

# Midterm Examination

CS 525 - Spring 2002

Tuesday, March 19, 2002

Write out your final solution to each problem clearly and unambiguously.

**Instructions (Please read carefully):** If a problem has no solutions, many solutions, or is infeasible or unbounded, you must clearly state this to be the case and justify your statement. In the case of multiple solutions, you must describe the full set of solutions. In the case of an unbounded linear program, give a direction of unboundedness. When linear dependence relations are present in a system of linear equations, they should be clearly described.

Each problem can be solved in three pivots or fewer.

1. Solve the following problems.

(a)

$$\begin{array}{rccccrcr} x_1 & + & x_2 & + & 3x_3 & + & 2x_4 & = & 5 \\ & & - & x_2 & & & + & x_4 & = & -1 \\ 2x_1 & + & x_2 & + & 6x_3 & + & 5x_4 & = & 6. \end{array}$$

- (b) Change the right-hand side of the last equation in part (a) from “6” to “9”, and solve the resulting problem.

2. Solve the following linear program.

$$\begin{array}{ll} \min & -x_1 - 2x_2 + x_3 \\ \text{subject to} & 3x_1 - x_2 + x_3 \geq -5, \\ & -x_1 + x_2 - 2x_3 \geq -2, \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

3. Solve the following linear program.

$$\begin{array}{ll} \max & -x_1 - 4x_2 + x_3 \\ \text{subject to} & 2x_1 + 4x_2 - x_3 \geq 4, \\ & x_1 + x_2 = 8, \\ & 2x_1 + 6x_2 \leq 18, \\ & x_1 \text{ unrestricted} \\ & x_2, x_3 \geq 0. \end{array}$$

4. Consider the following linear program:

$$\begin{array}{ll} \min & 3x_1 + 2x_2 + 5x_3 \\ \text{subject to} & x_1 + x_2 - x_3 \geq 2, \\ & 2x_1 + x_2 + 6x_3 \geq 6, \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

- (a) Write down the dual of this problem, and find a feasible point for the dual.
- (b) Using the information from part (a), and without performing any pivots, determine a lower bound on the optimal objective value of the problem given above.