Midterm Examination

CS 525 - Spring 2004

Wednesday, March 25, 2004, 7:15pm-9:15pm

Each question is worth 20 points. Each problem that involves tableaus can be solved in three pivots or fewer.

1. Solve the following problem. If the problem has multiple solutions, describe them all. If linear dependence relations exist between rows of the coefficient matrix for this system, state them clearly.

$$\begin{array}{rcrcrcr} -x_1 & + & 3x_2 & + & 5x_3 & = & 1 \\ x_1 & + & x_2 & - & x_3 & = & 2 \\ -x_1 & + & 7x_2 & + & 9x_3 & = & 4. \end{array}$$

2. Solve the following linear program. If it is unbounded, give a direction of unboundedness.

$$\begin{array}{lrcrcrcr} \min & -x_1 & - & x_2 & + & 2x_3 & & \\ & & & & & & & \\ \text{subject to} & -x_1 & + & x_2 & - & x_3 & \geq & -8, \\ & x_1 & - & x_2 & + & 2x_3 & \geq & -6, \\ & x_1 & & x_2 & , & x_3 & \geq & 0. \end{array}$$

3. Solve the following linear program using "Scheme II" and the two-phase simplex method. If it is unbounded, give a direction of unboundedness.

$$\begin{array}{lrcrcrcr} \min & 4x_1 & + & 6x_2 & + & 2x_3 & & \\ & & & & & & & \\ \text{subject to} & 2x_1 & - & 3x_2 & + & x_3 & = & 4, \\ & 7x_1 & - & 10x_2 & + & x_3 & \geq & 9, \\ & x_1 & & x_2 & & & \geq & 0, \\ & & & & & x_3 & & \text{free.} \end{array}$$

4. Consider the following linear program:

$$
\begin{array}{llcrcrcl}
\min & 2x_1 & + & x_2 & - & x_3 & & \\
\\
\text{subject to} & x_1 & + & x_2 & & & \leq & 1, \\
 & & & 6x_2 & + & x_3 & \geq & 3, \\
 & 2x_1 & + & x_2 & + & x_3 & = & -2, \\
\\
 & x_3 & \multicolumn{4}{l}{\text{unrestricted},} & & \\
 & x_1 & , & x_2 & & & \geq & 0.
\end{array}
$$

(a) Write down the dual of this problem. (It should have three variables).

(b) By inspection, find a feasible point for the dual and evaluate the dual objective at this point.

(c) Determine a lower bound on the optimal objective of the original problem without constructing any tableaus. (Explain any theorems that you used.)

5. Consider the following linear program:

$$
\begin{array}{llcrcrcl}
\min & 3x_1 & + & 2x_2 & + & 9x_3 & & \\
\\
\text{subject to} & x_1 & + & x_2 & - & x_3 & \geq & -1, \\
 & 2x_1 & + & x_2 & + & 6x_3 & \geq & 6, \\
 & x_1 & & x_2 & , & x_3 & \geq & 0.
\end{array}
$$

(a) Solve this problem using the dual simplex method.

(b) Find two more solutions to this problem, one of which is a vertex solution.