

$$A = \begin{bmatrix} 1 & 4 & 7 \\ -1 & 2 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

   (b) Repeat part (a) for the following choice of A and b:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ -3 & -1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}.$$

2. Solve the following linear program. If it infeasible, say so. If it is unbounded, give a direction of unboundedness. If there are multiple solutions, describe the full set of solutions.

$$\begin{array}{lrcl} \min & x_1 - 2x_2 + 2x_3 & & \\ & -x_1 + x_2 - 3x_3 & \geq & 1, \\ \text{subject to} & x_1 + 4x_2 + 4x_3 & \leq & 2, \\ & 4x_1 + x_2 - 6x_3 & \leq & 5, \\ & x_1, x_2, x_3 & \geq & 0. \end{array}$$

3. Solve the following linear program. (Hint: Use Scheme II.) If it infeasible, say so. If it is unbounded, give a direction of unboundedness. If there are multiple solutions, describe the full set of solutions.

$$\begin{array}{lrcl} \min & 4x_1 + 6x_2 + 2x_3 & & \\ & 2x_1 - 3x_2 + x_3 & = & 4, \\ \text{subject to} & 3x_1 - 5x_2 + x_3 & \geq & 9, \\ & x_1, x_2 & \geq & 0, \quad x_3 \text{ free.} \end{array}$$

4. (a) Explain why the following linear program cannot be infeasible, and give an *upper* bound on its optimal objective value:

$$\min p'x \text{ subject to } Ax \geq 0, \quad x \geq 0.$$

(b) Write down conditions under which the problem in part (a) has a solution. (Hint: Use the dual.) Assuming these conditions hold, write down a solution to this problem without doing any computations at all. (Justify your answers by citing the appropriate theorems.)

(c) Write down the dual of the following linear program in n unknowns, and find the solution of the dual:

$$\begin{array}{rl} \max\limits_{x_1, x_2, \ldots, x_n} & x_1 + 2x_2 + 3x_3 + \ldots + nx_n \\ \text{subject to} & x_1 + x_2 + x_3 + \ldots + x_n \leq 1, \\ & x_1, x_2, x_3, \ldots, x_n \geq 0. \end{array}$$