Midterm Examination

CS 525 - Fall 2010

Thursday, October 28, 2010, 7:15-9:15pm

Each question is worth the same number of points.

No electronic devices, notes, or books allowed, except that you may bring one standard-size sheet of paper, handwritten on both sides, into the test. **You need to give reasoning and justify all your answers**, citing the appropriate theorems where necessary.

1. (a) For the following choice of A and b, solve the system of equations $Ax = b$ by using tableaus. If there are multiple solutions, describe the full solution set.

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 6 & -1 \\ 0 & 2 & 2 \end{bmatrix}, \qquad b = \begin{bmatrix} 7 \\ -4 \\ 2 \end{bmatrix}.$$

   (b) Using Jordan exchanges, find the inverse of this matrix. If the matrix has no inverse, find a dependency relationship between its rows.

$$C = \begin{bmatrix} 0 & 1 & -2 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}.$$

2. Consider the following linear program:

$$\begin{array}{lrcl} \min & 2x_1 + x_2 & & \\ & x_1 - x_2 & \geq & 4, \\ \text{subject to} & 3x_1 + 2x_2 & \geq & 10, \\ & x_1, x_2 & \geq & 0. \end{array}$$

(a) Write down the dual of this problem.

(b) Find solutions for the primal and dual.

(c) Suppose the right-hand side of the first constraint is changed from 4 to 10. Without performing any additional simplex iterations or referring to the tableau, give a lower bound on the optimal primal objective for the modified problem. Explain.

3. Solve the following linear program, using any of techniques we have learnt about in class (or any combination thereof). If it infeasible, say so. If it is unbounded, give a direction of unboundedness. If there are multiple solutions, describe the full set of solutions.

$$\begin{array}{lrcl} \min & 6x_1 + 6x_2 + 2x_3 & & \\ & 4x_1 - 5x_2 + x_3 & \geq & 10, \\ \text{subject to} & 3x_1 - 3x_2 + x_3 & = & 5, \\ & x_1, x_2 & \geq & 0, \quad x_3 \text{ free.} \end{array}$$

4. Consider the following linear program:

$$\begin{aligned} \min_{x_1, x_2, \ldots, x_n} \quad & -x_1 - x_2 - x_3 - \ldots - x_n \\ \text{subject to} \quad & a_1 x_1 + a_2 x_2 + a_3 x_3 + \ldots a_n x_n \leq 1, \\ & x_1, x_2, x_3, \ldots, x_n \geq 0. \end{aligned}$$

where $a_1, a_2, \ldots, a_n$ are (strictly) positive constants.

(a) Write down the dual of this problem.

(b) Find the solution of the dual, by inspection. Is this solution unique?

(c) Write down the KKT conditions for this problem.

(d) By using the KKT conditions (or by any other means), find a solution of the original (primal) linear program. (It if sufficient to write down just one solution.)

(e) How do the primal and dual solutions change if one or more of the constants a_i, $i = 1, 2, \ldots, n$ is allowed to be zero?