

Midterm Examination

CS 525 - Spring 2003

Tuesday, March 25, 2003, 7:15pm-9:15pm

Each question is worth 20 points. Each problem that involves tableaux can be solved in three pivots or fewer.

1. Solve the following problem. If the problem has multiple solutions, describe them all. If linear dependence relations exist between rows of the coefficient matrix for this system, state them clearly.

$$\begin{array}{rclcl} x_1 & + & x_2 & - & 3x_3 & = & 3 \\ 2x_1 & - & x_2 & + & x_3 & = & 2 \\ -2x_1 & + & 7x_2 & - & 15x_3 & = & -1. \end{array}$$

2. Solve the following linear program using the two-phase simplex method.

$$\begin{array}{ll} \min & -x_1 + 3x_2 + x_3 \\ \text{subject to} & -2x_1 + x_2 - x_3 \geq 1, \\ & 2x_1 + x_2 - 2x_3 \geq -4, \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

3. Solve the following linear program. Explain the reasons for each of your choices of pivots. (Hint: Use Scheme II.)

$$\begin{array}{ll}
\max & -x_1 - 4x_2 + x_3 \\
\text{subject to} & 2x_1 + 4x_2 - x_3 \geq 4, \\
& x_1 + x_2 = 8, \\
& 2x_1 + 6x_2 \leq 16, \\
& x_1 \text{ unrestricted,} \\
& x_2, x_3 \geq 0.
\end{array}$$

4. Solve the following linear program using the dual simplex algorithm:

$$\begin{array}{ll}
\min & x_1 + 2x_2 + 2x_3 \\
\text{subject to} & -2x_1 + x_2 + x_3 \geq 2, \\
& x_1 - x_2 - 2x_3 \geq -3, \\
& x_1, x_2, x_3 \geq 0.
\end{array}$$

5. Consider the following linear program:

$$\begin{array}{ll}
\min & x_1 + x_2 - x_3 \\
\text{subject to} & x_1 + x_2 \leq 1, \\
& 5x_2 + x_3 \geq 3, \\
& x_1 + x_2 + x_3 = -1, \\
& x_3 \text{ unrestricted,} \\
& x_1, x_2 \geq 0.
\end{array}$$

- Write down the dual of this problem. (It should have three variables).
- By inspection, find a feasible point for the dual and evaluate the dual objective at this point.
- Determine a lower bound on the optimal objective of the original problem without constructing any tableaus. (Explain any theorems that you used.)