SIMULATION OF COMPLEX NONLINEAR ELASTIC BODIES USING LATTICE DEFORMERS

Taylor Patterson

Department of Computer Sciences
University of Wisconsin-Madison

February 8, 2013

INTRODUCTION: SOFT BODIES

Soft Bodies

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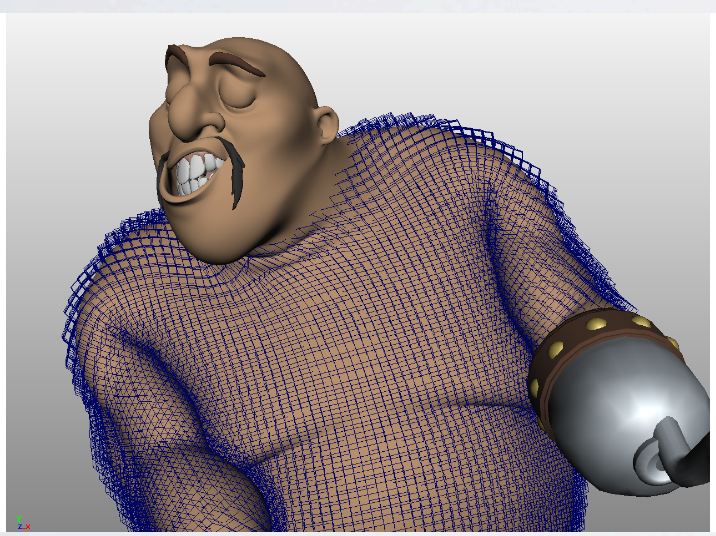
Soft Bodies

Movies



[McAdams, et al. 2011]

Research

[Lee, et al. 2009]

Games

[Parker, et al. 2009]

INTRODUCTION: SOFT BODIES

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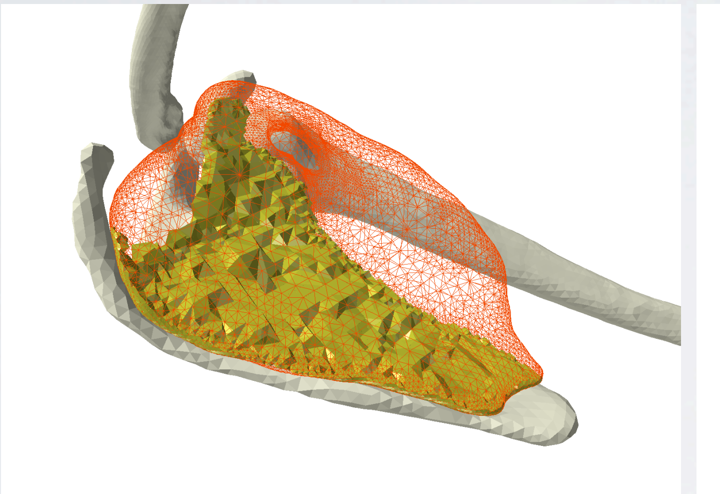
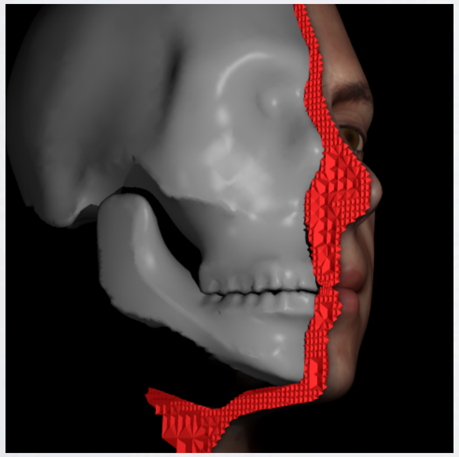
Conforming Meshes



[Teran et. al. 2005]



[Sifakis et. al. 2005]

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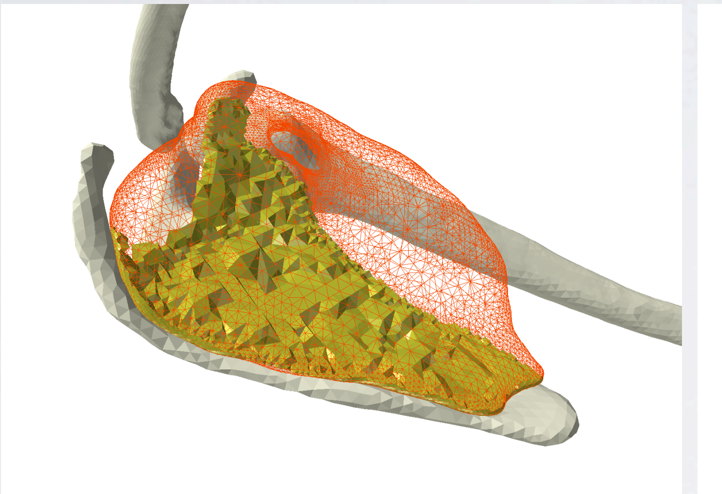
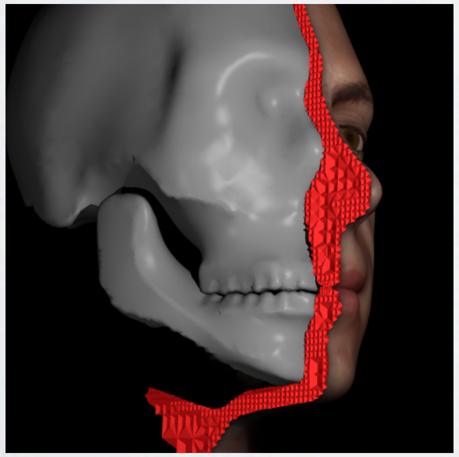


[Teran et. al. 2005]

VS.



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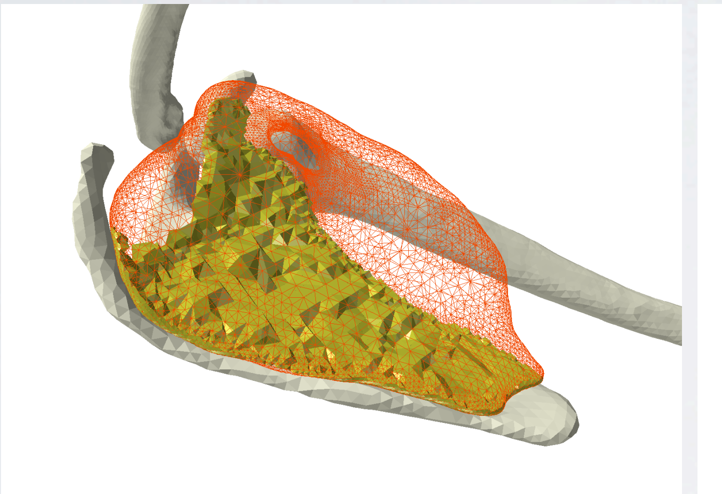

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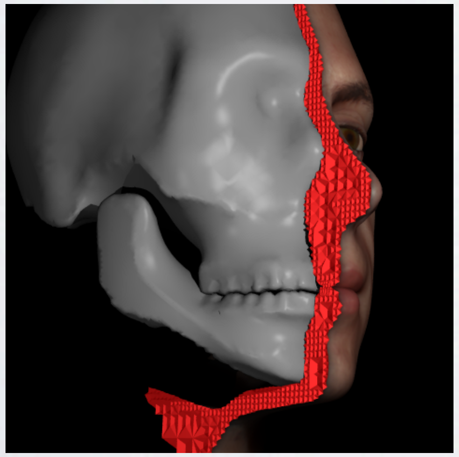
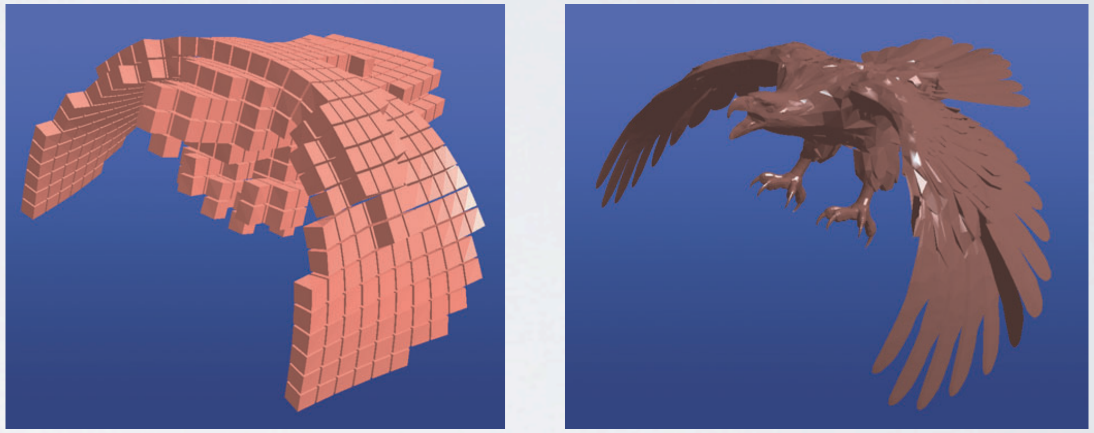


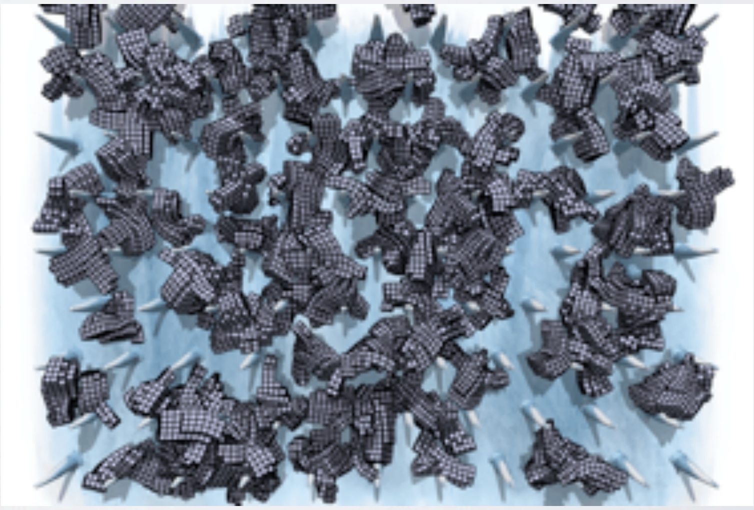
[Sifakis et. al. 2005]

vs.

Embedded Deformers



[Müller et. al. 2004]



[Rivers, A. & James, D. 2007]

FEATURES

Arbitrary Materials

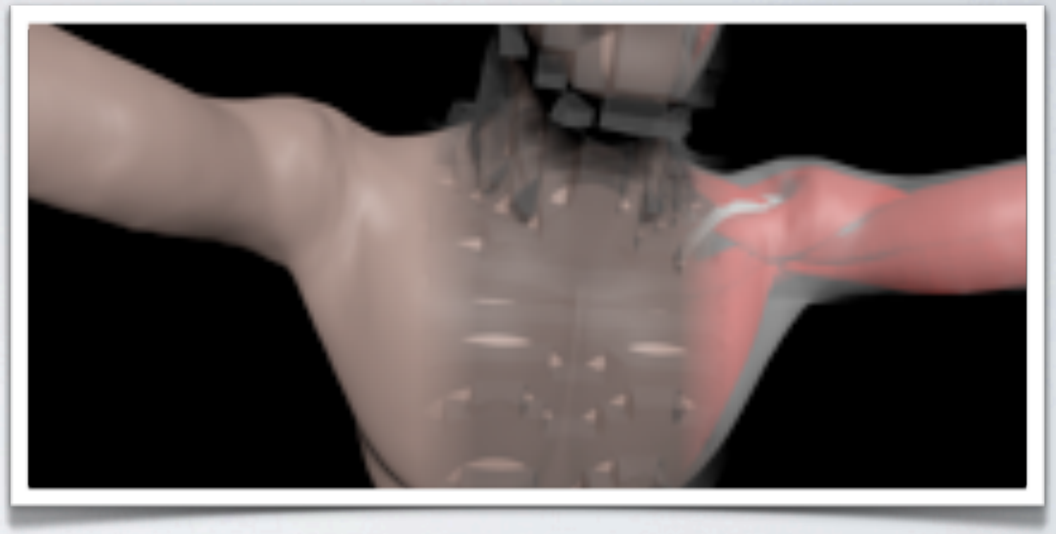
Incompressibility

Sub-voxel Precision

Robust

Parallelism

FEATURES



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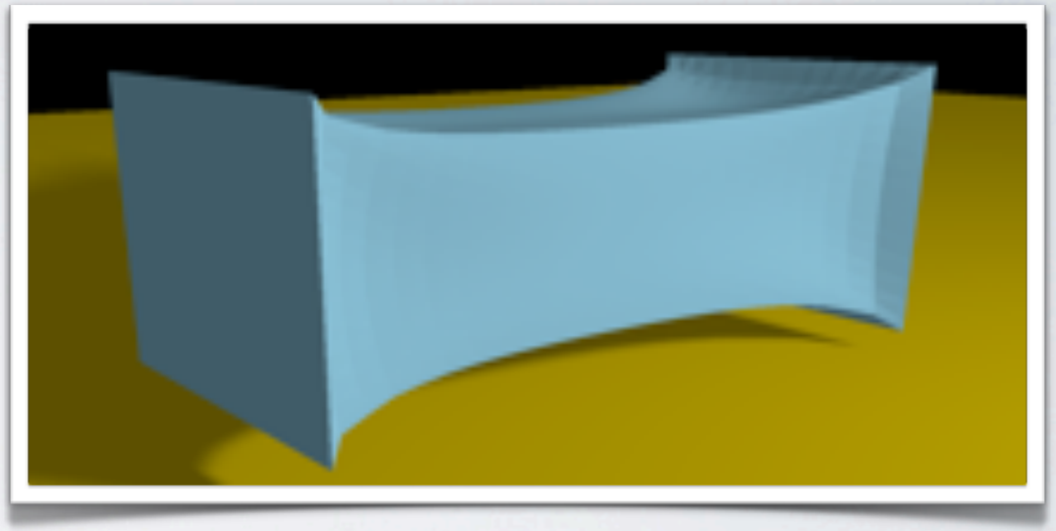

Sub-voxel Precision

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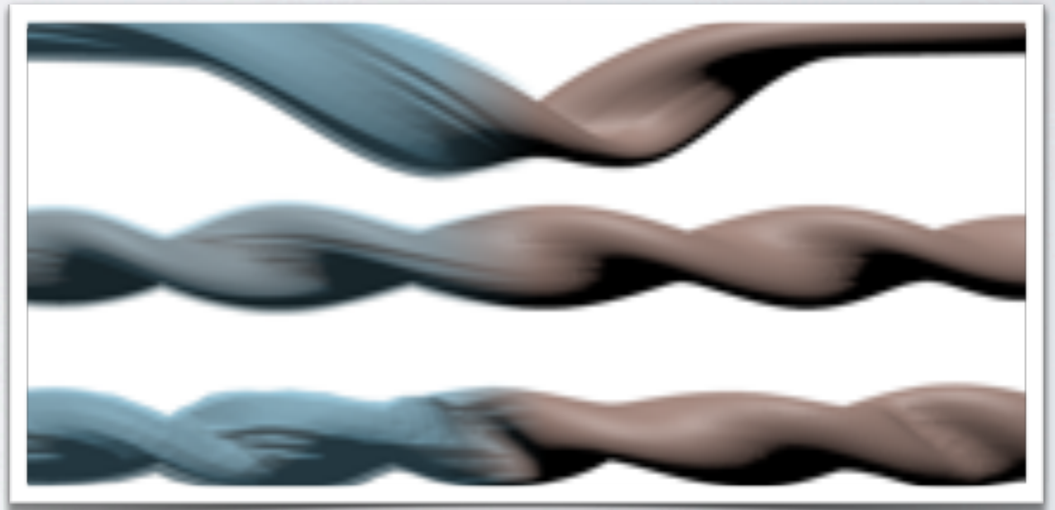

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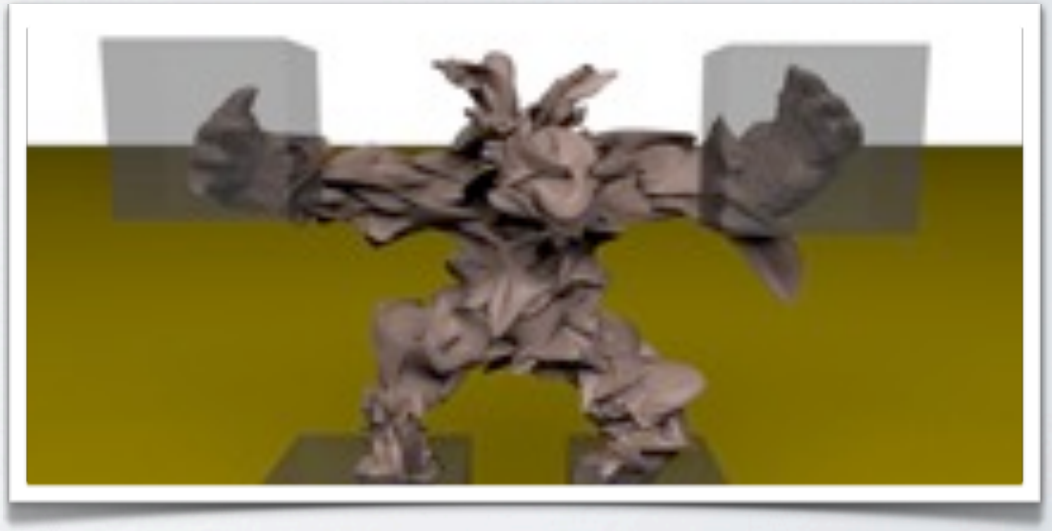

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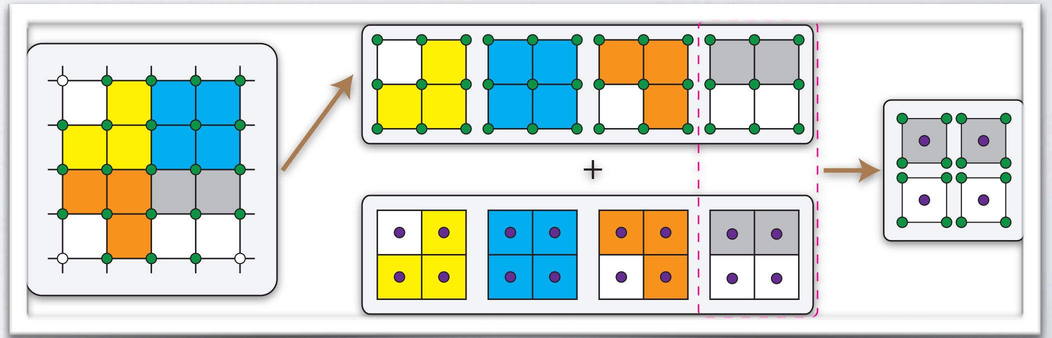

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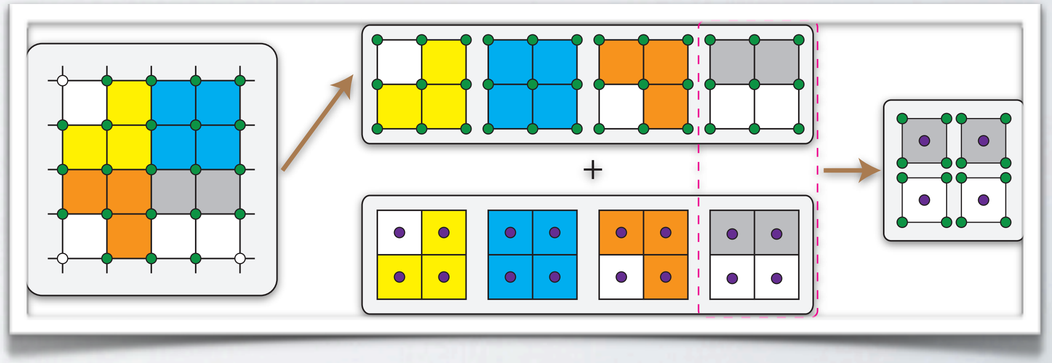

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MOTIVATION

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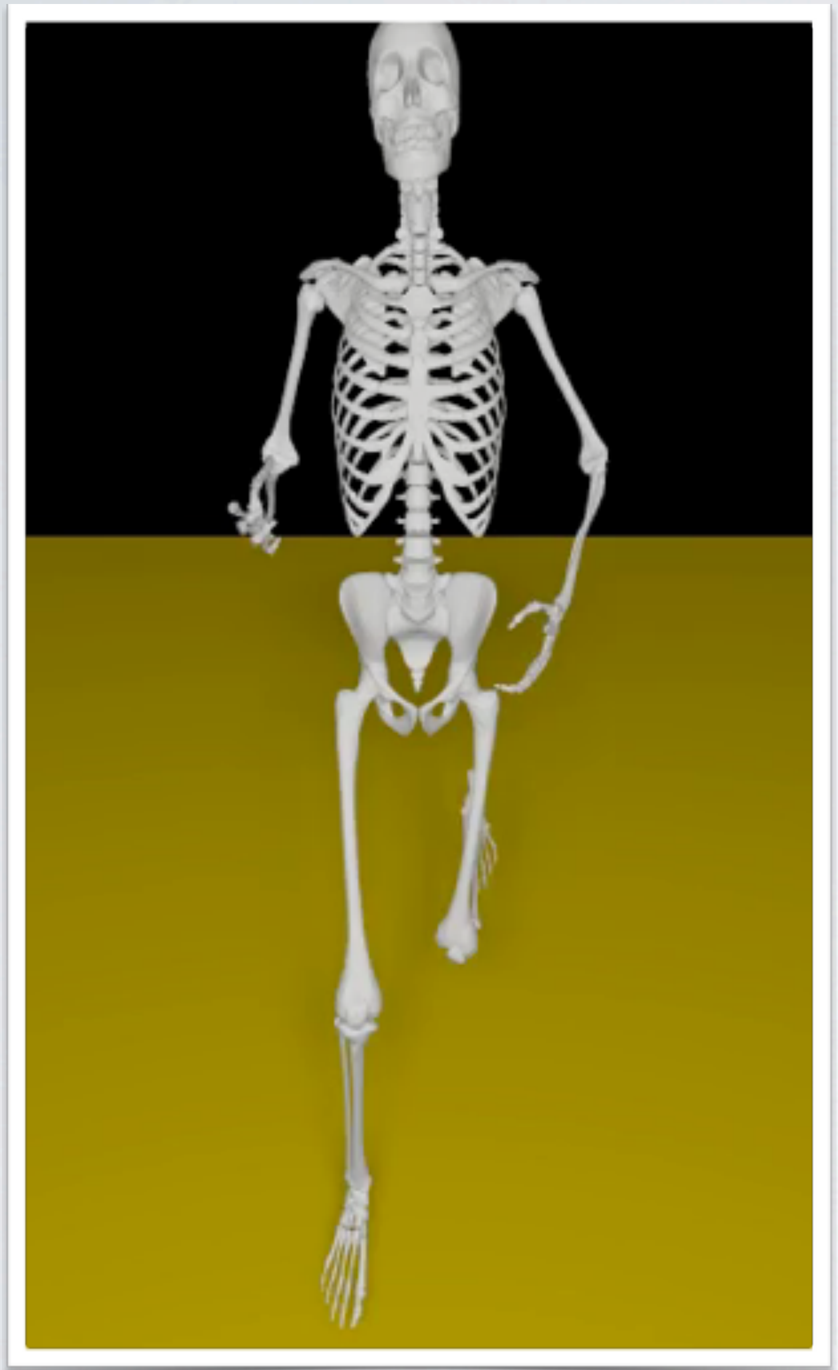

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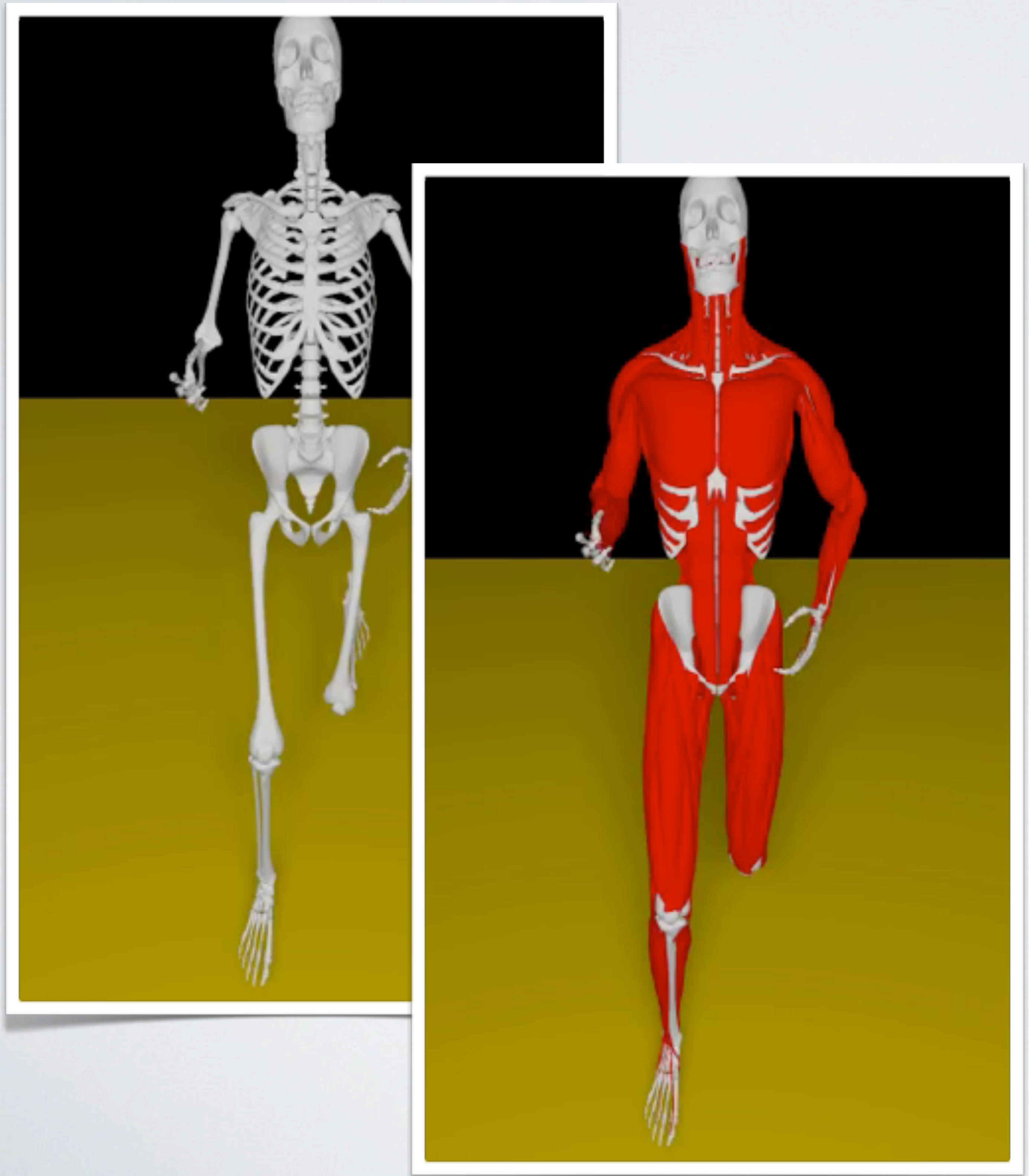

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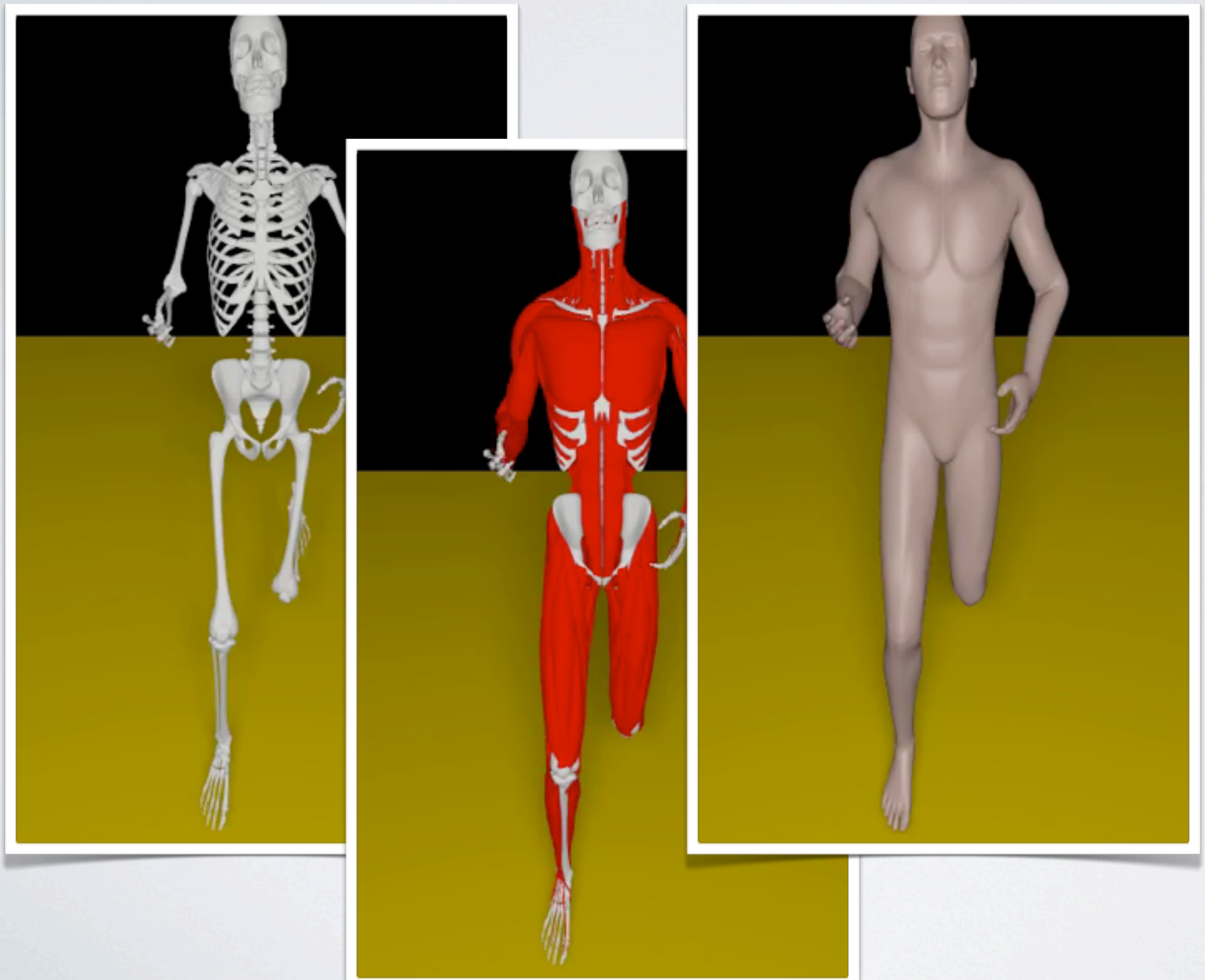

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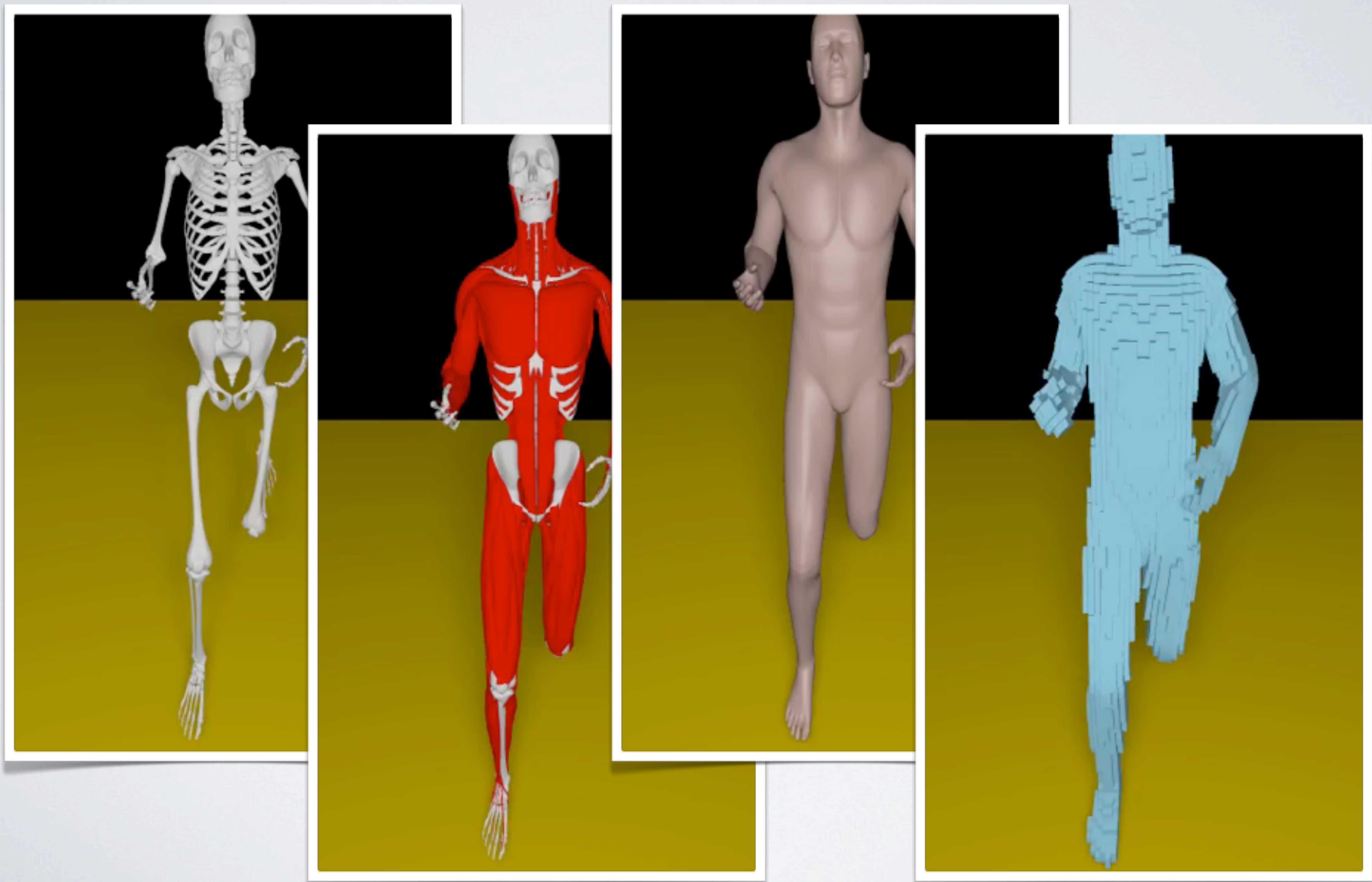
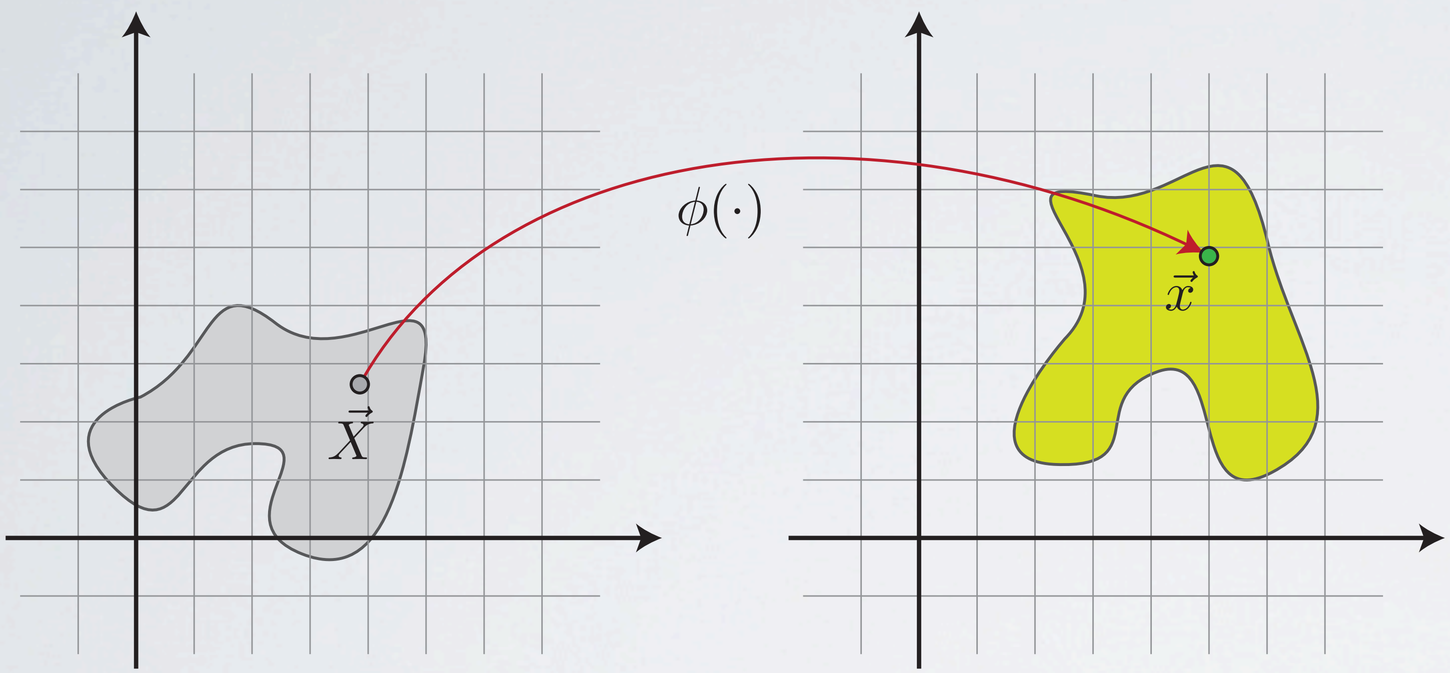


THE BASICS



Deformation Map

$$\phi = \mathbb{R}^3 \to \mathbb{R}^3$$

Deformation Gradient

$$F = \partial\phi / \partial X$$

Energy Density

$$\Psi = \Psi(F)$$

Elastic Energy

$$E = \int_\Omega \Psi(F)\delta X$$

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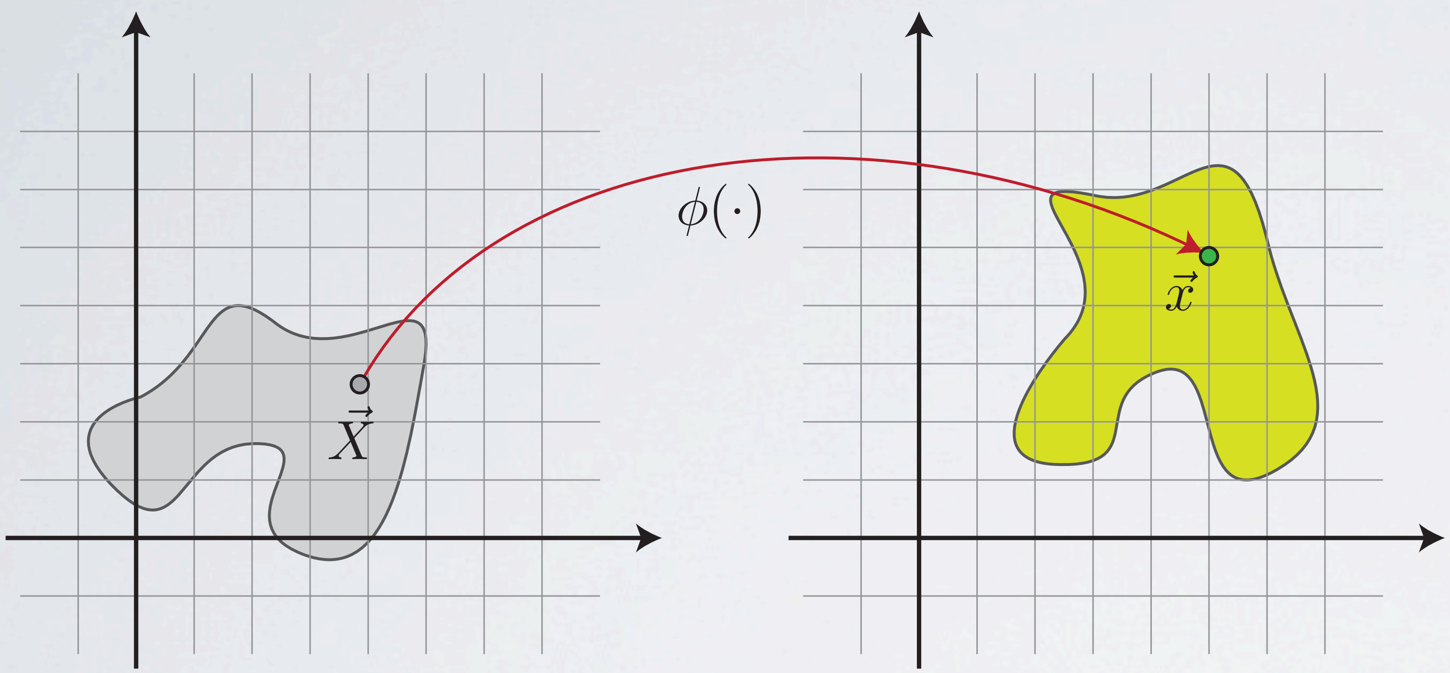

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$$\Psi = \frac{\mu}{2}(\|F\|_F^2 - 3) - \mu \log(J) + \frac{\kappa}{2}\log^2(J)$$

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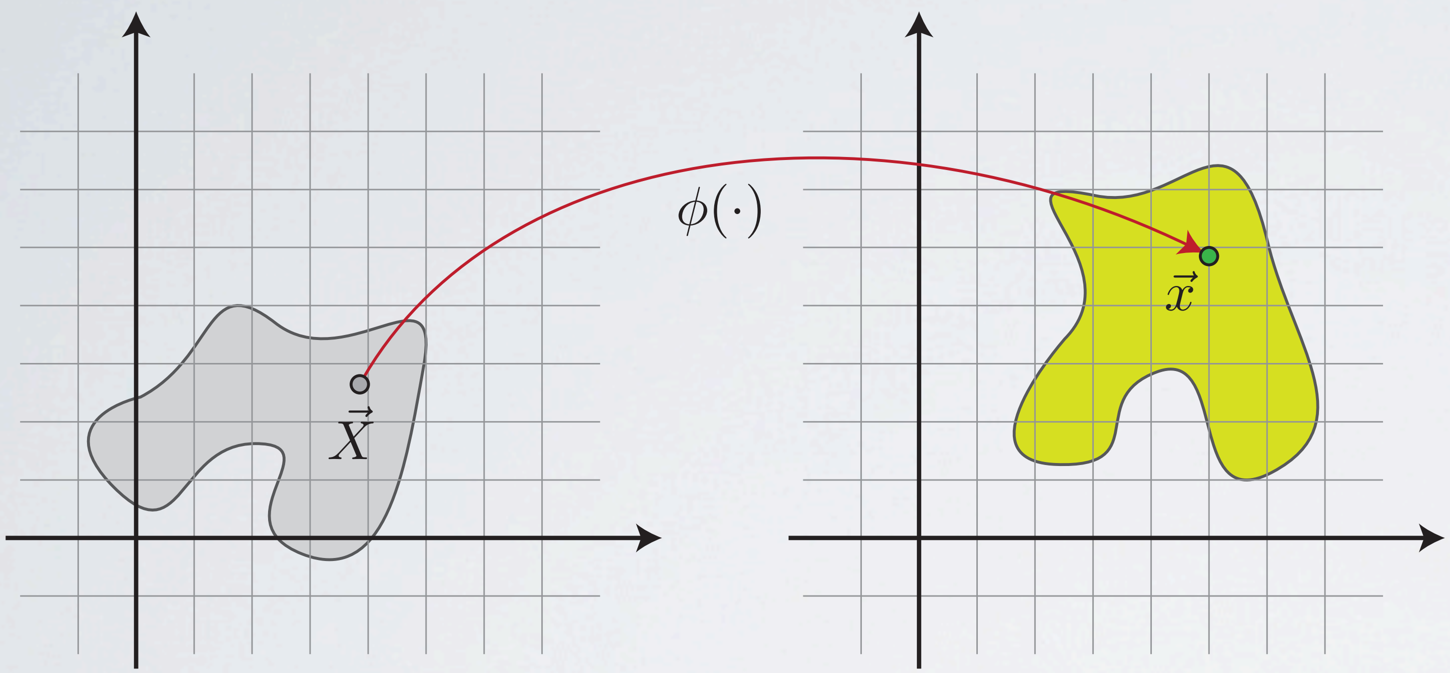

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↓

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↓

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$\downarrow$

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Compute Discrete Energy $\hat{E}$

$\downarrow$

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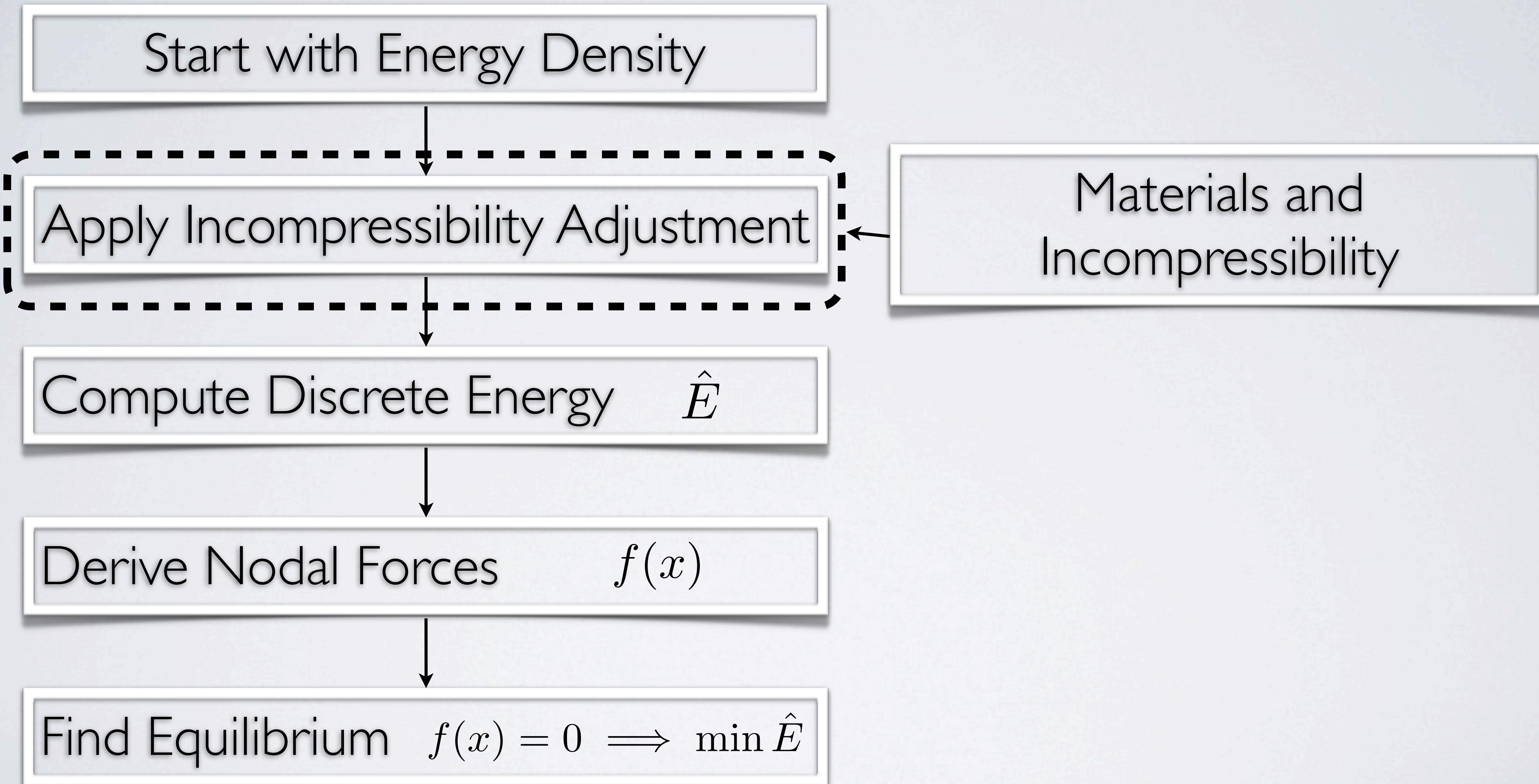
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Materials and Incompressibility
Compute Discrete Energy $\hat{E}$
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Boundary Cell Treatment

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Newton-Raphson Iteration

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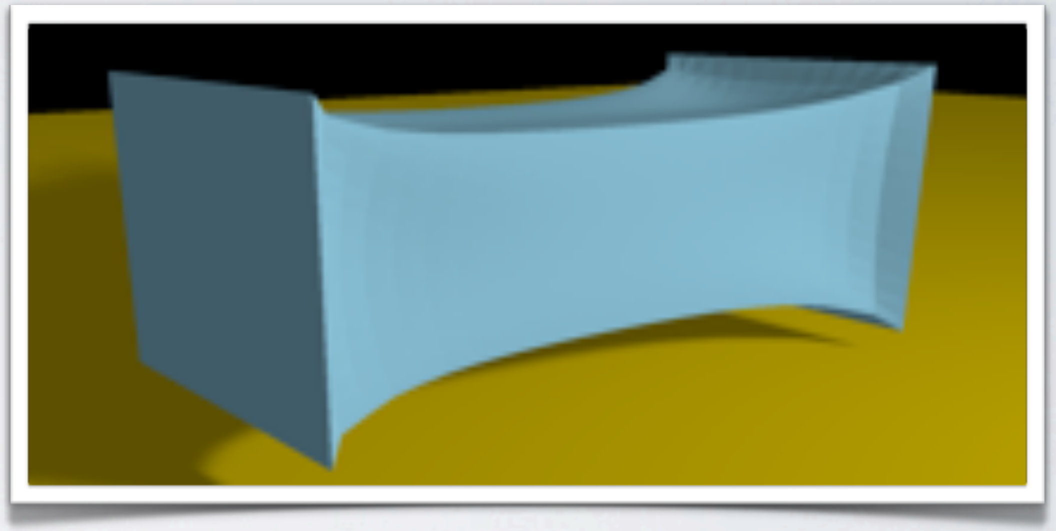

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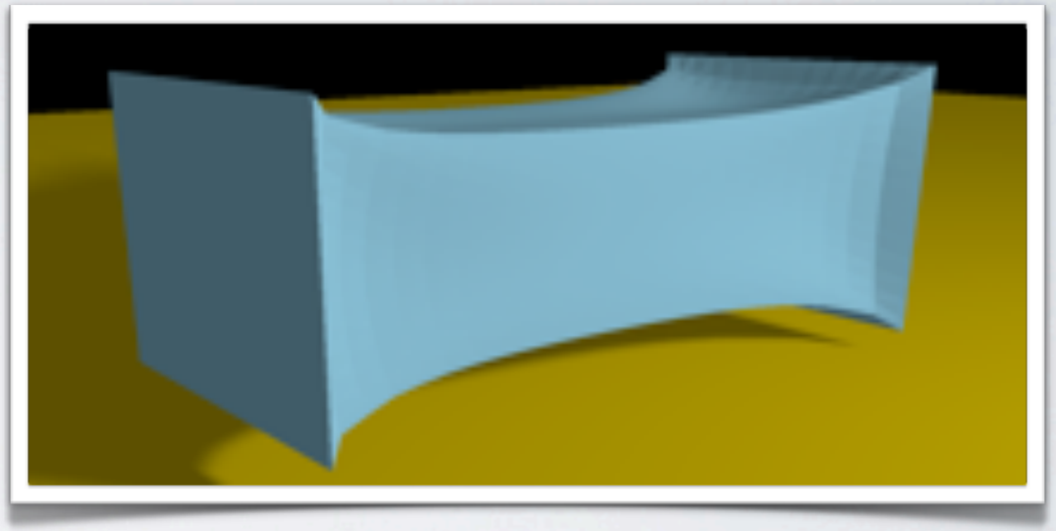

INCOMPRESSIBILITY: MOTIVATION

Importance

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$$\Psi = \mu \|\epsilon\|_F^2 + \frac{\kappa}{2} tr^2(\epsilon)$$

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$$\Psi = \mu \|\mathbf{F} - \mathbf{R}\|_F^2 + \frac{\kappa}{2} \mathrm{tr}^2(\mathbf{S} - \mathbf{I})$$

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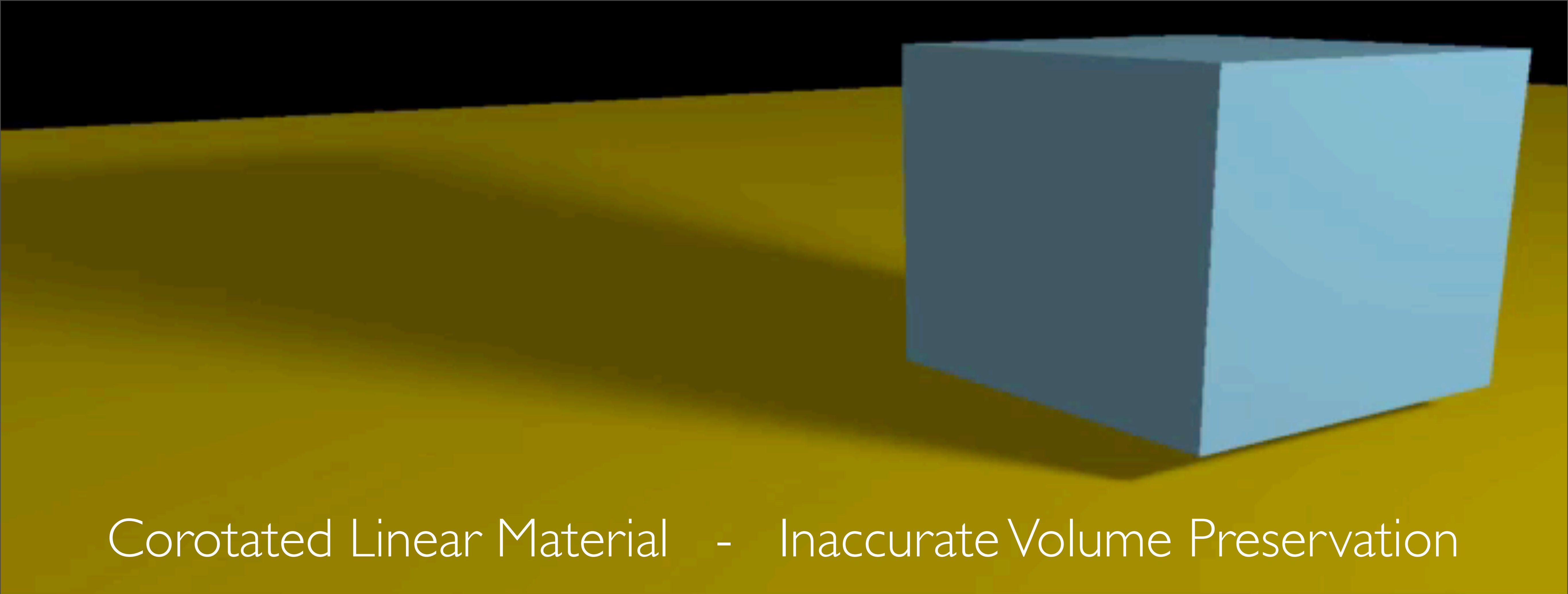
Raise κ to enforce incompressibility.

$$\kappa \to \infty$$

Implies complete incompressibility

Corotated Linear Material - Inaccurate Volume Preservation

Neohookean Material - Accurate Volume Preservation



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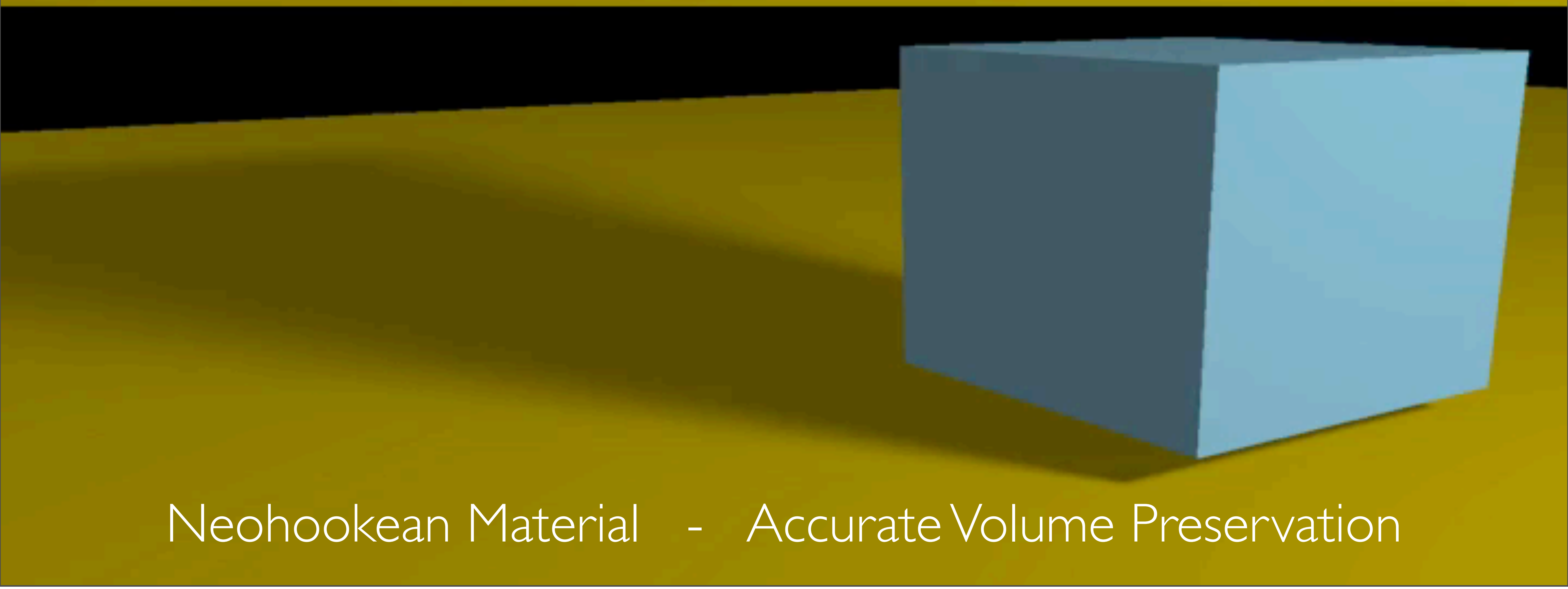

Neohookean Material - Accurate Volume Preservation

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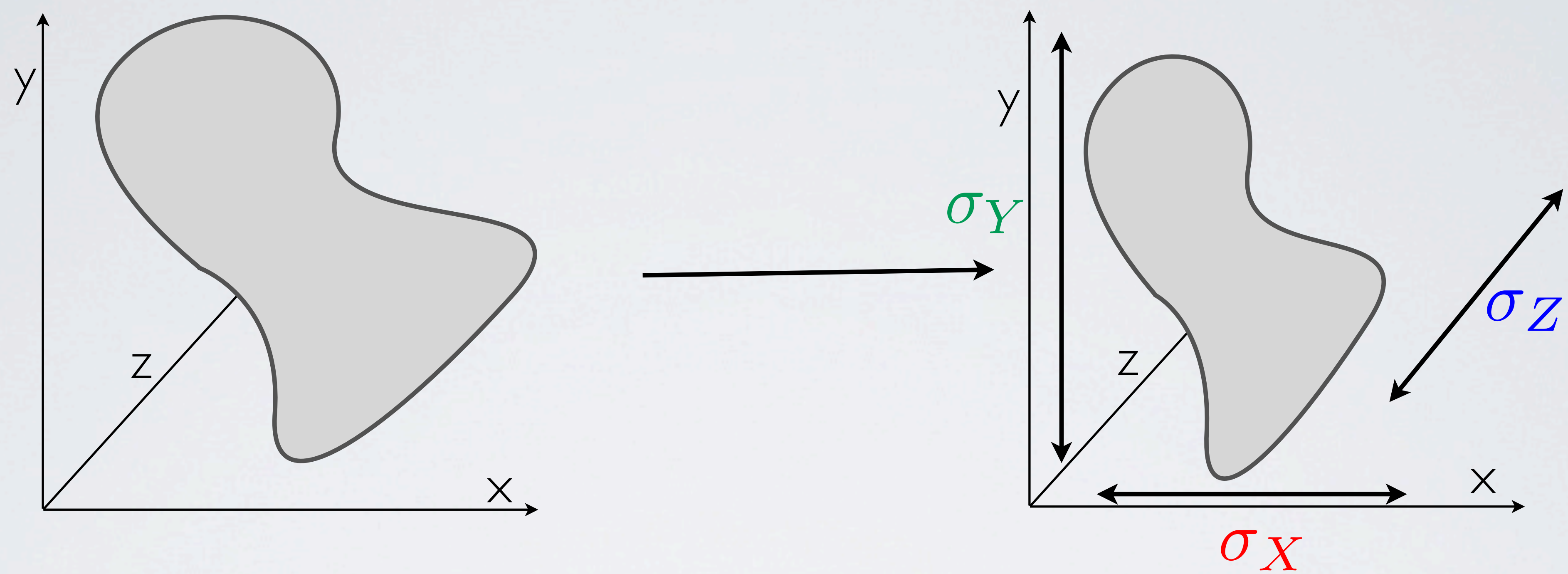

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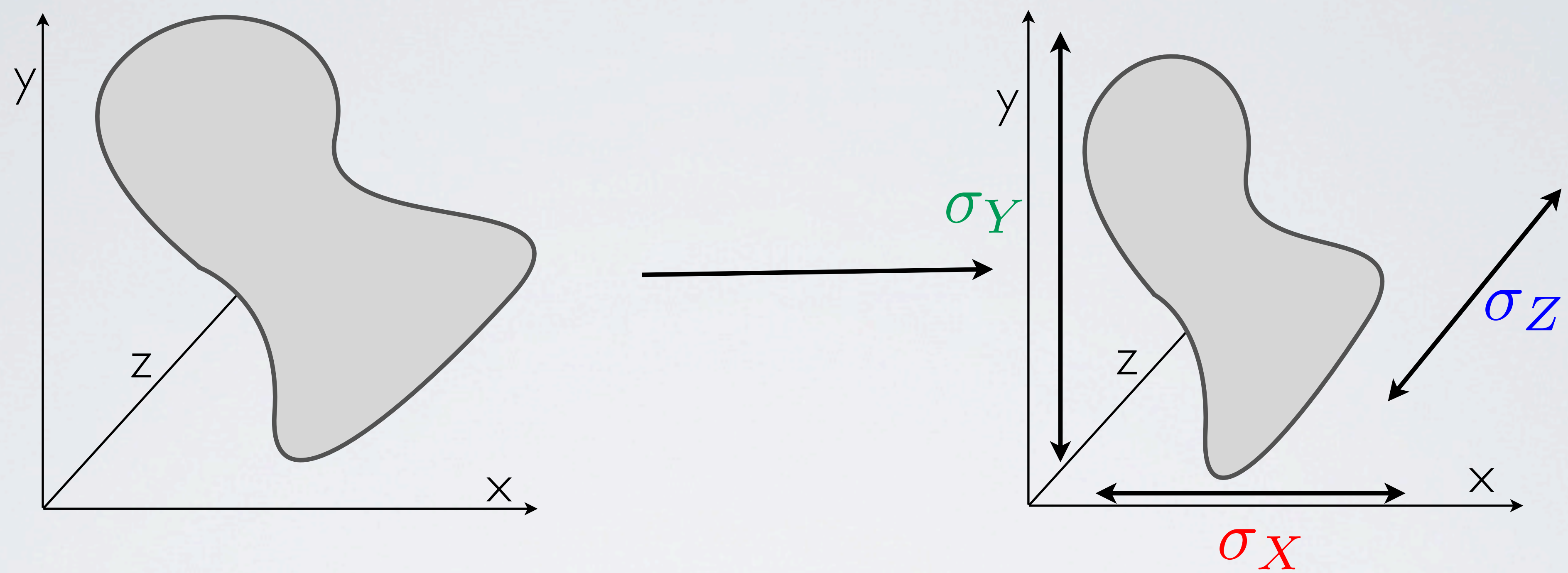
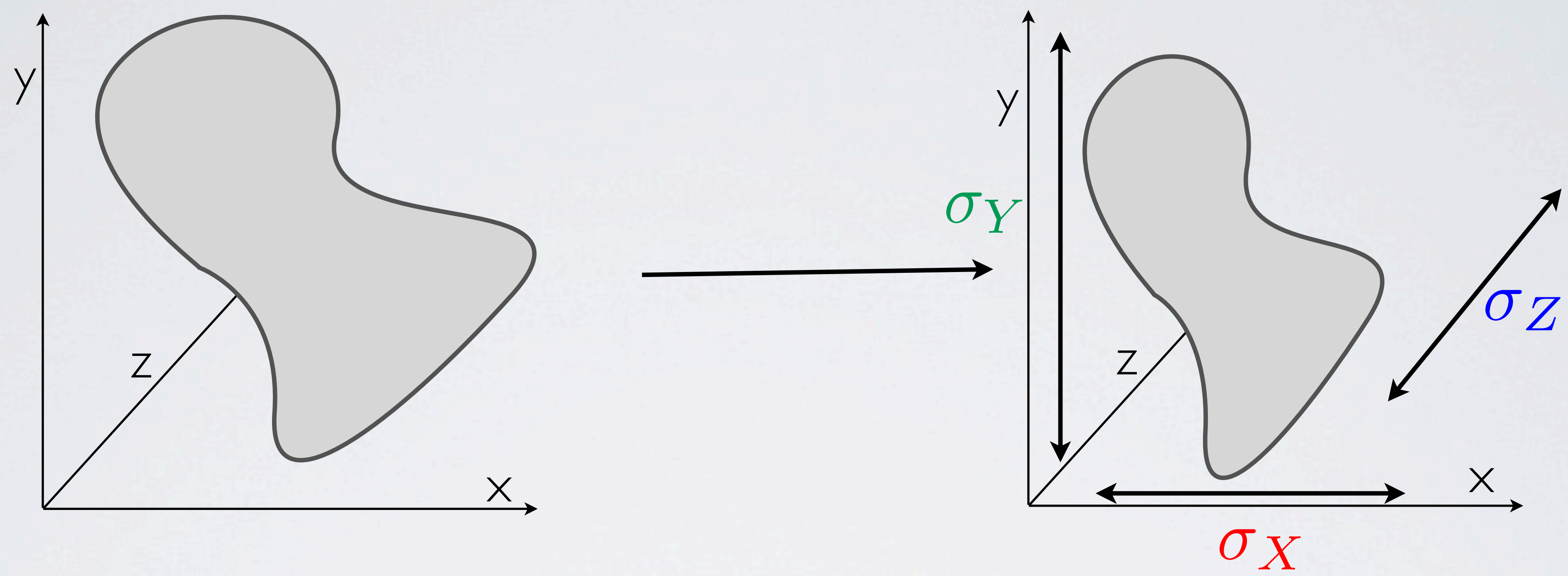


Corotated linear elasticity

$$tr^2(S - I) = 0 \implies \frac{\sigma_X + \sigma_Y + \sigma_Z}{3} \approx 1$$

INCOMPRESSIBILITY: MATERIALS



Neohookean elasticity

$$log^2(J) = 0 \implies \sigma_X \cdot \sigma_Y \cdot \sigma_Z \approx 1$$

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INCOMPRESSIBILITY: LOCKING

[Irving et al. 2007]

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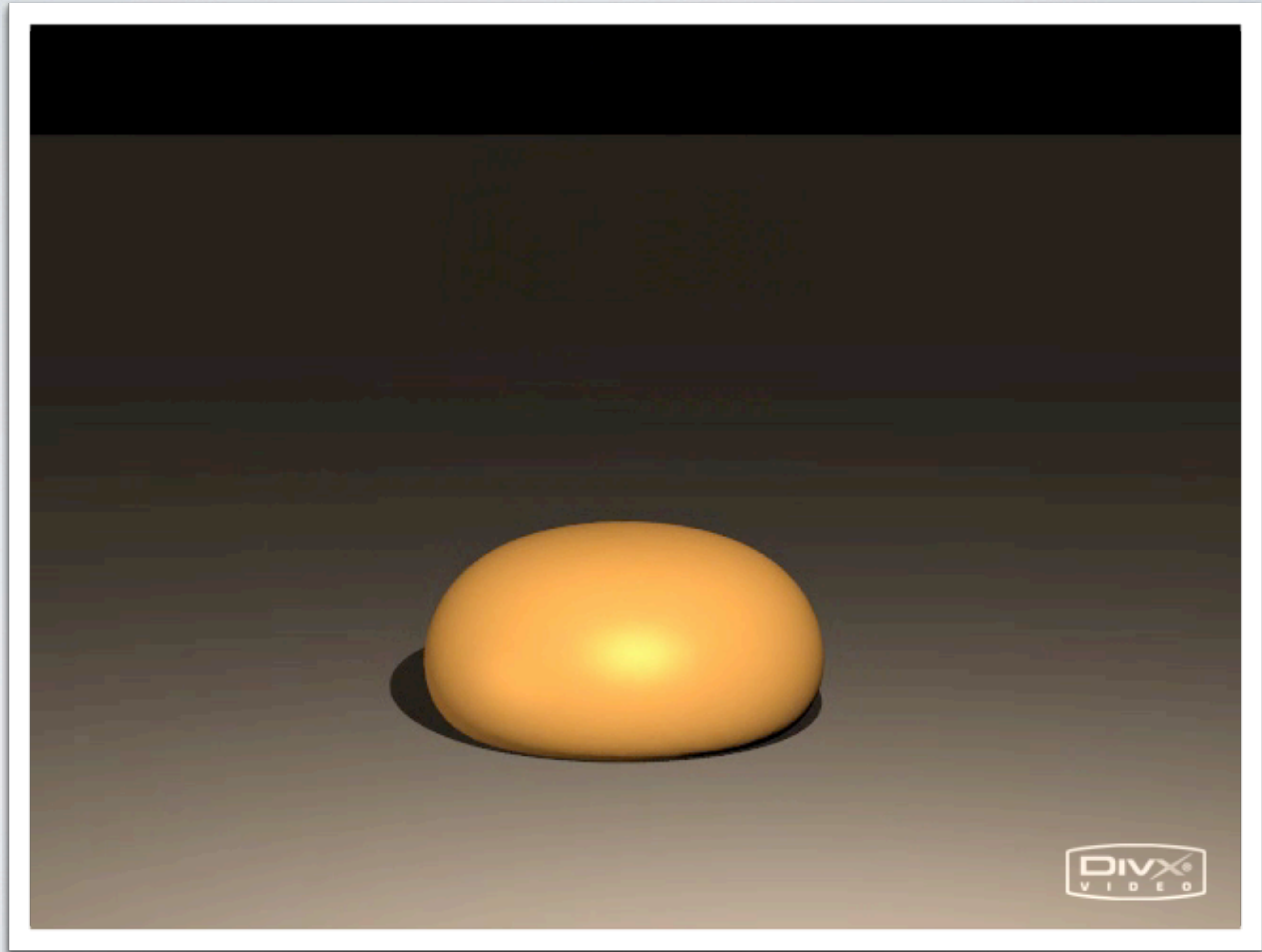

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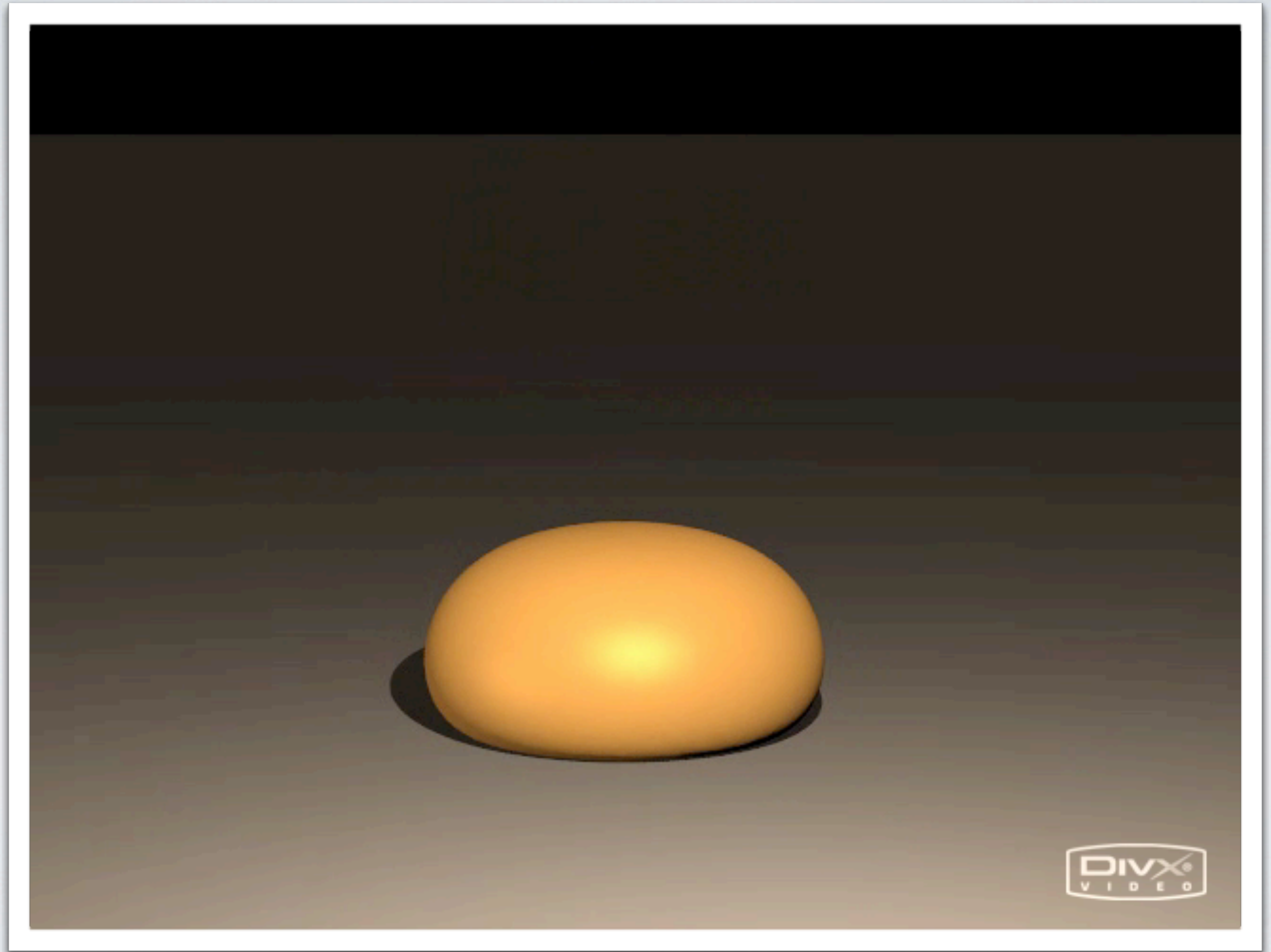

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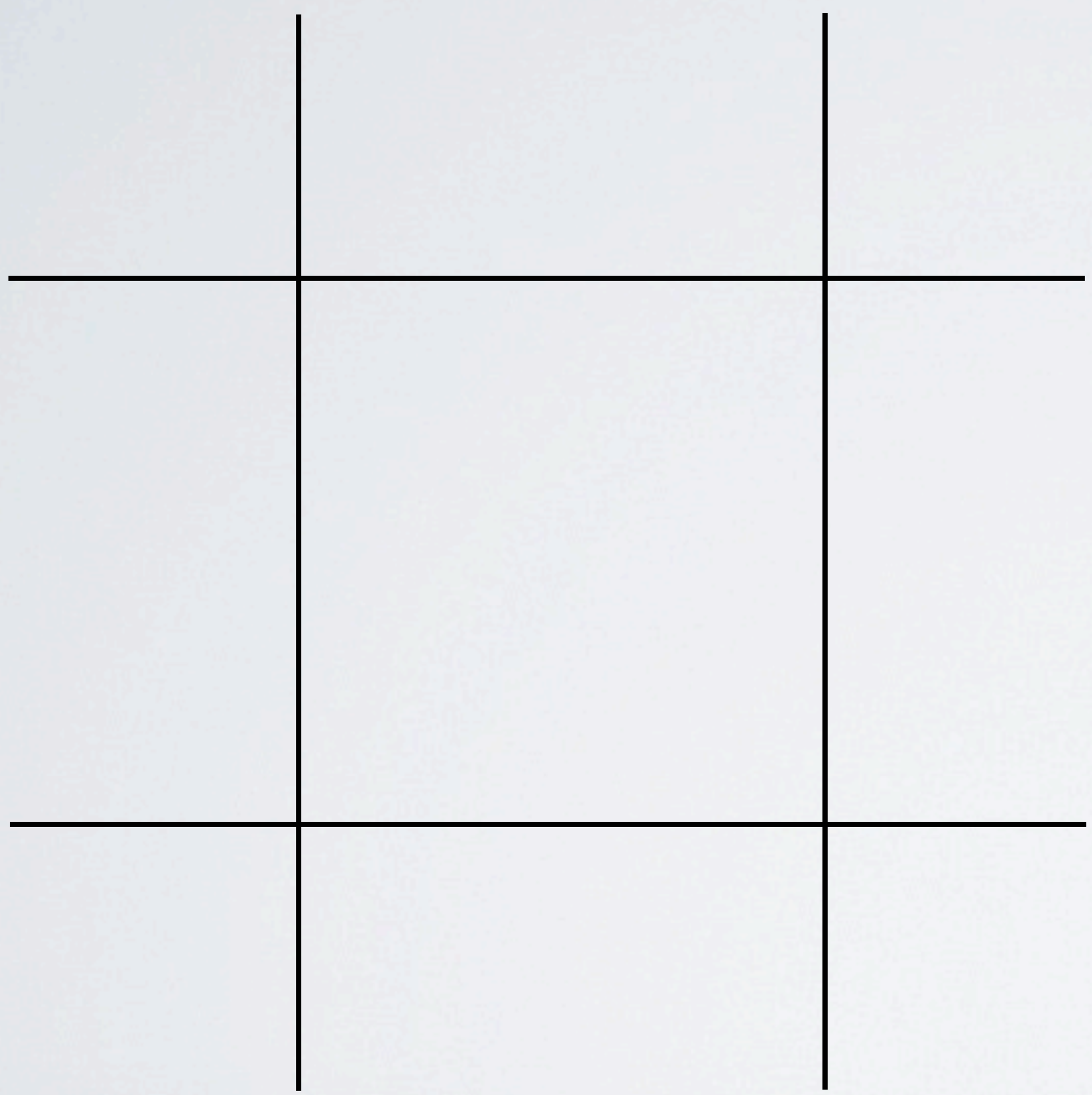
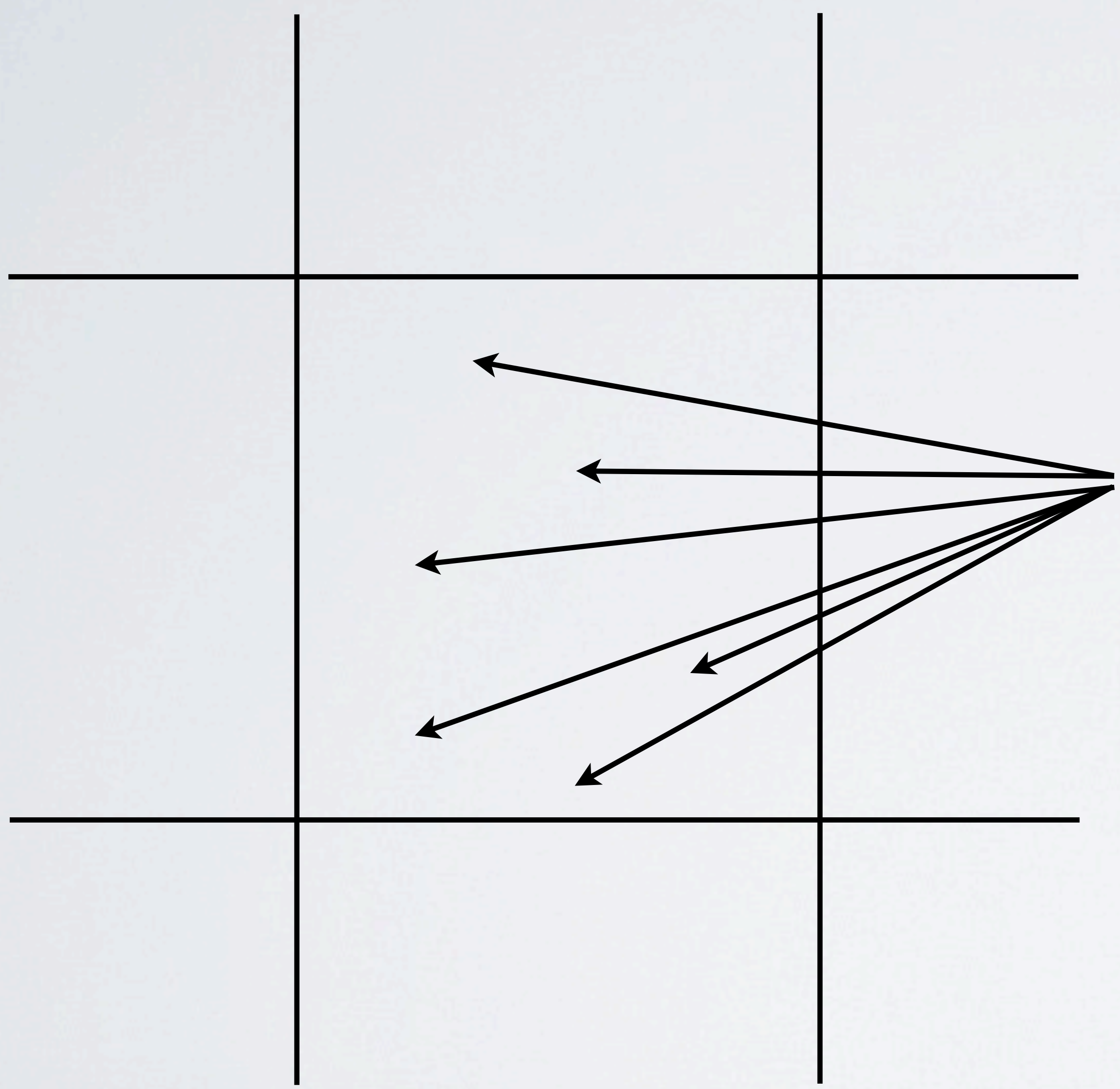


Elastic Energy

$$E = E_0 + \frac{\kappa}{2}\int_\Omega M^2(F)\delta X$$

INCOMPRESSIBILITY: LOCKING

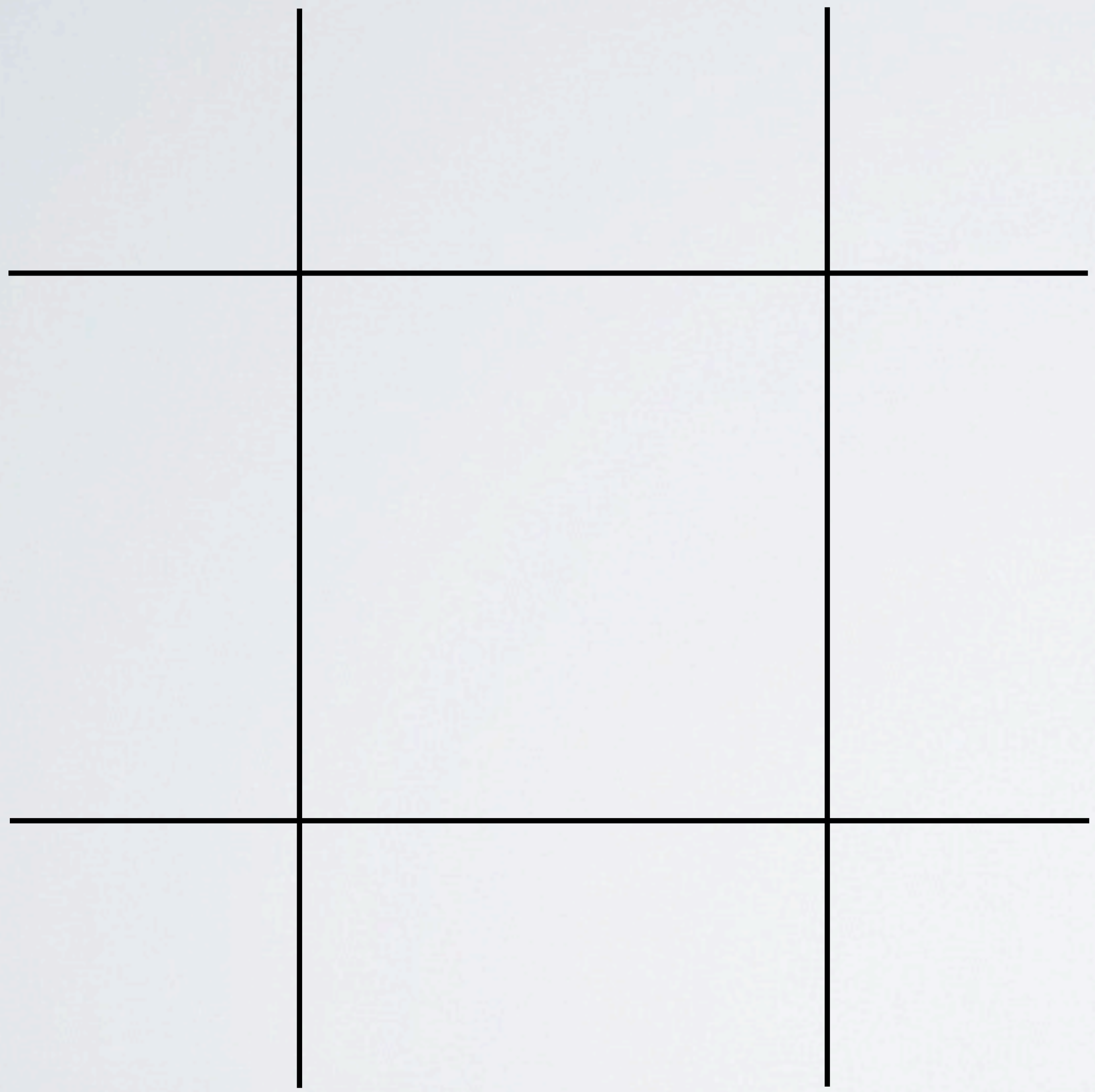


Require
Volume
Preservation
Everywhere

Elastic Energy

$$E = E_0 + \frac{\kappa}{2}\int_{\Omega} M^2(F)\delta X$$

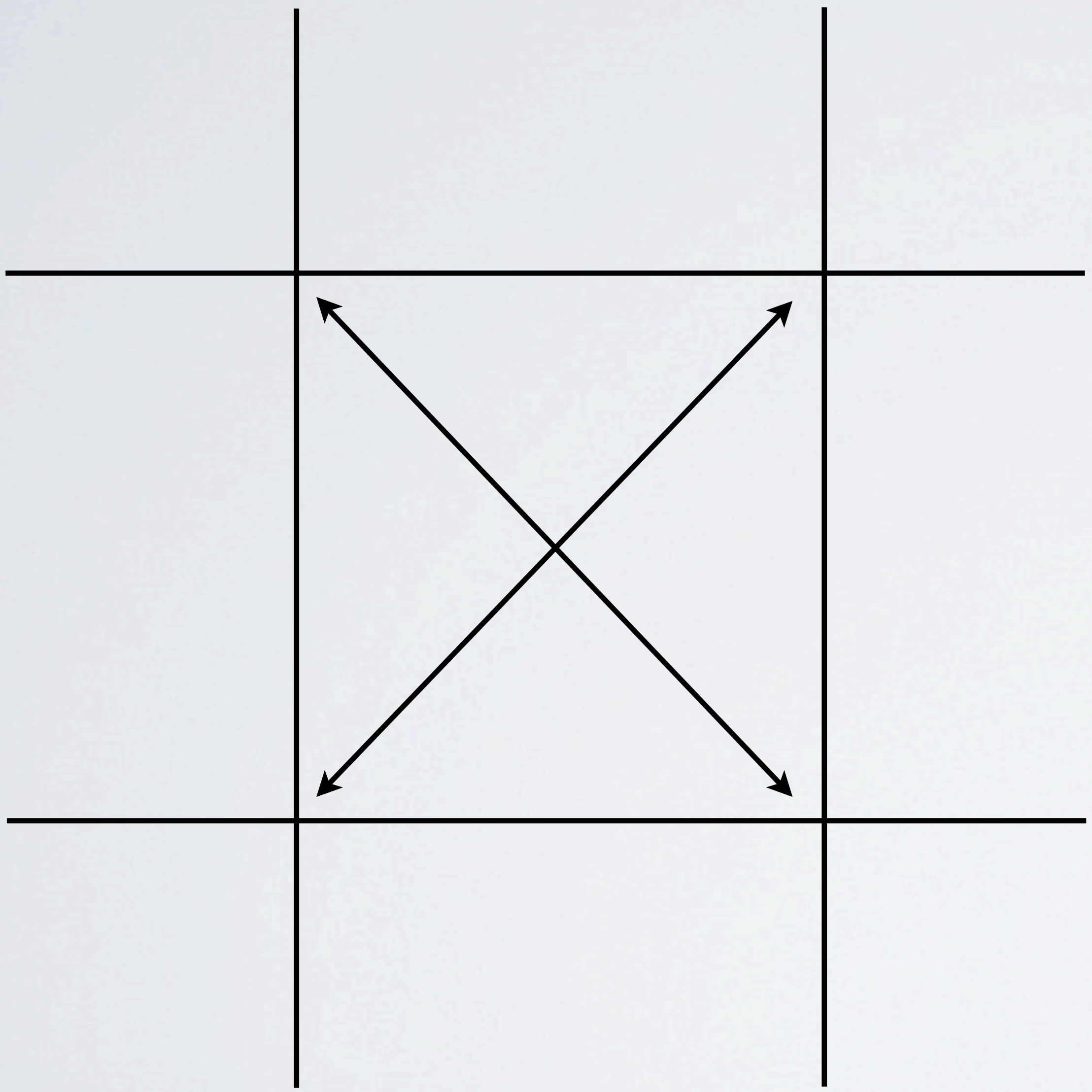
INCOMPRESSIBILITY: LOCKING



Elastic Energy

$$E = E_0 + \cancel{\frac{\kappa}{2}\int_\Omega M^2(F)\delta X}$$

INCOMPRESSIBILITY: LOCKING



Require
Volume
Preservation
On Average

Elastic Energy

$$\cancel{E = E_0 + \frac{\kappa}{2}\int_\Omega M^2(F)\delta X}$$

$$E = E_0 + \frac{\kappa}{2}\int_\Omega \overline{M}^2(F)\delta X$$

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Find Equilibrium $f(x) = 0 \implies \min \hat{E}$

INCOMPRESSIBILITY: CONDITIONING

Find Equilibrium $f(x) = 0 \implies \min \hat{E}$

INCOMPRESSIBILITY: CONDITIONING

$\boxed{\text{Find Equilibrium} \quad f(x) = 0 \implies \min \hat{E}}$

No internal forces implies equilibrium after deformation

INCOMPRESSIBILITY: CONDITIONING

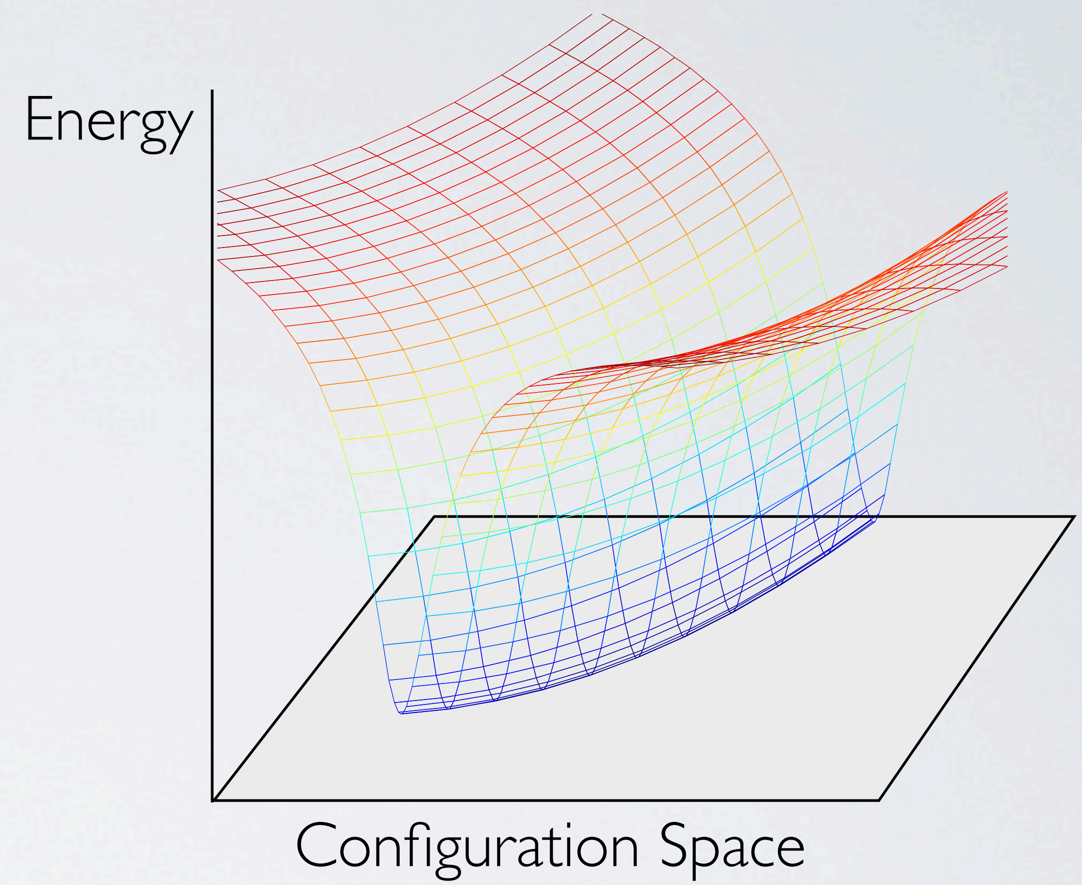
Find Equilibrium $f(x) = 0 \implies \min \hat{E}$

No internal forces implies equilibrium after deformation

Zero forces implies a minimum in the potential energy

INCOMPRESSIBILITY: CONDITIONING

$$\Psi(F) = \Psi_0 + \frac{\kappa}{2}\overline{M}^2(F)$$



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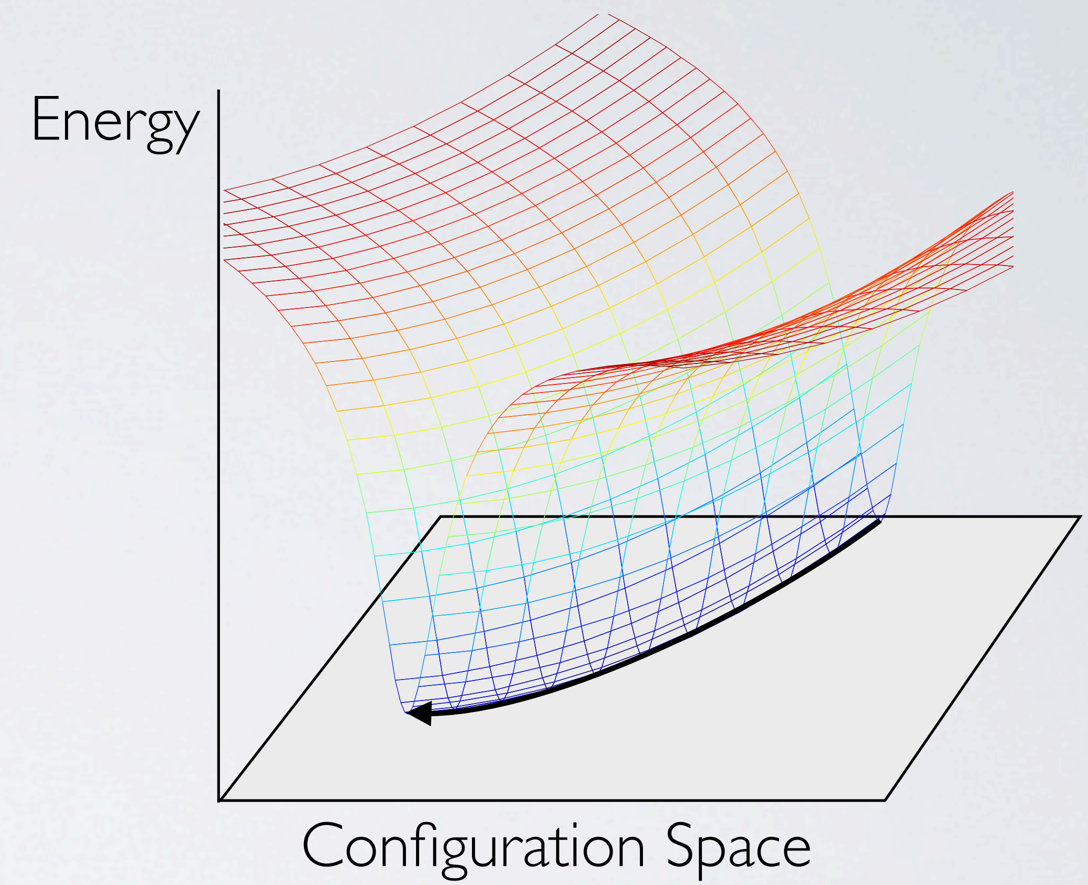

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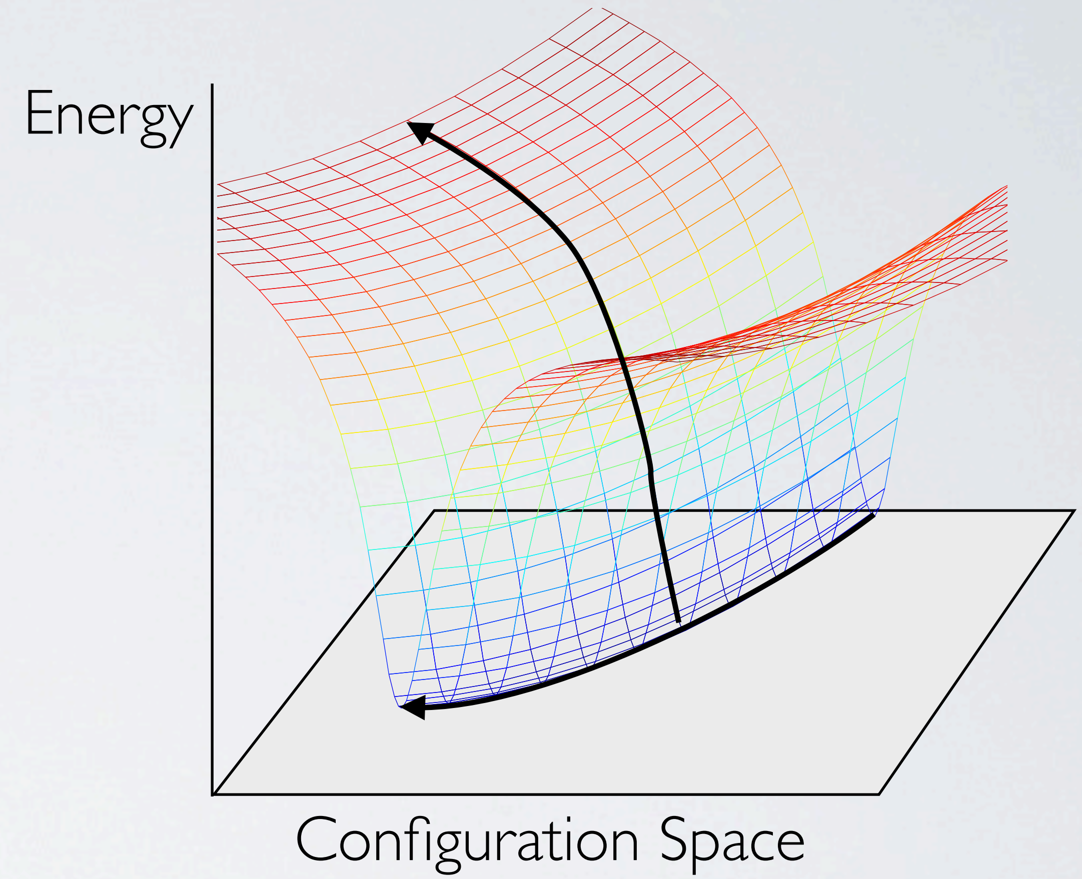

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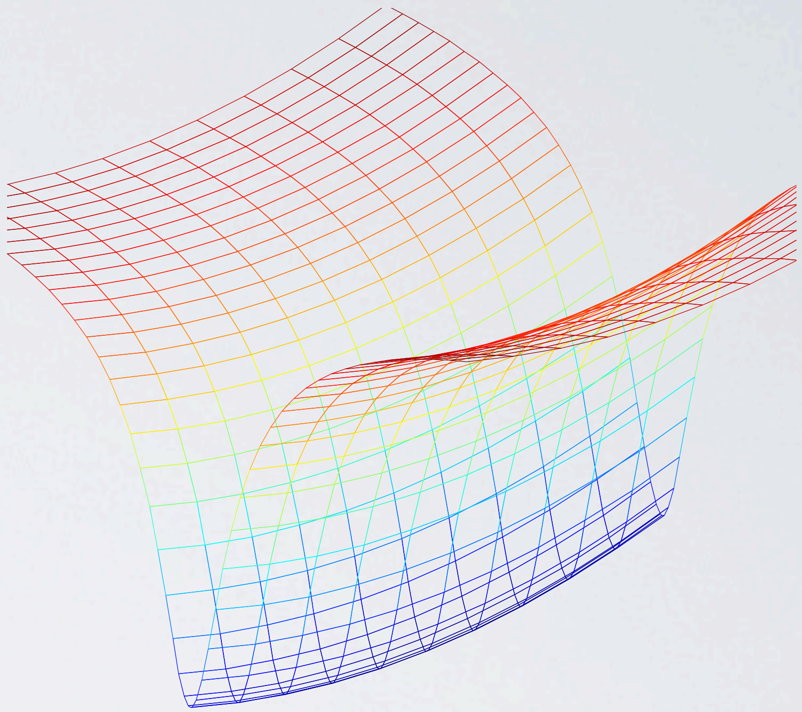

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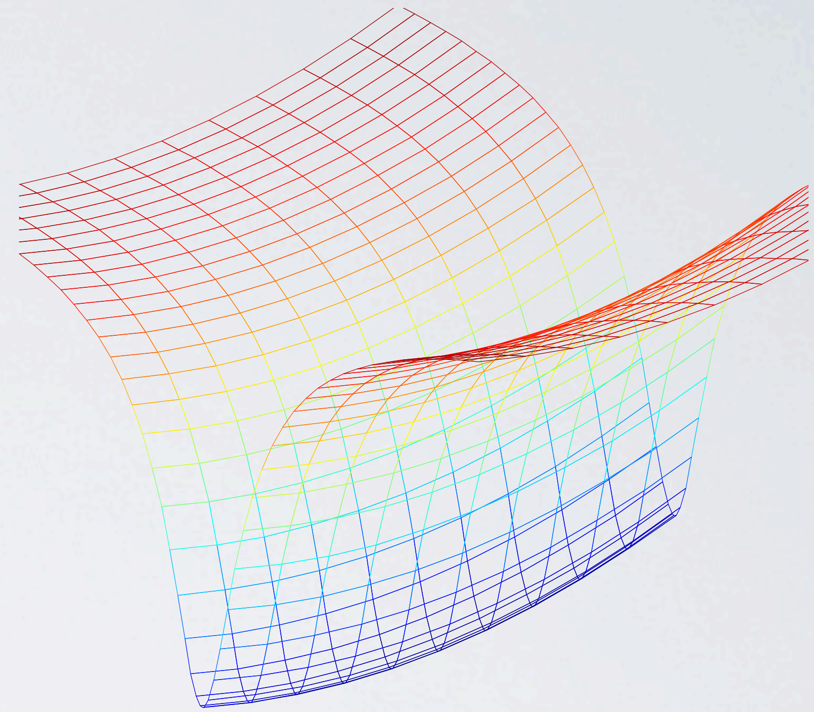
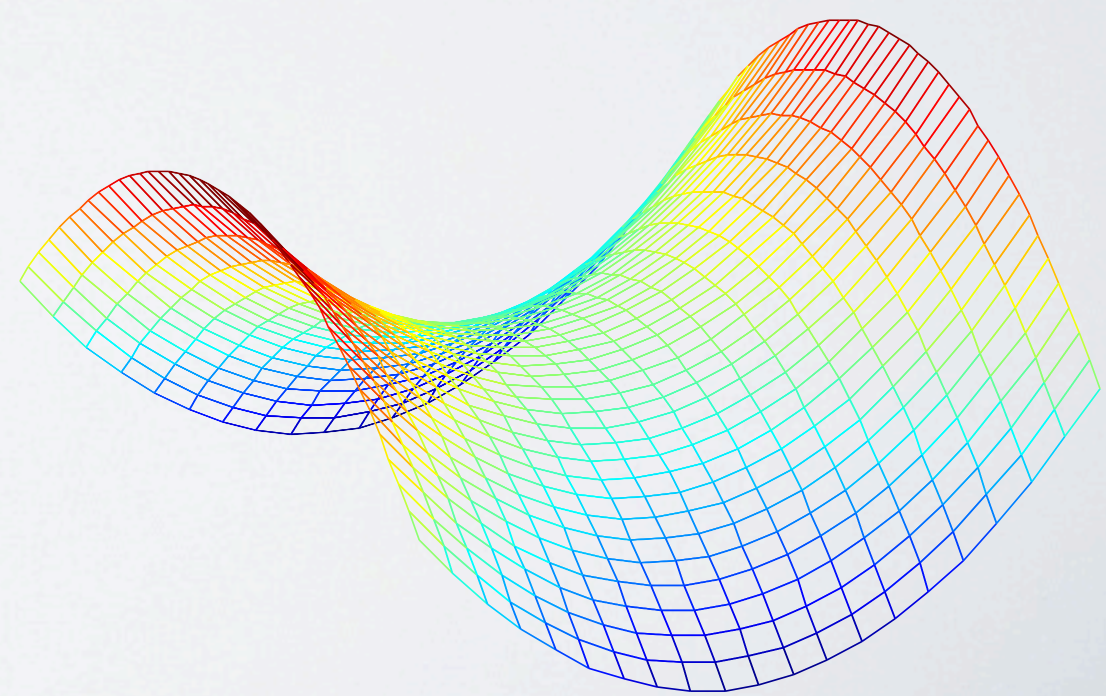


$$\downarrow$$

$$\Psi(F,p) = \Psi_0(F) + \alpha p \overline{M}(F) - \frac{\alpha^2 p^2}{2\kappa}$$



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$$\downarrow$$

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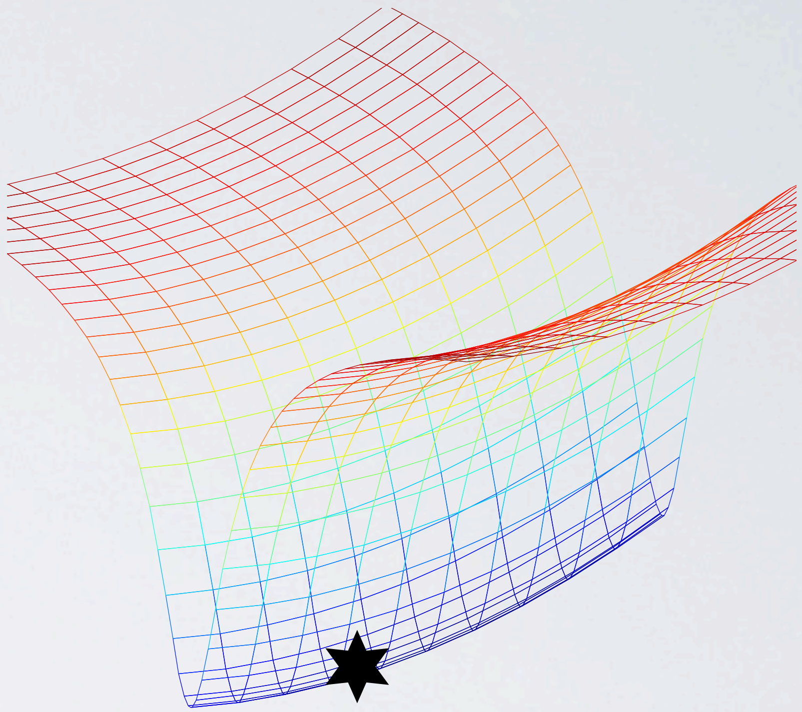
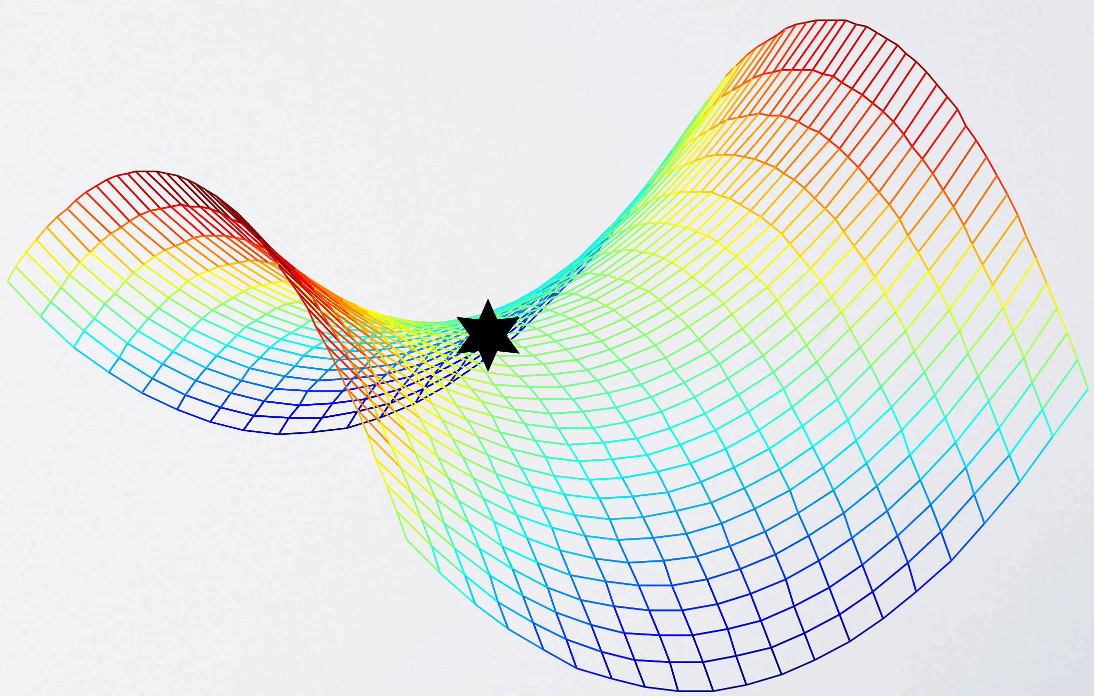


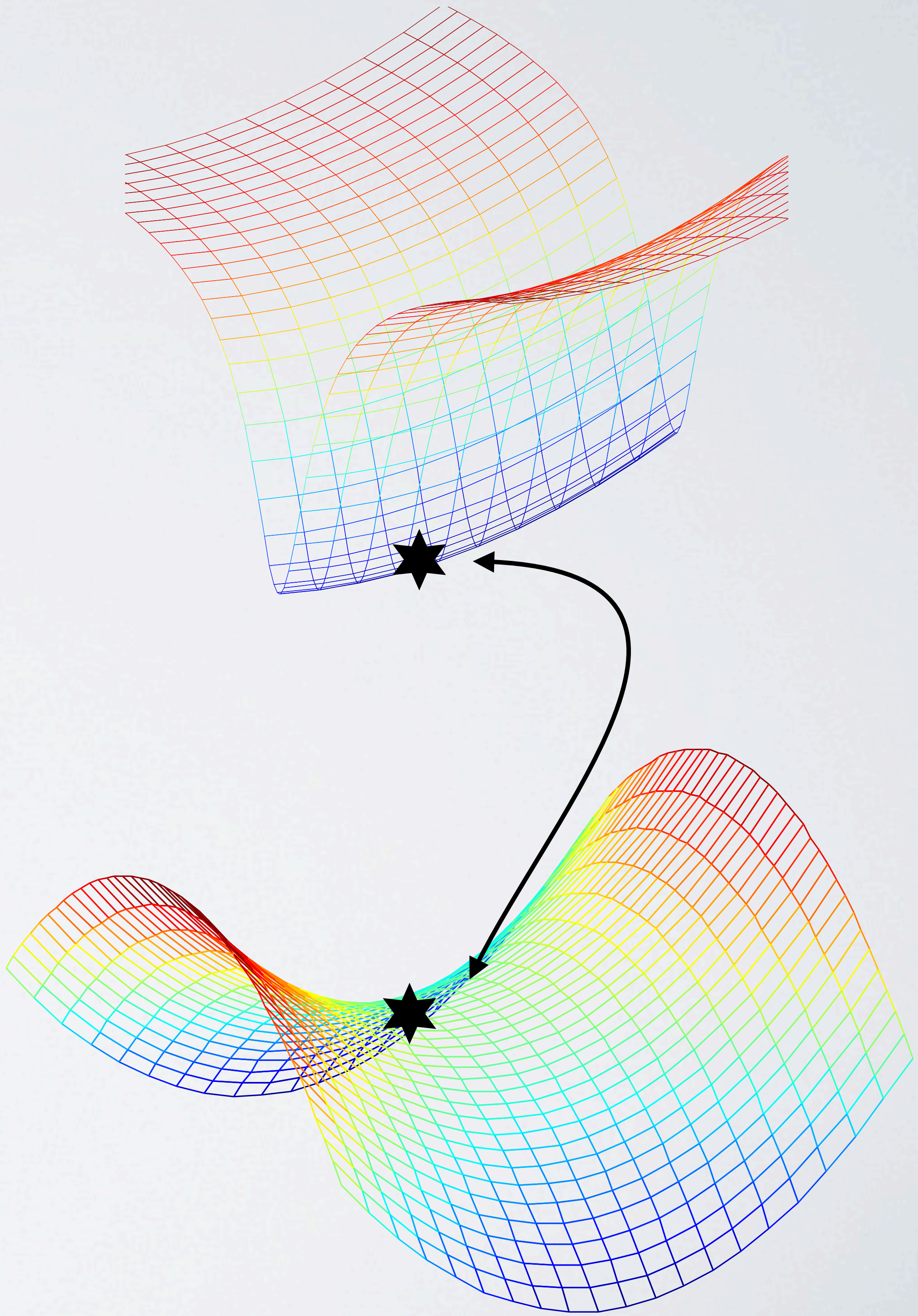


INCOMPRESSIBILITY: CONDITIONING

$$\Psi(F) = \Psi_0 + \frac{\kappa}{2}\overline{M}^2(F)$$

$$\downarrow$$

$$\Psi(F,p) = \Psi_0(F) + \alpha p \overline{M}(F) - \frac{\alpha^2 p^2}{2\kappa}$$



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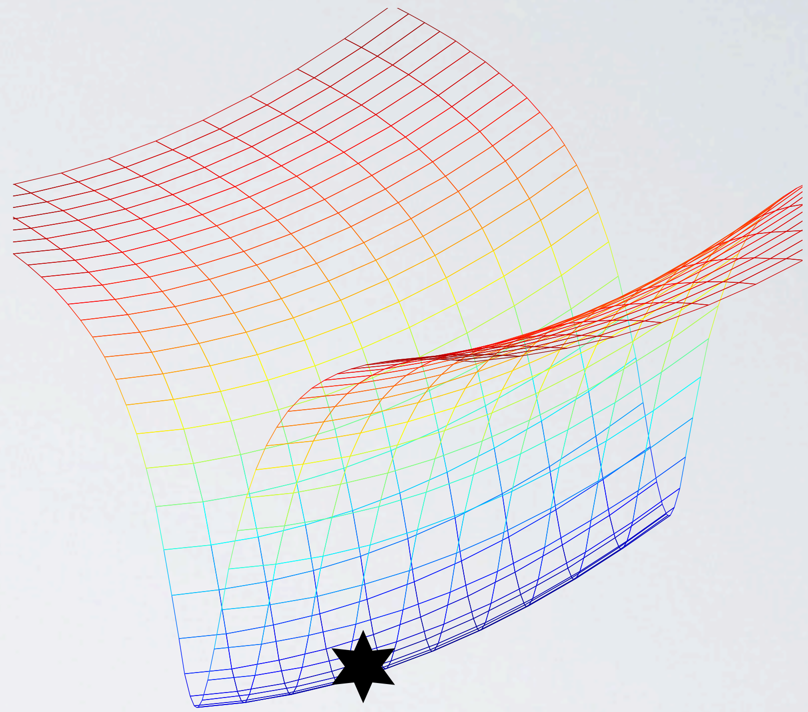

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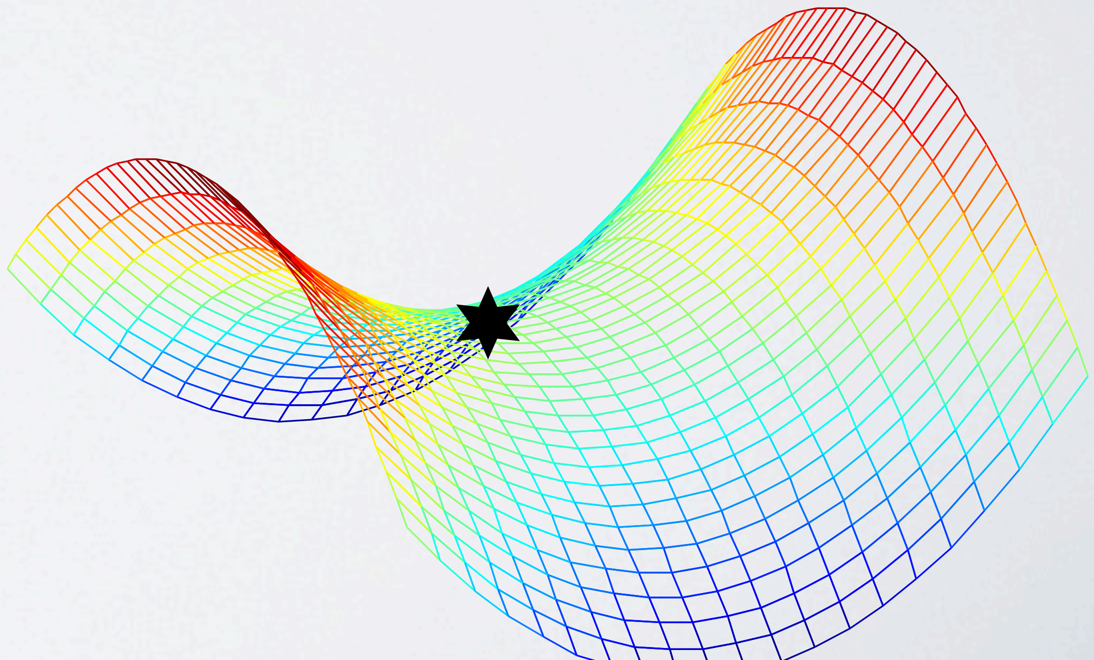

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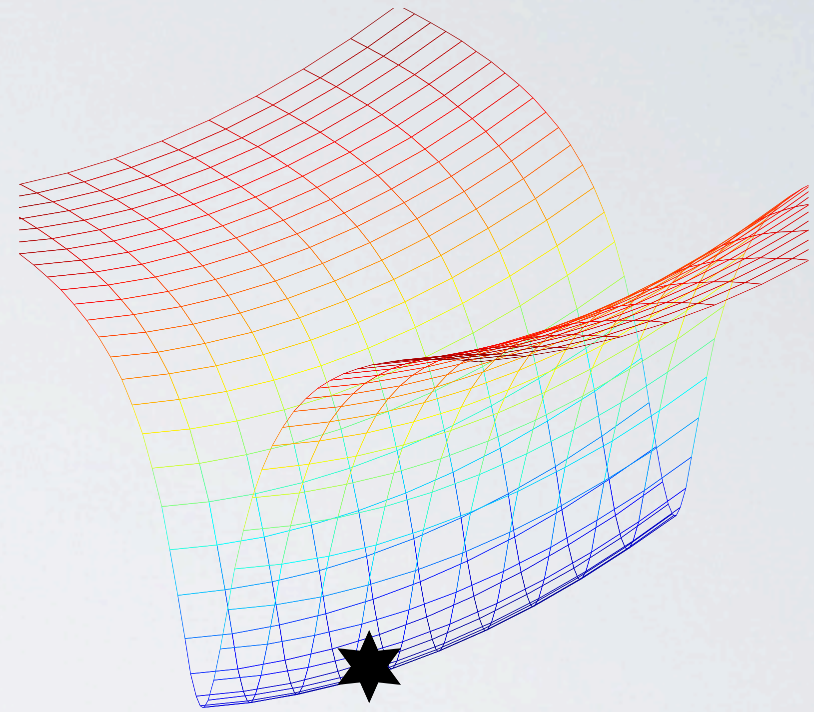

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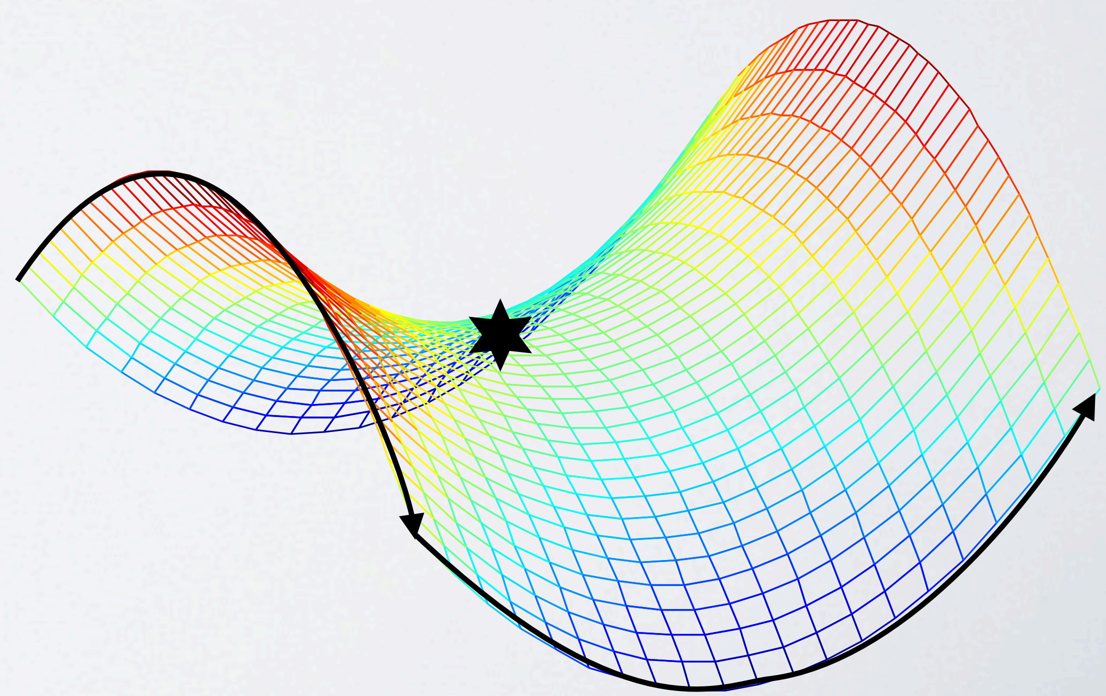
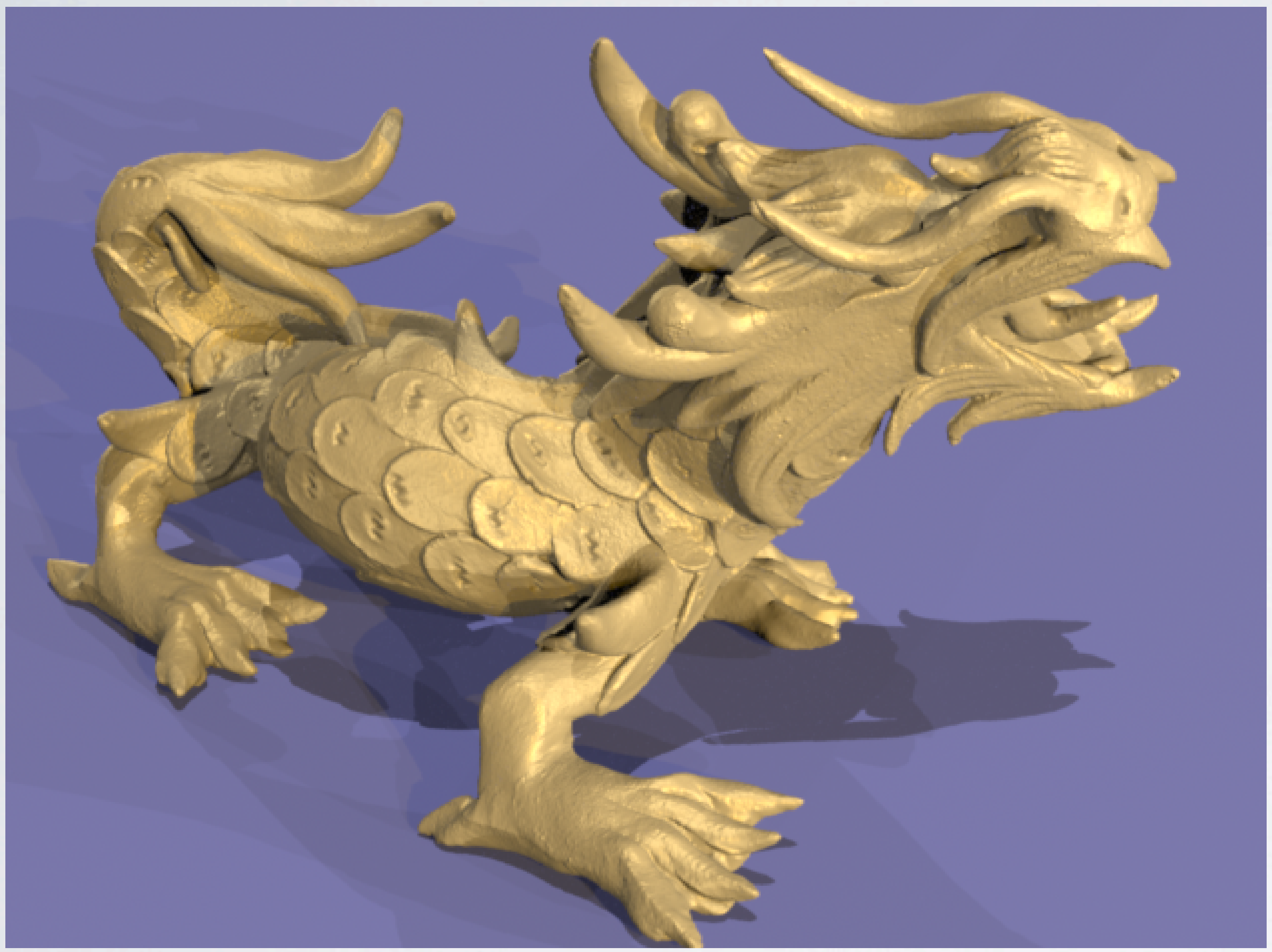
$$\Psi(F,p) = \Psi_0(F) + \alpha p \overline{M}(F) - \frac{\alpha^2 p^2}{2\kappa}$$



INCOMPRESSIBILITY: CONDITIONING



[Zhu et al. 2010]

Implemented at PDE level

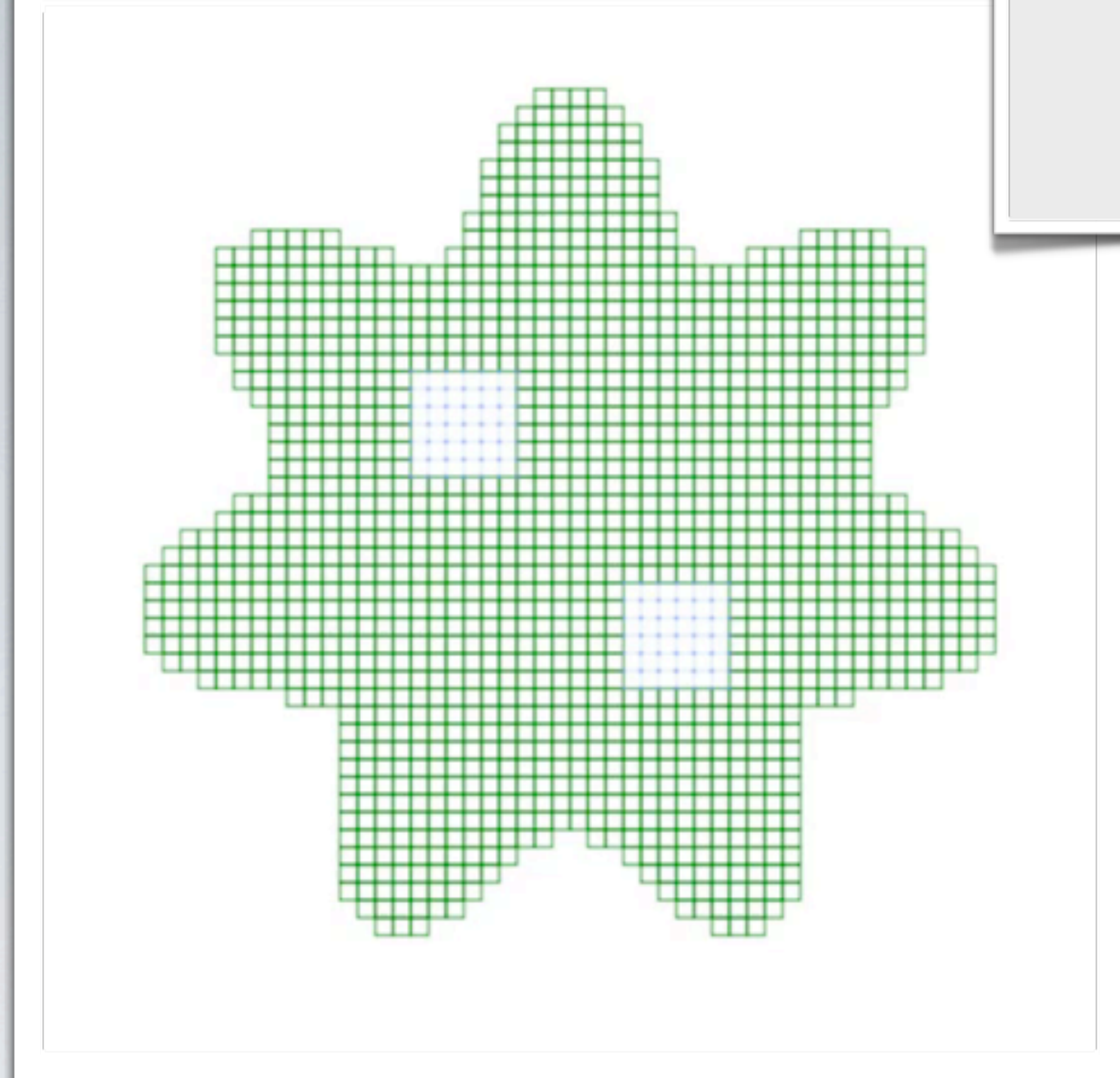
More flexiblity at energy level

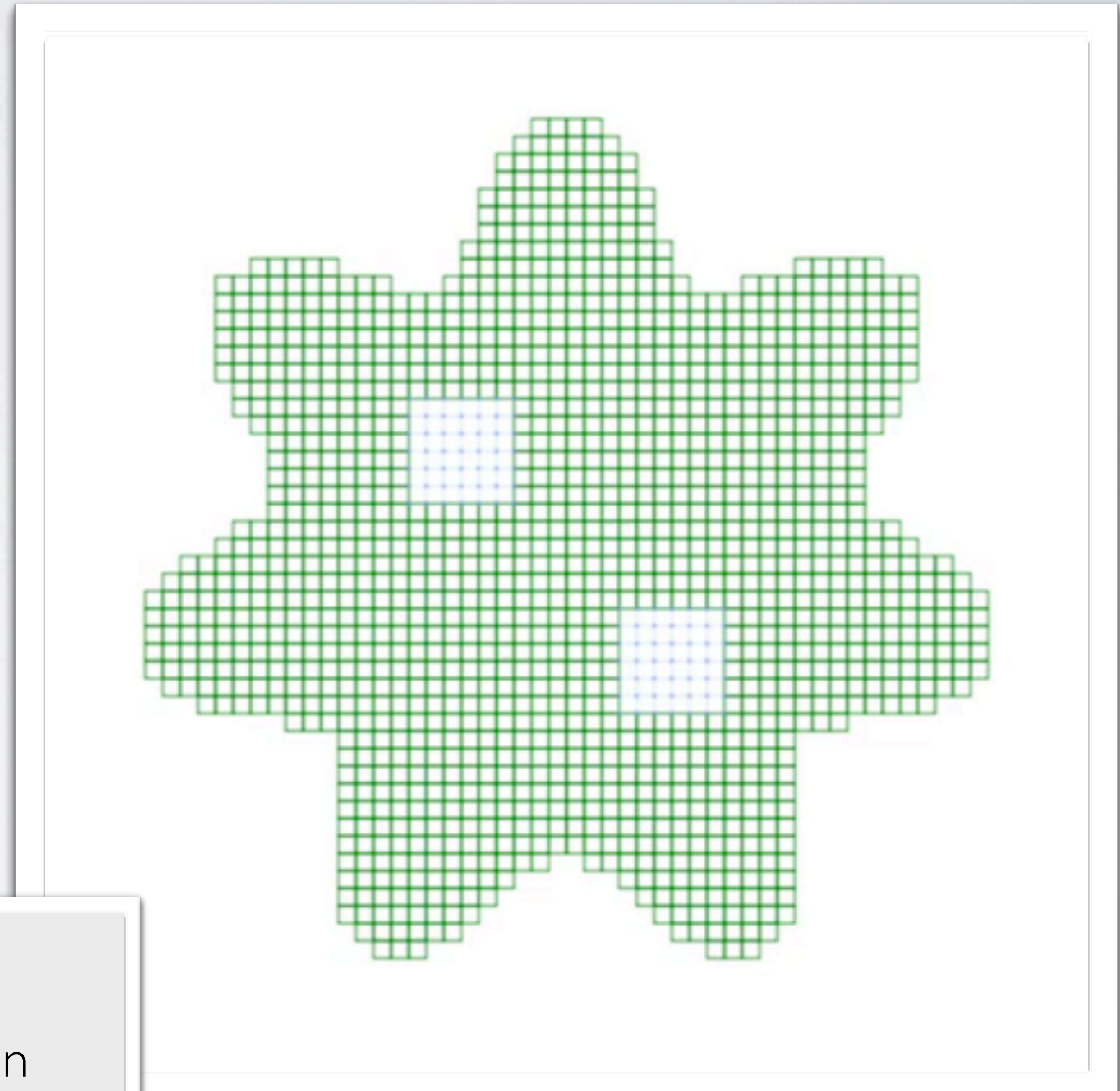
Limited set of materials

Using conventional
minimization formulation

Using saddle
point formulation

Using conventional
minimization formulation





Using saddle
point formulation

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minimization formulation

Using saddle
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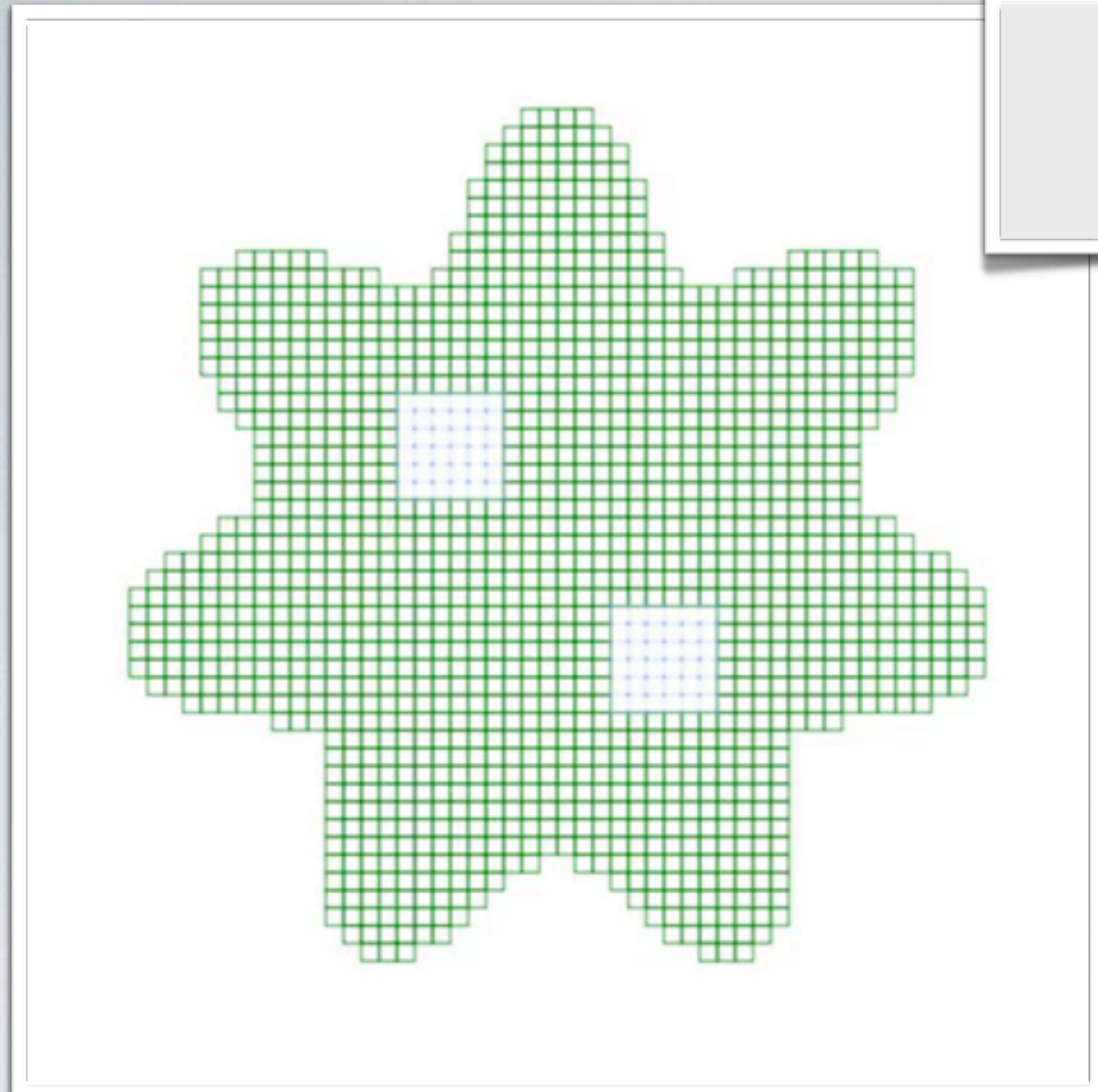
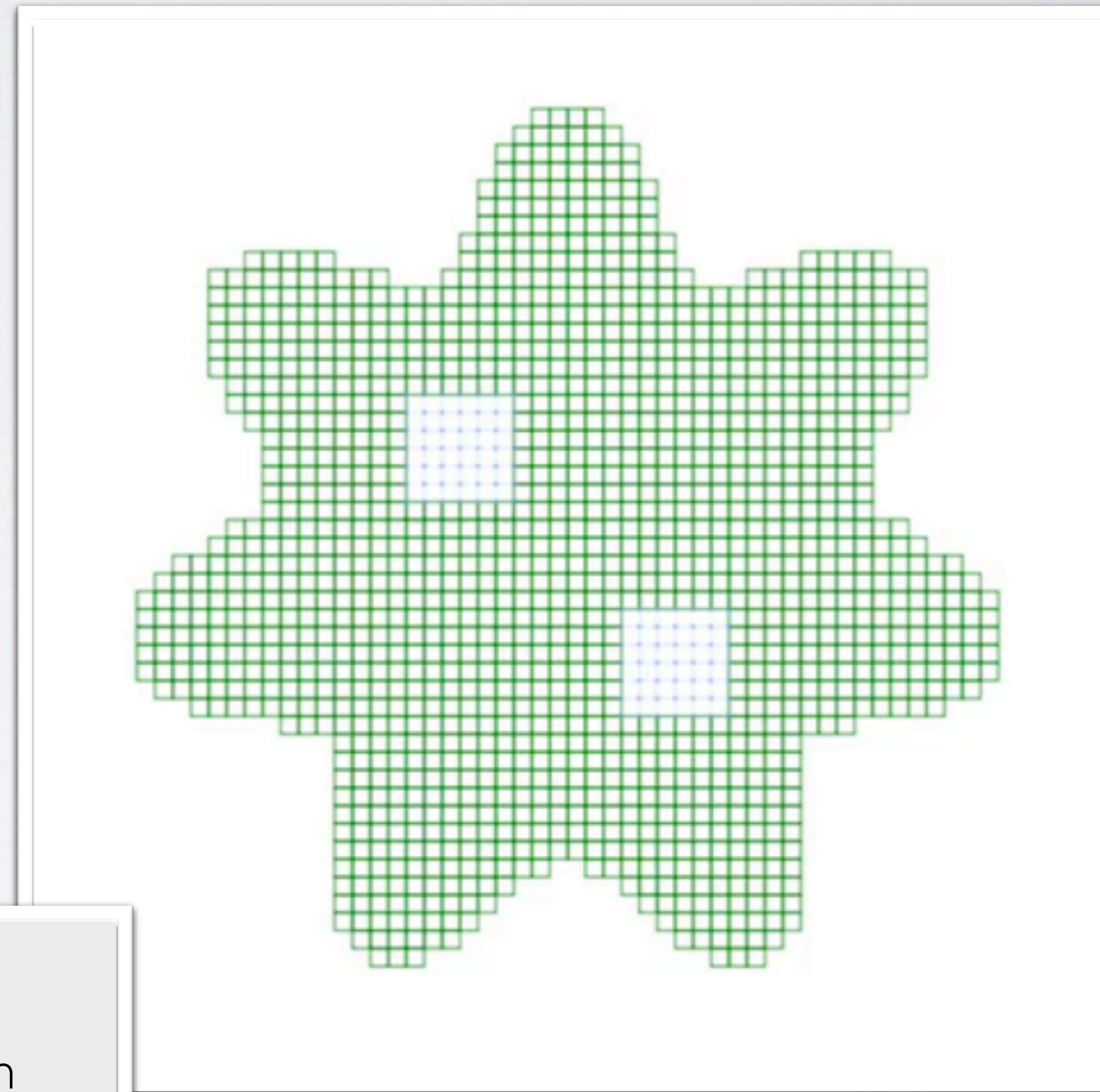


Using conventional minimization formulation



Using saddle point formulation

FEATURES

Arbitrary Materials

Incompressibility

Sub-voxel Precision

Robust

Parallelism

FEATURES

Arbitrary Materials

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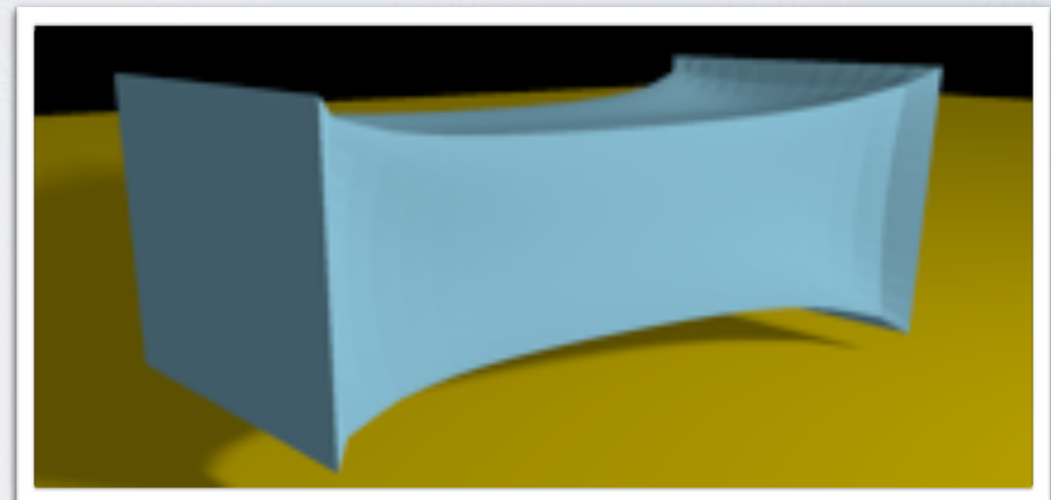

FEATURES

Arbitrary Materials

✓ Incompressibility

Sub-voxel Precision

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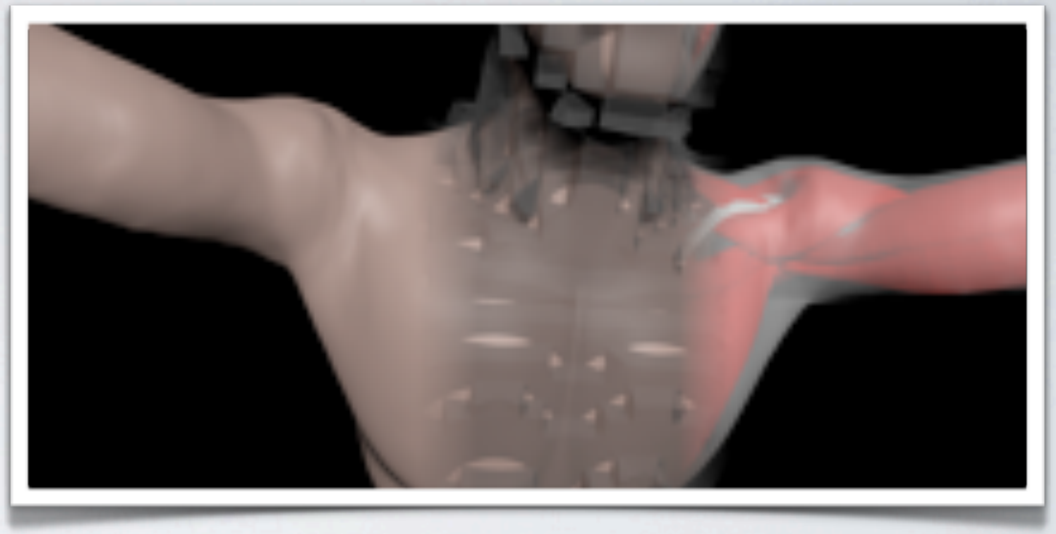
FEATURES



Arbitrary Materials

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FEATURES

✓ Arbitrary Materials

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FEATURES

✓ Arbitrary Materials

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BOUNDARY CELLS

Start with Energy Density

↓

Apply Incompressibility Adjustment

↓

Compute Discrete Energy $\hat{E}$

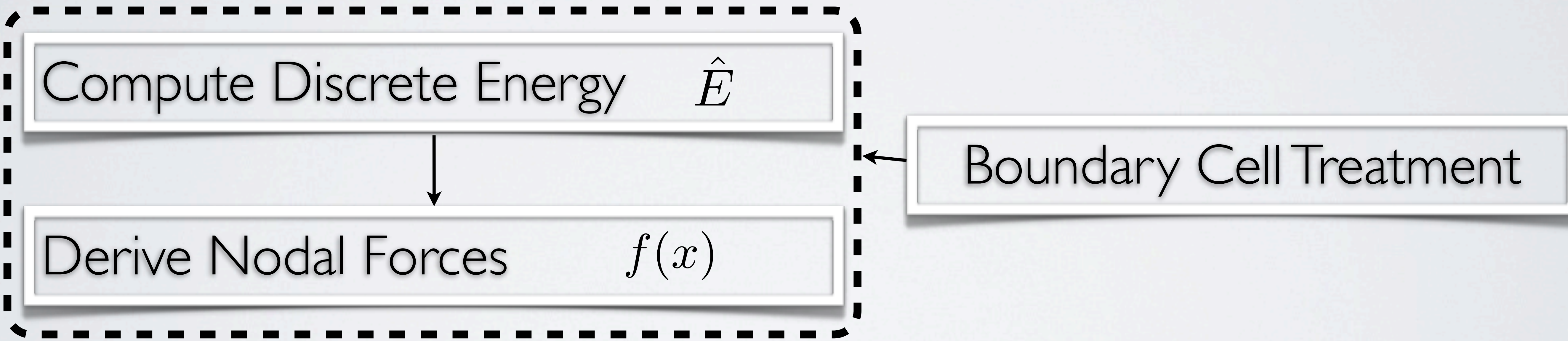
↓

Derive Nodal Forces $f(x)$

↓

Find Equilibrium $f(x) = 0 \implies \min \hat{E}$

BOUNDARY CELLS



BOUNDARY CELLS: MOTIVATION

Importance

BOUNDARY CELLS: MOTIVATION

Importance

- Objects are not made of cubes

BOUNDARY CELLS: MOTIVATION

Importance

- Objects are not made of cubes
- Visible grid artifacts (discontinuities)

BOUNDARY CELLS: MOTIVATION

Importance

- Objects are not made of cubes
- ~~Visible grid artifacts (discontinuities)~~

> Interpolation discontinuities can be avoided by using tricubic interpolation

BOUNDARY CELLS: MOTIVATION

Importance

- Objects are not made of cubes
- ~~Visible grid artifacts (discontinuities)~~

Challenges

BOUNDARY CELLS: MOTIVATION

Importance

Objects are not made of cubes

~~Visible grid artifacts (discontinuities)~~

Challenges

How accurate are we?

BOUNDARY CELLS: MOTIVATION

Importance

Objects are not made of cubes

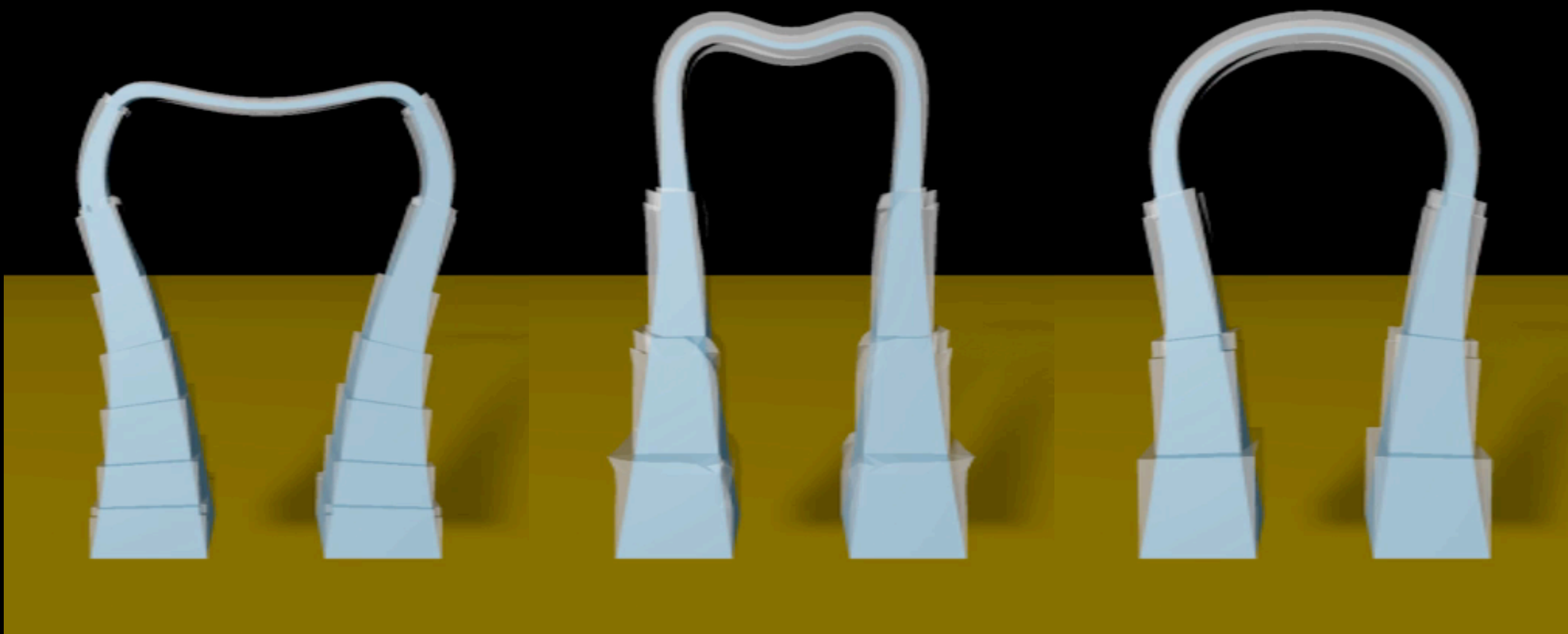
~~Visible grid artifacts (discontinuities)~~

Challenges

How accurate are we?

Can we make it go fast?

Reference Treated Not Treated



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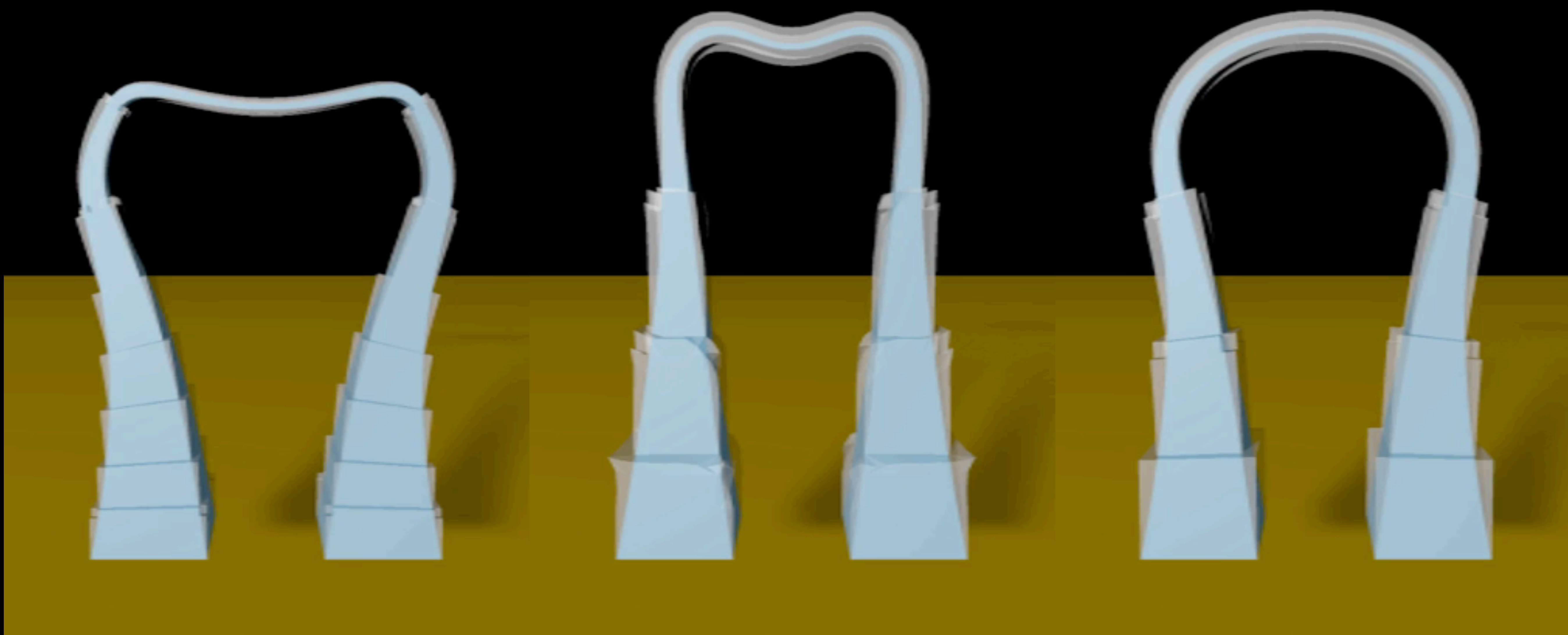

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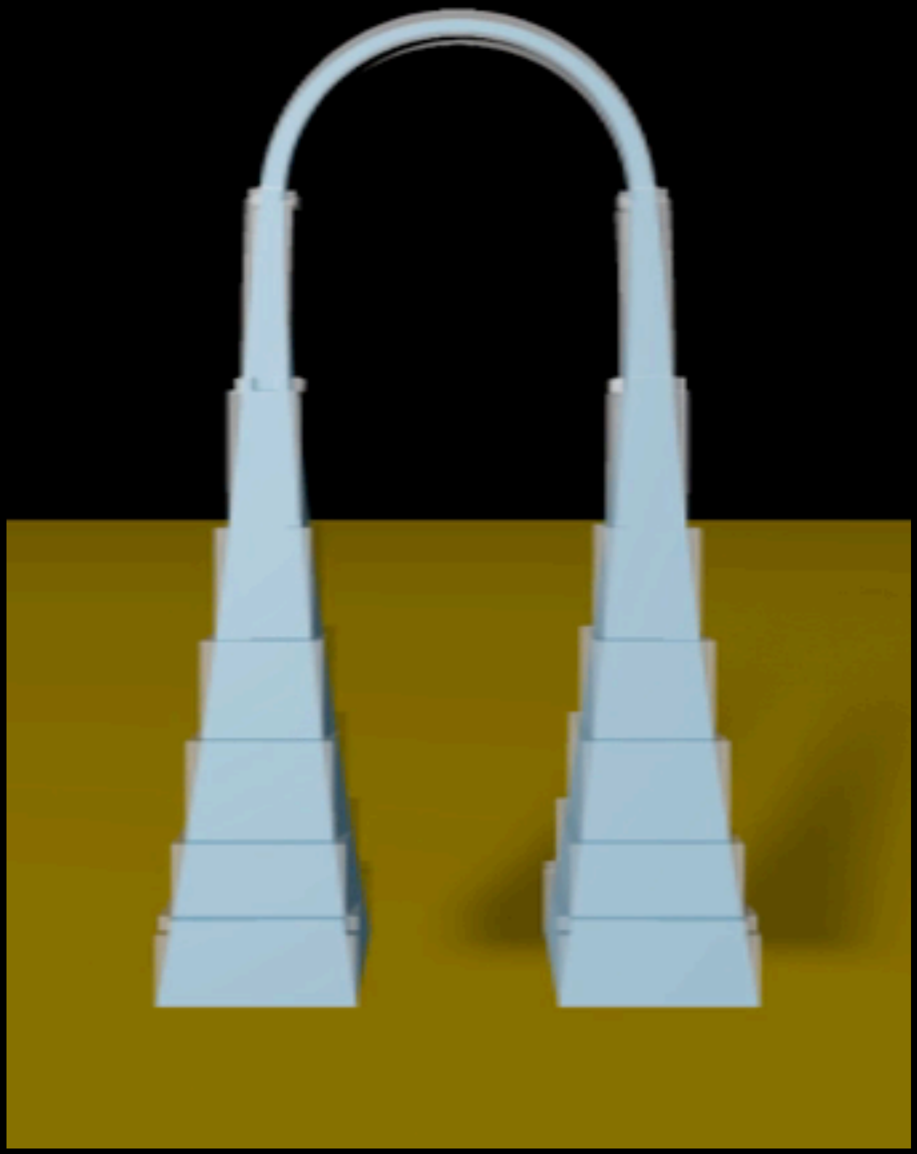
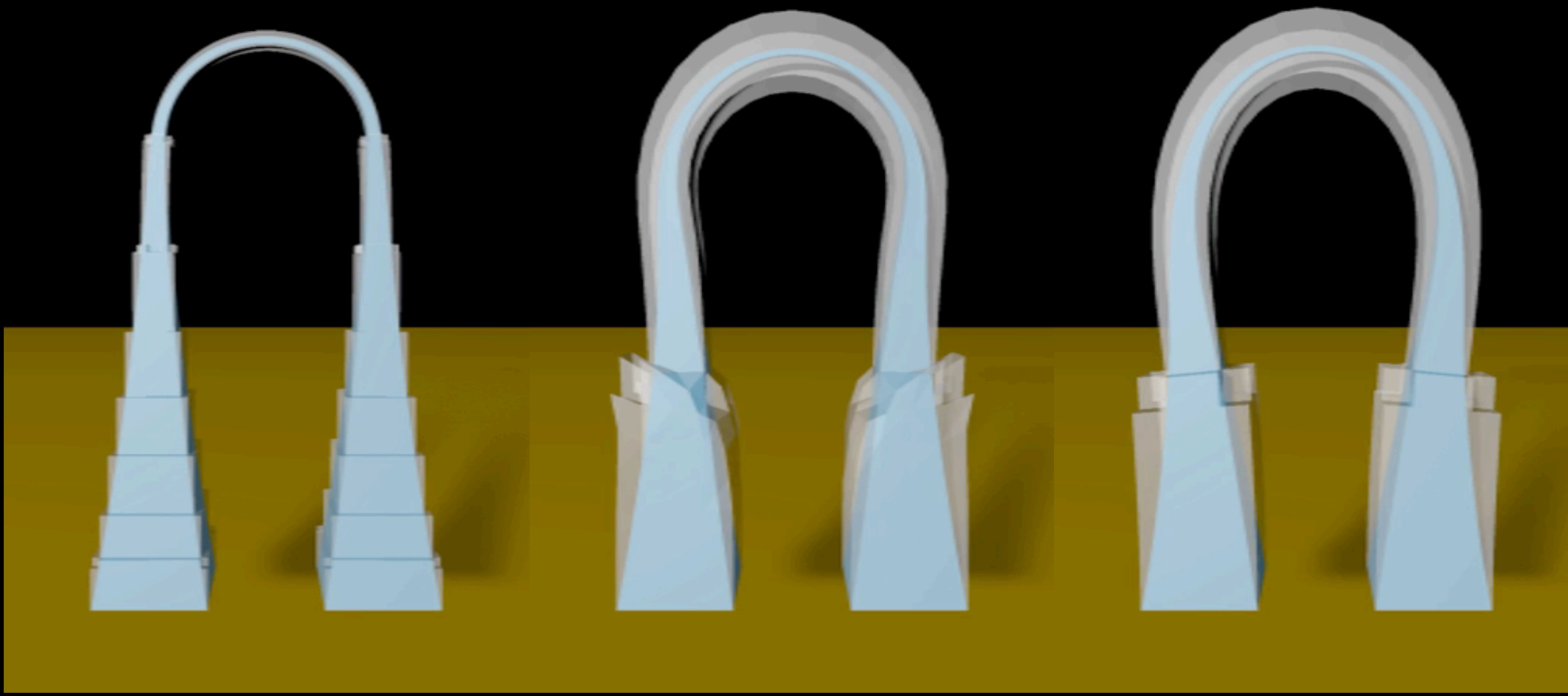


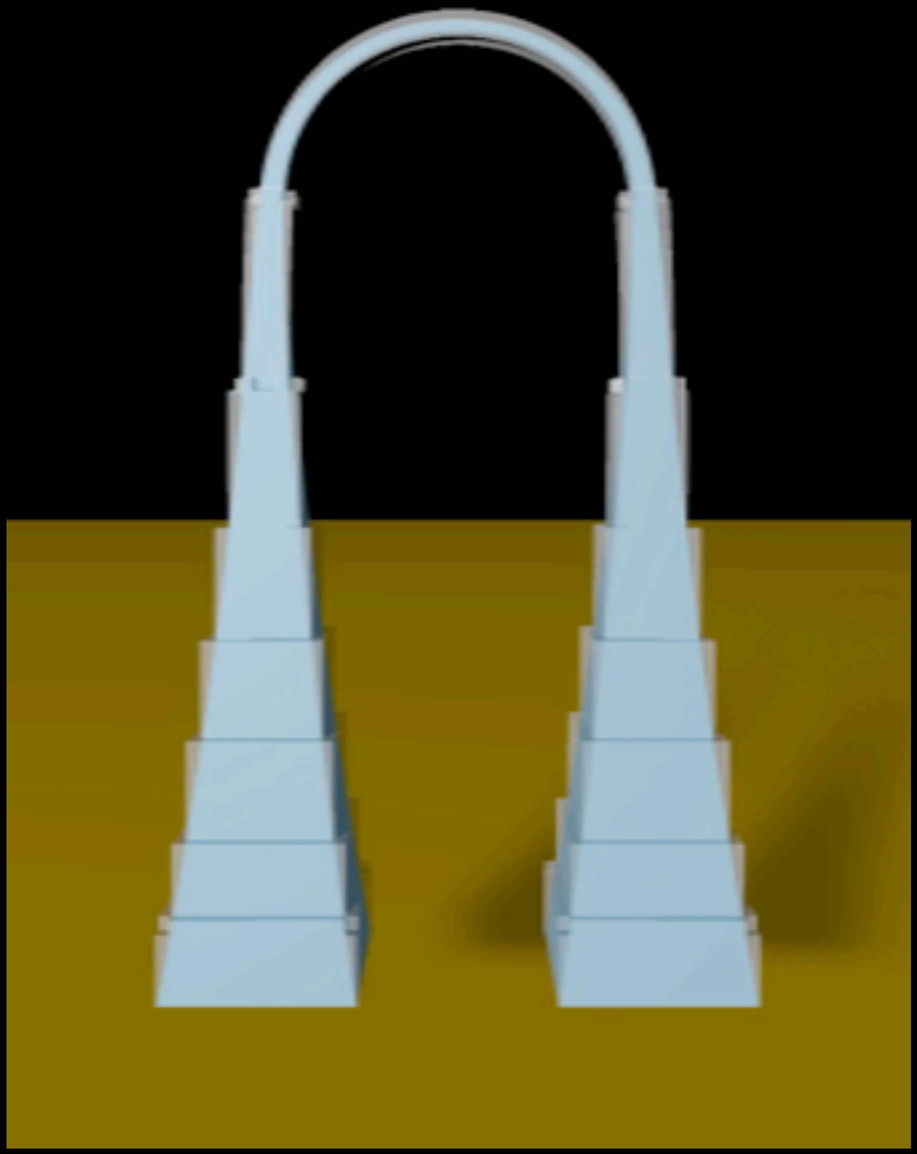
Reference

Treated

Not Treated



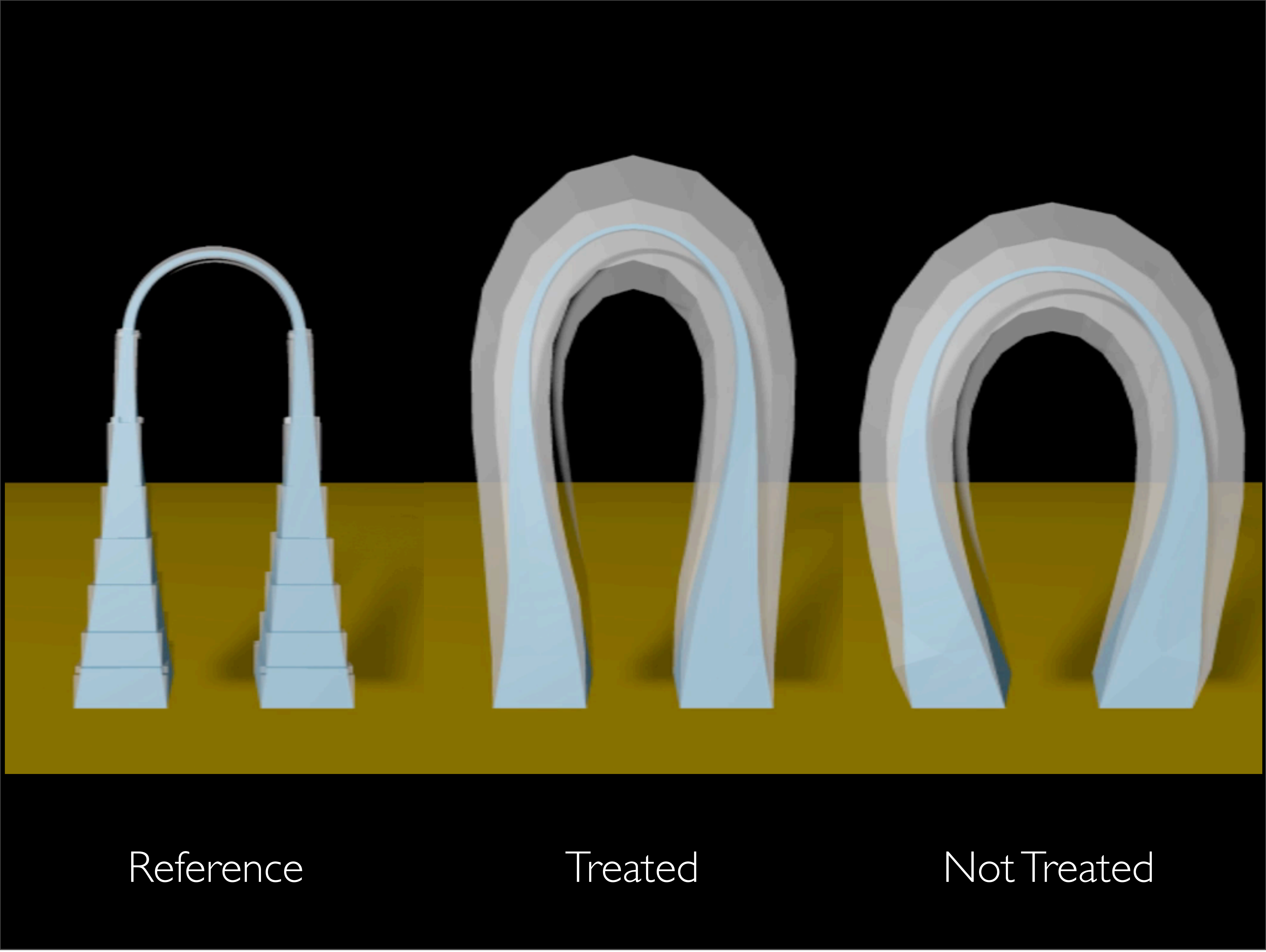
Reference Treated Not Treated



Reference

Treated

Not Treated


Reference
Treated
Not Treated

BOUNDARY CELL TREATMENT

Start with Energy Density

$\downarrow$

Apply Incompressibility Adjustment

$\downarrow$

Compute Discrete Energy $\hat{E}$

$\downarrow$

Derive Nodal Forces $f(x)$

$\downarrow$

Find Equilibrium $f(x) = 0 \implies \min \hat{E}$

BOUNDARY CELL TREATMENT

Start with Energy Density

↓

Apply Incompressibility Adjustment

↓

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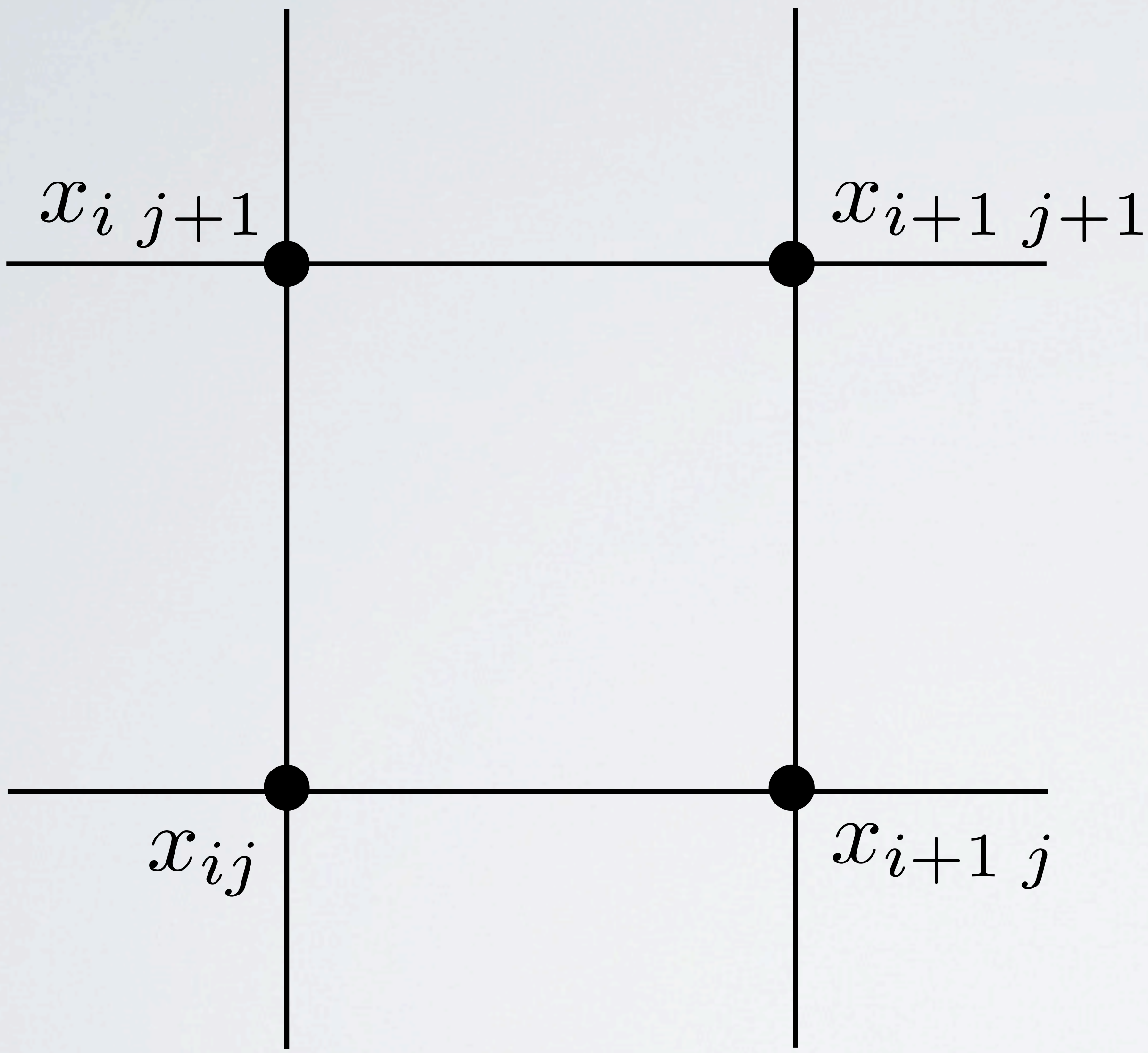
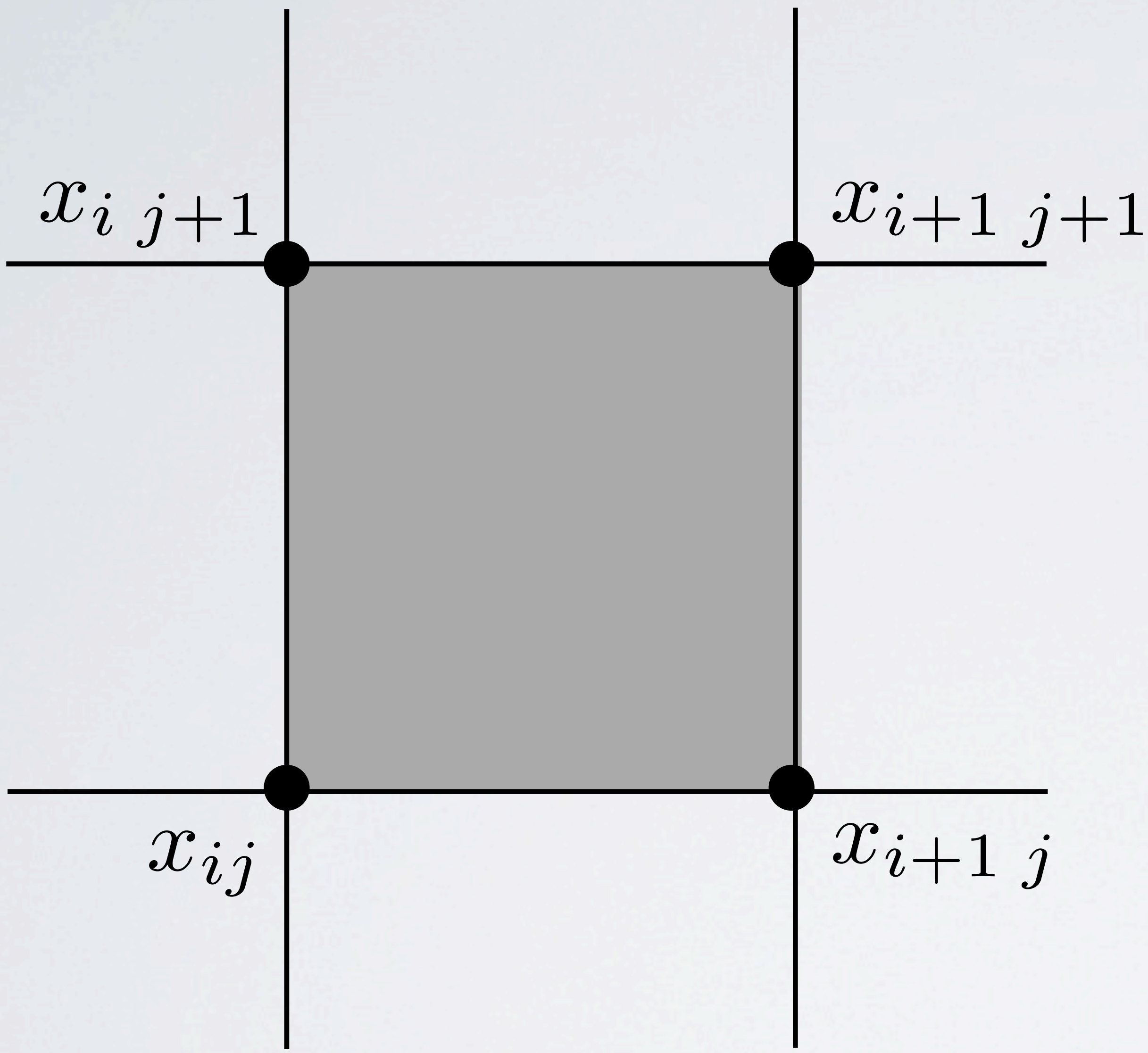
BOUNDARY CELL TREATMENT



$$E_{obj} = \sum E_{cell}$$

$$E_{cell} = \int_{cell} \Psi(F)\partial X$$

BOUNDARY CELL TREATMENT

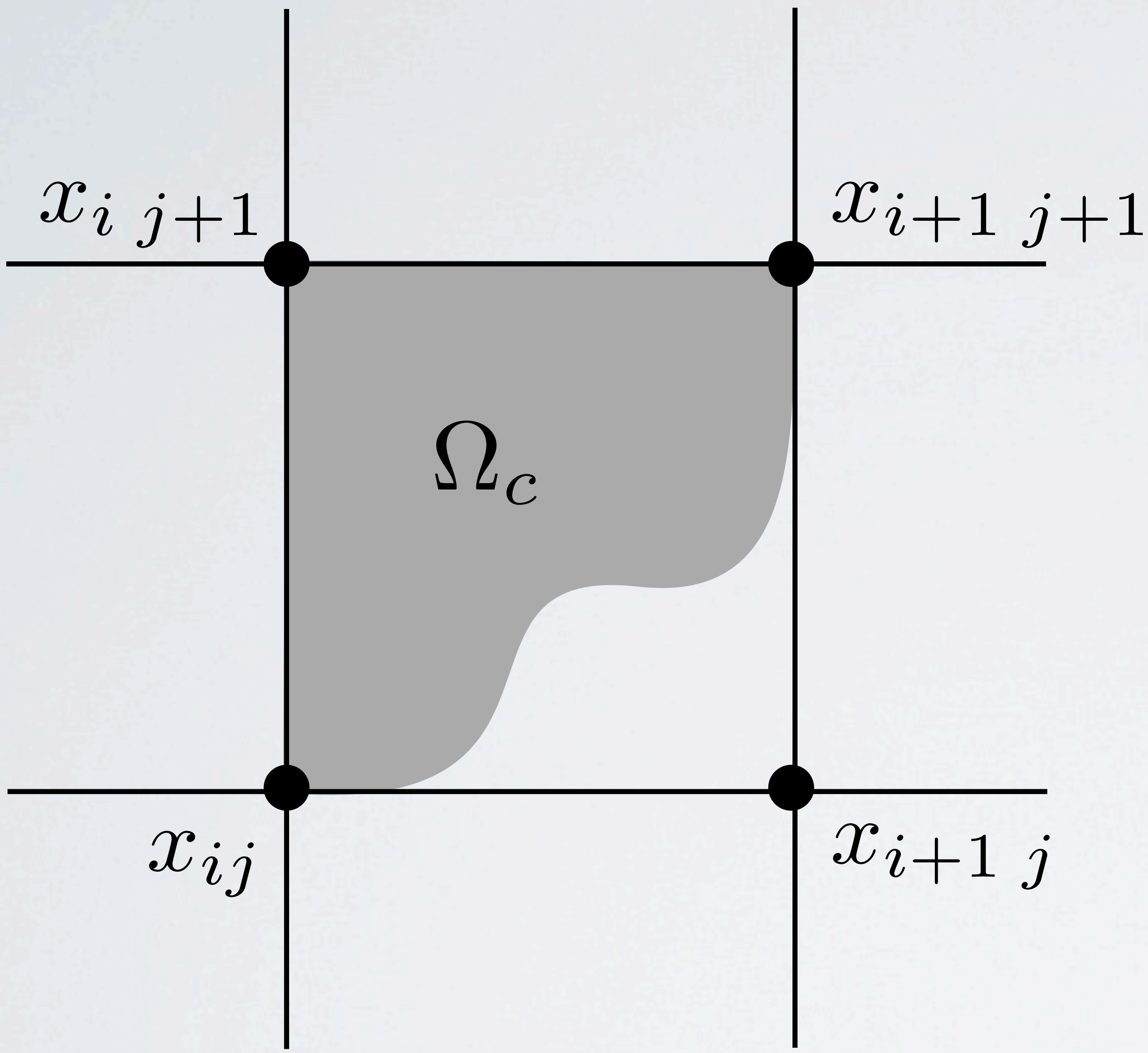


$$E_{obj} = \sum E_{cell}$$

Whole cells are straight-forward.

$$E_{cell} = \int_{[0,h]^3} \Psi(F)\partial X$$

BOUNDARY CELL TREATMENT



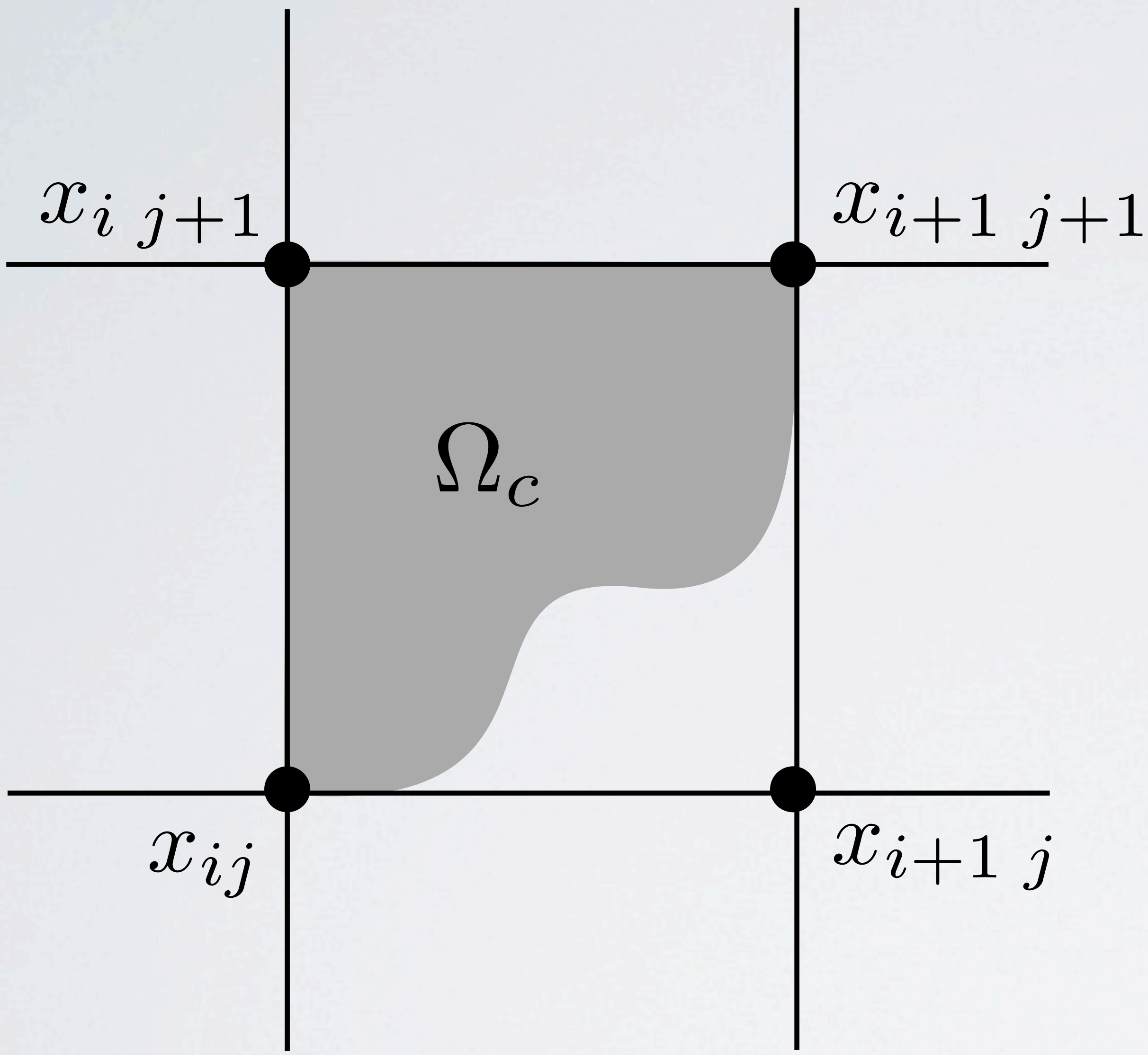
$$E_{obj} = \sum E_{cell}$$

Fractional cells are tricky.

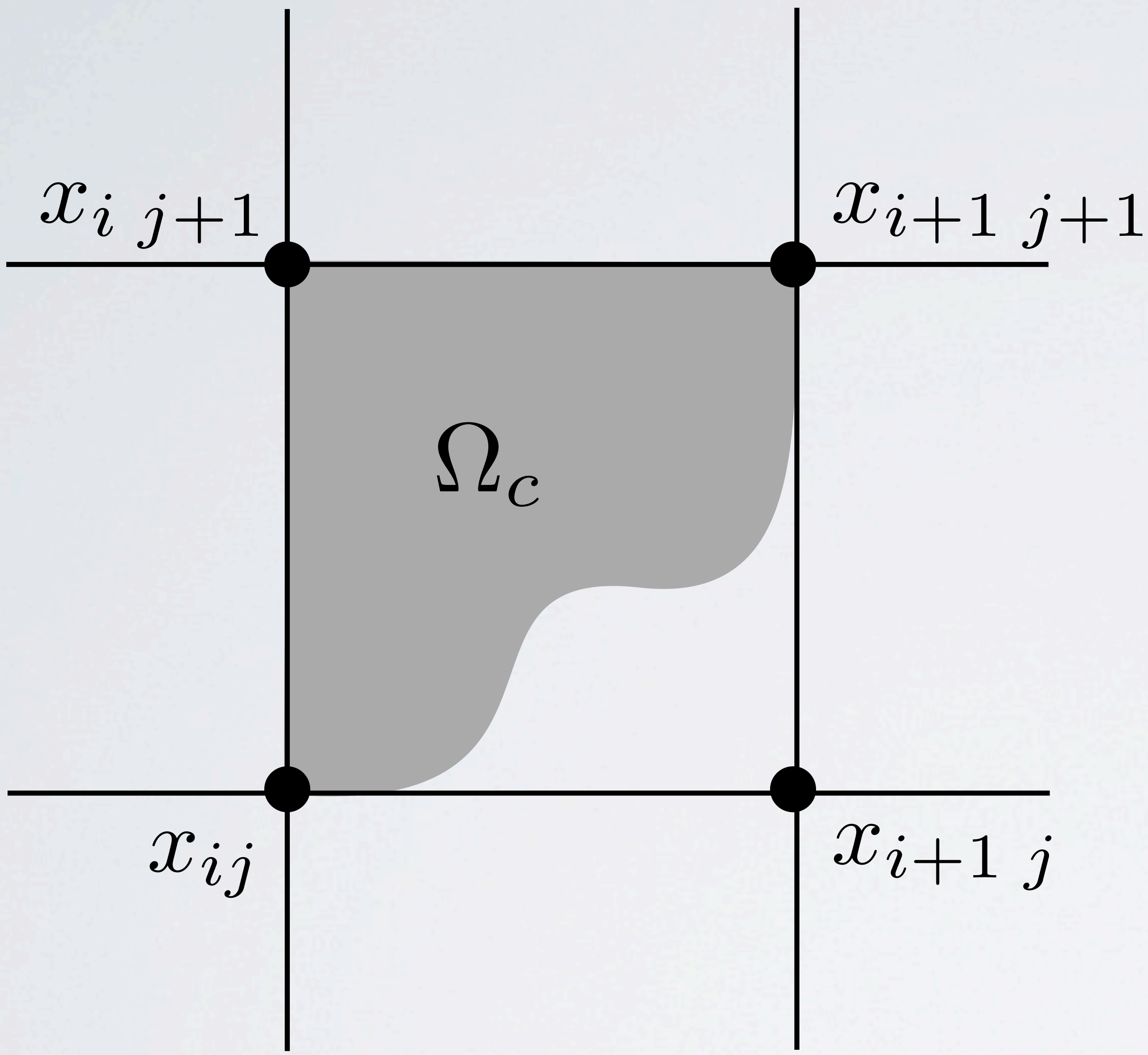
$$E_{cell} = \int_{\Omega_c} \Psi(F)\partial X$$

$$= ???$$

BOUNDARY CELL TREATMENT

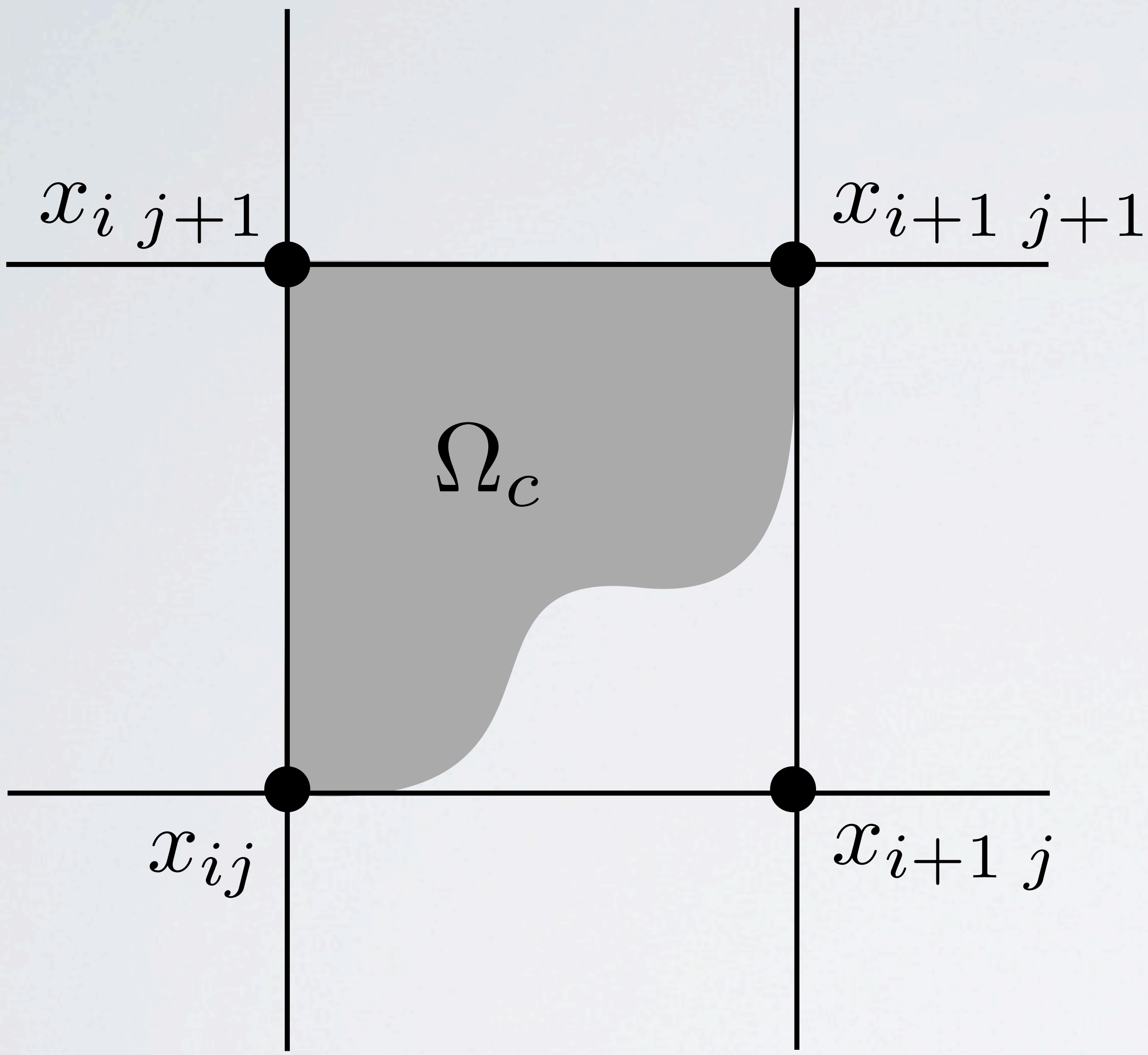


BOUNDARY CELL TREATMENT



$\Longrightarrow$ Compute analytically

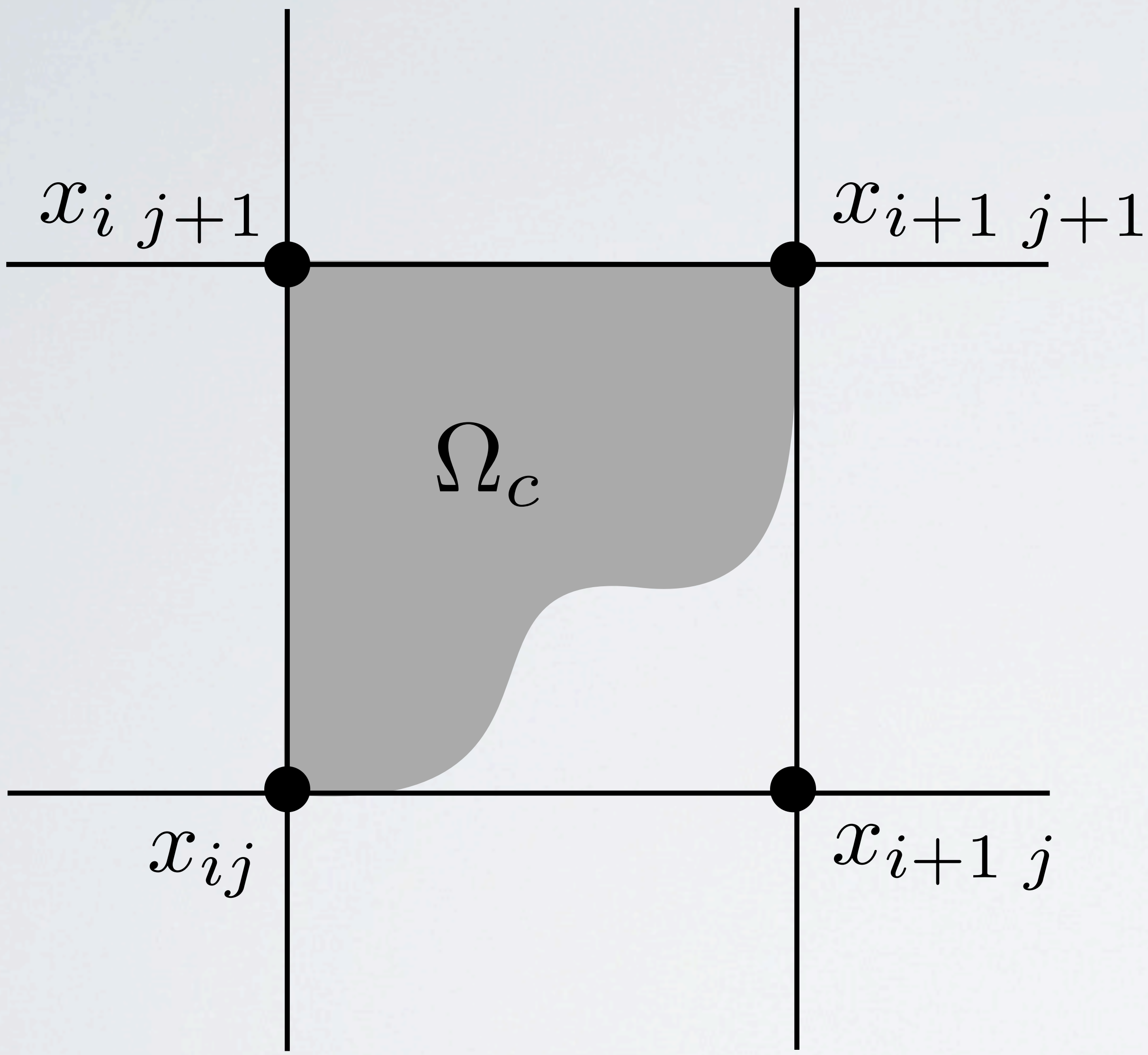
BOUNDARY CELL TREATMENT



$\Longrightarrow$ Compute analytically

Very simple $\Psi's$ only

BOUNDARY CELL TREATMENT

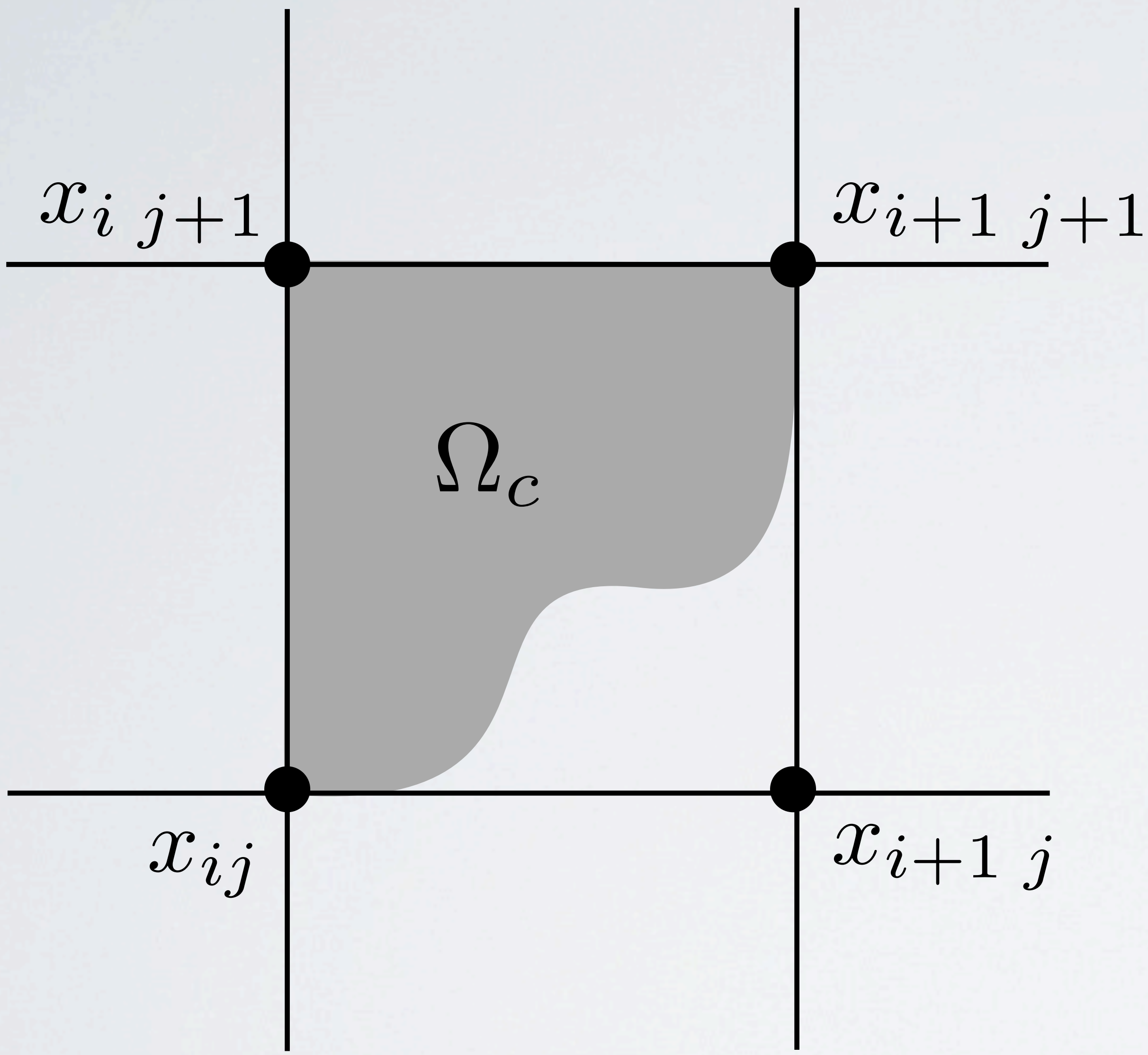


$\Longrightarrow$ Compute analytically

Very simple $\Psi's$ only

$\Longrightarrow$ Monte-Carlo

BOUNDARY CELL TREATMENT



$\Longrightarrow$ Compute analytically

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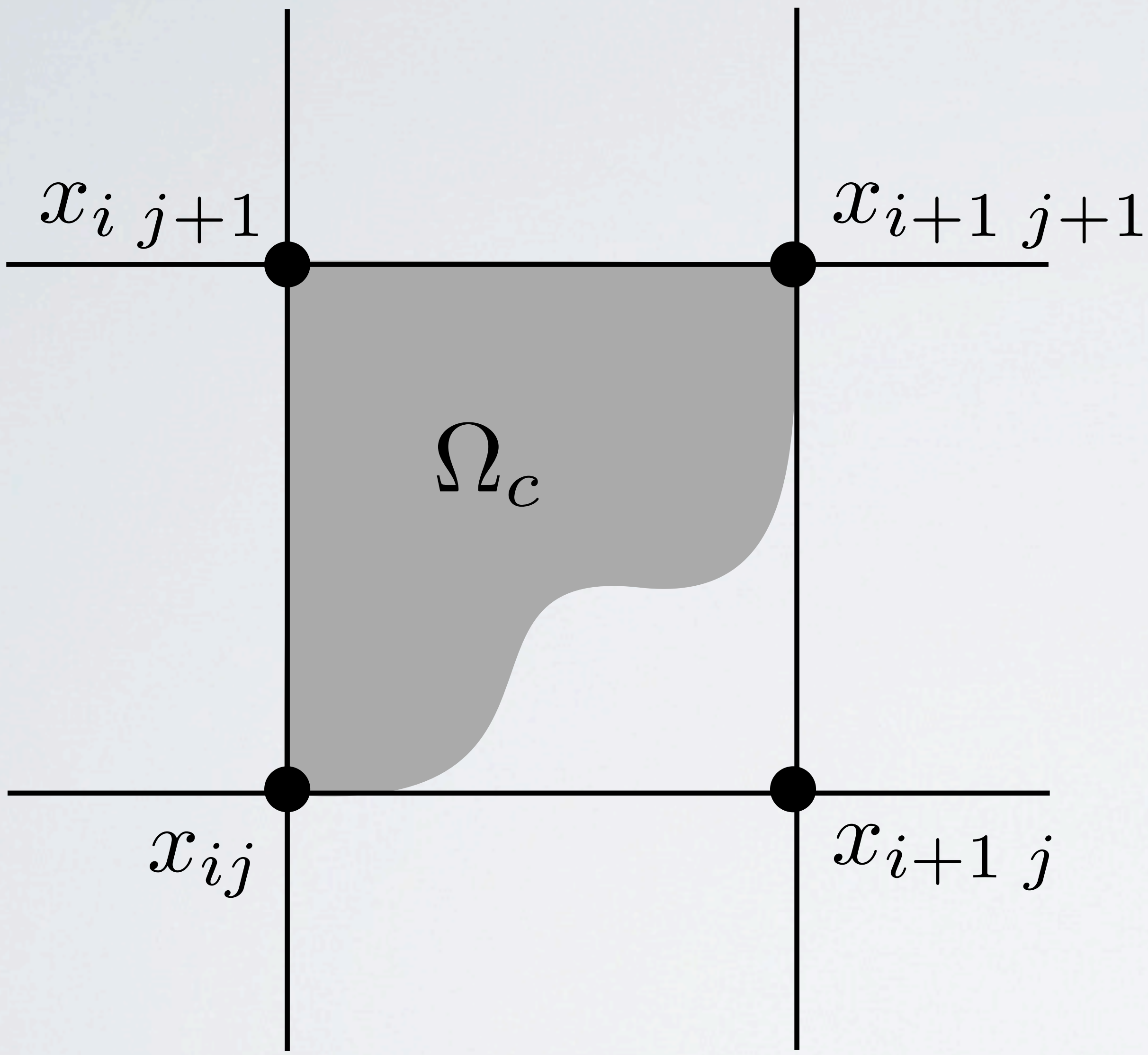
$\Longrightarrow$ Monte-Carlo

Expensive

BOUNDARY CELL TREATMENT



$\Longrightarrow$ Compute analytically

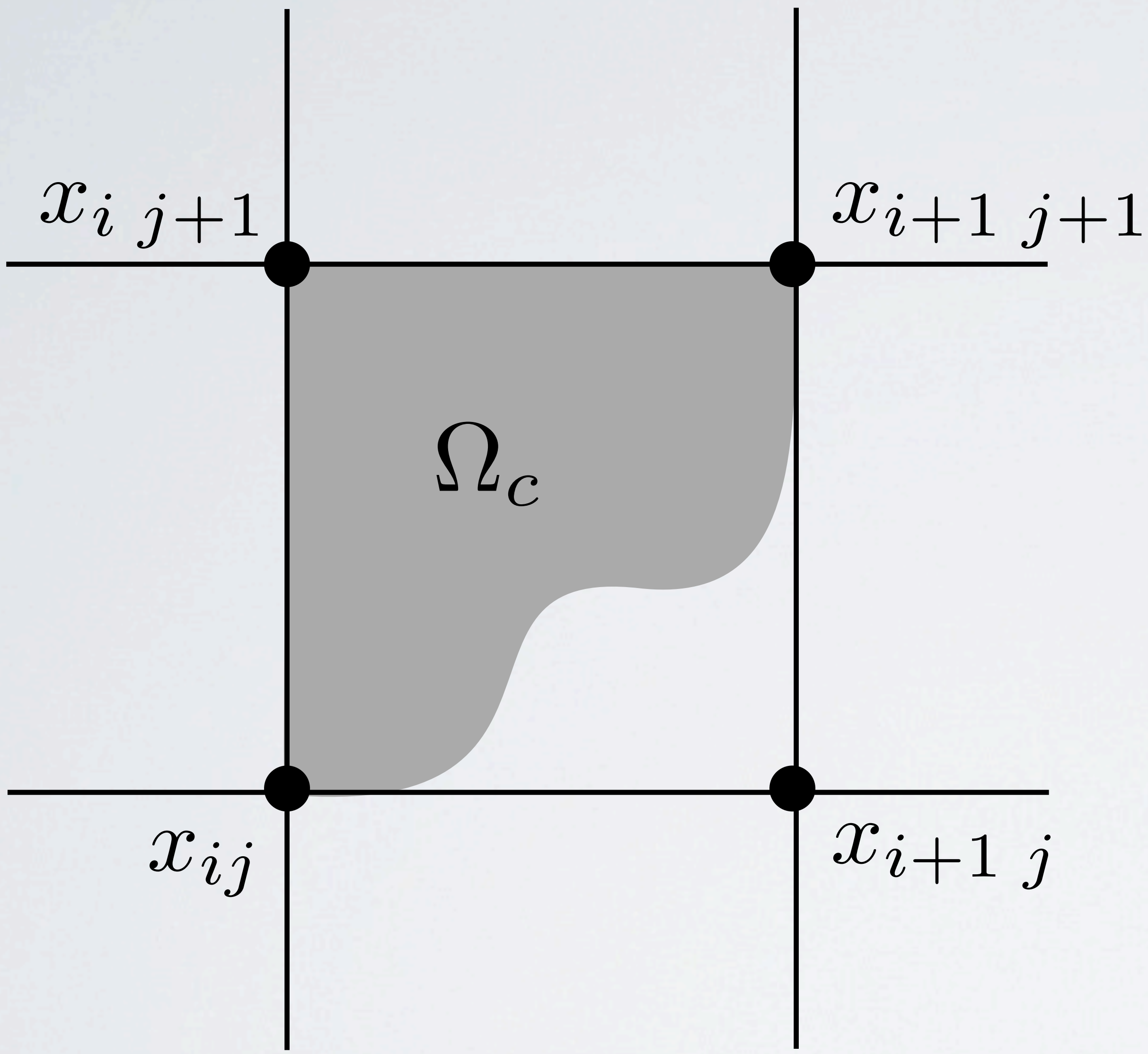
Very simple $\Psi's$ only

$\Longrightarrow$ Monte-Carlo

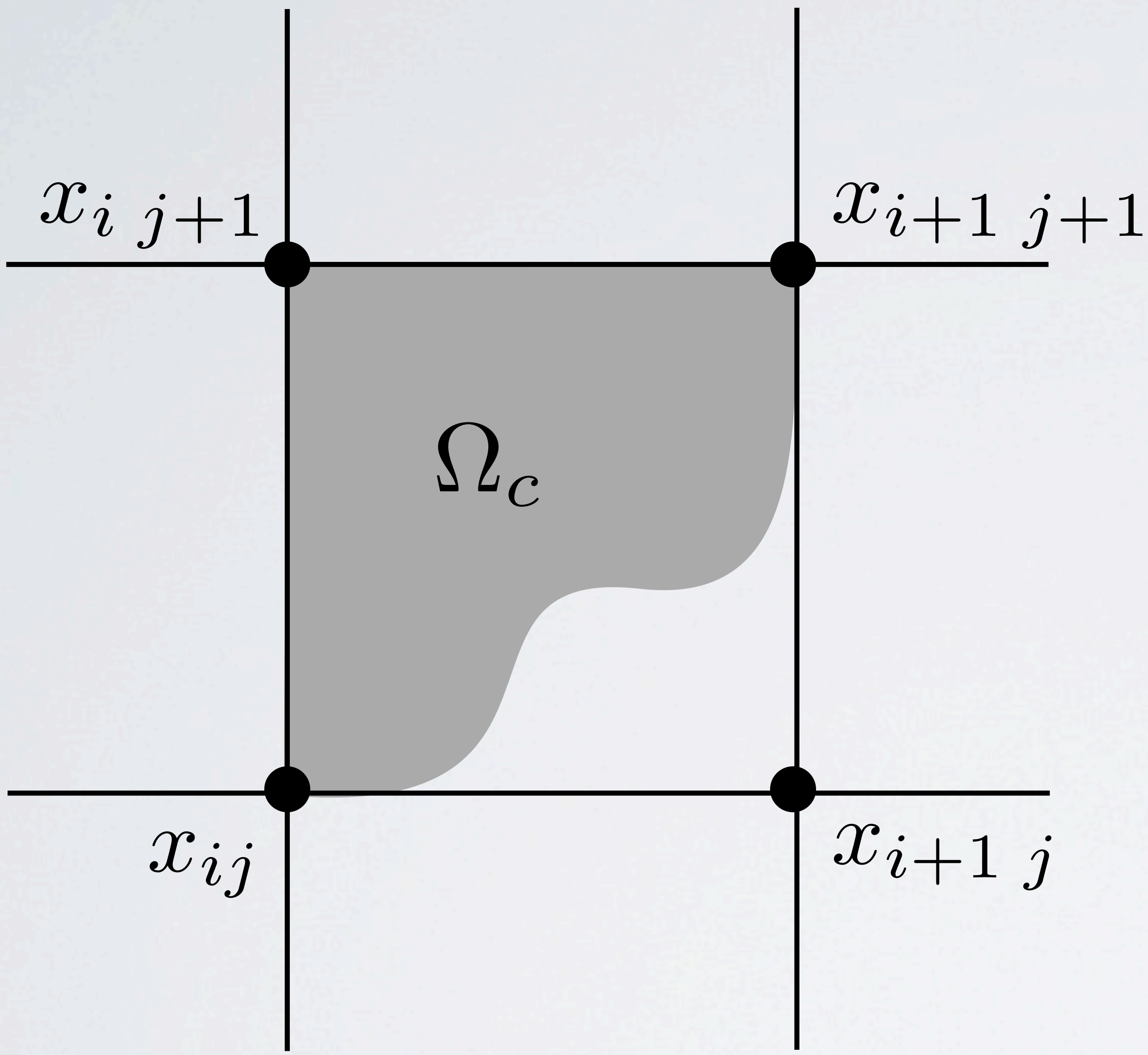
Expensive

$\Longrightarrow$ Our solution:
Efficient quadrature scheme

BOUNDARY CELL TREATMENT

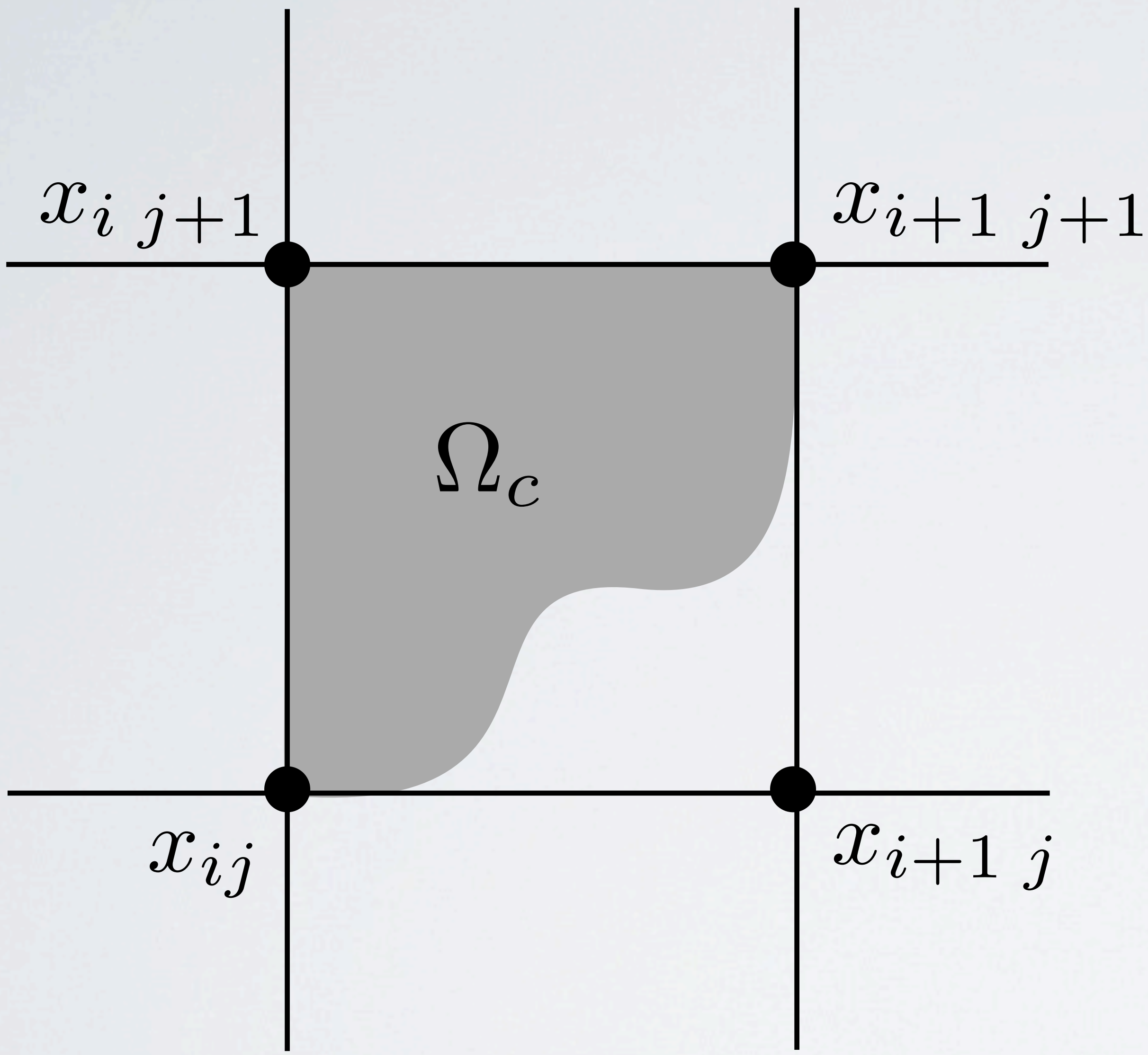


BOUNDARY CELL TREATMENT



$$E_{cell} = \int_{\Omega_c} \Psi(F)\partial X$$

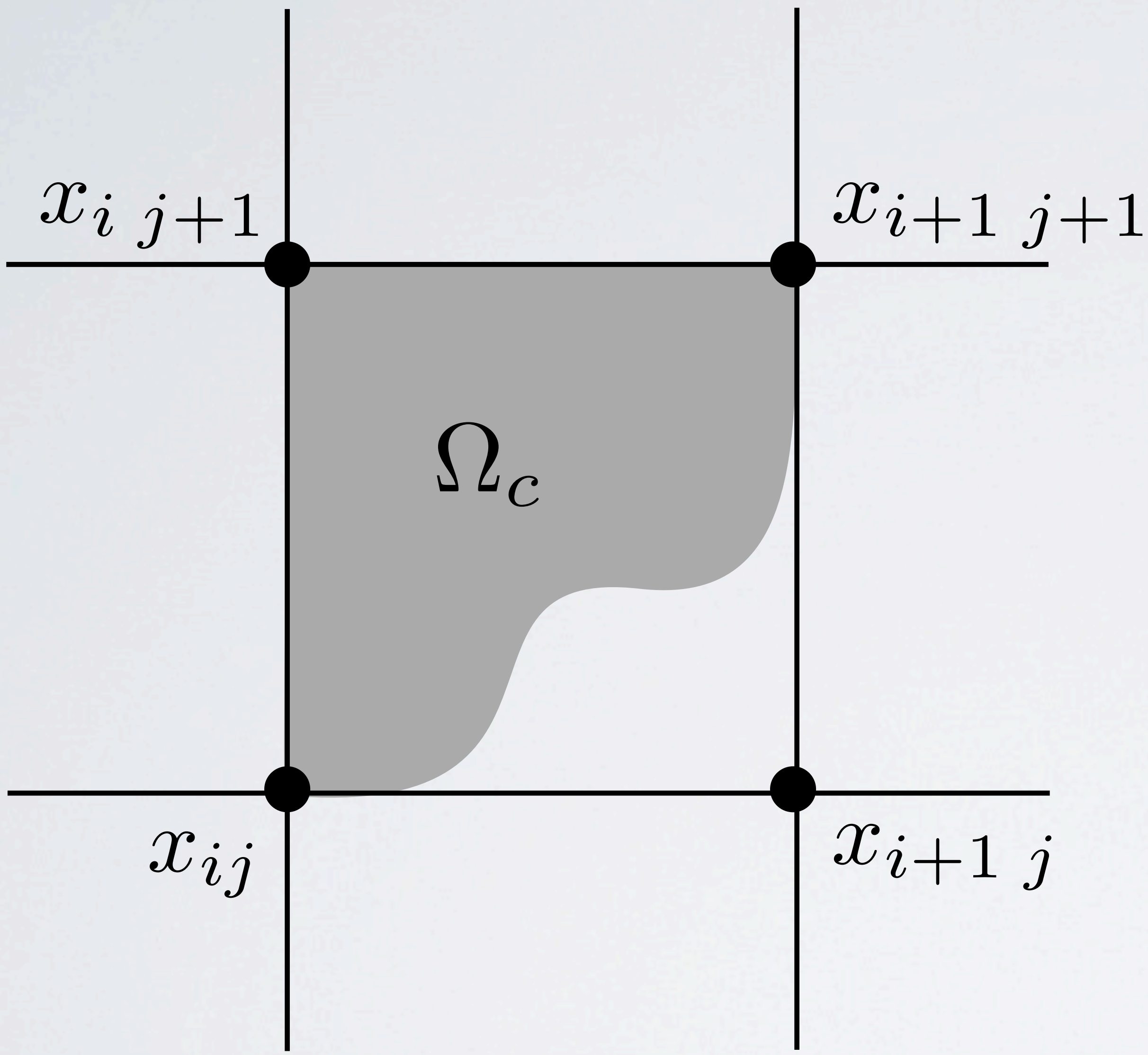
BOUNDARY CELL TREATMENT



$$E_{cell} = \int_{\Omega_c} \Psi(F)\partial X$$

$$\approx \sum_{i=1}^{k} c_i \Psi(X_i)$$

BOUNDARY CELL TREATMENT



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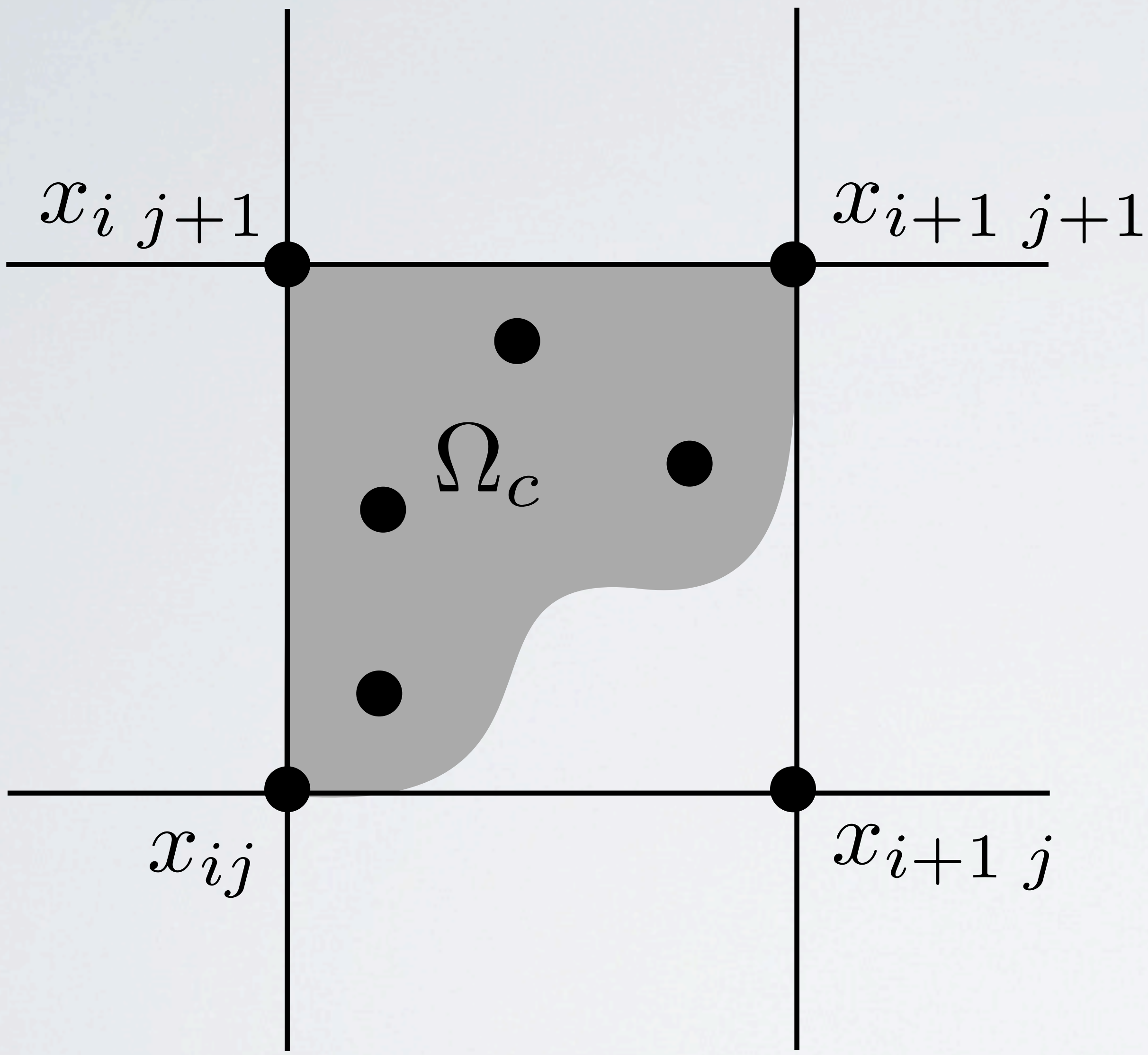
$$\sum c_i = \text{vol}\ (\Omega_c)$$

BOUNDARY CELL TREATMENT



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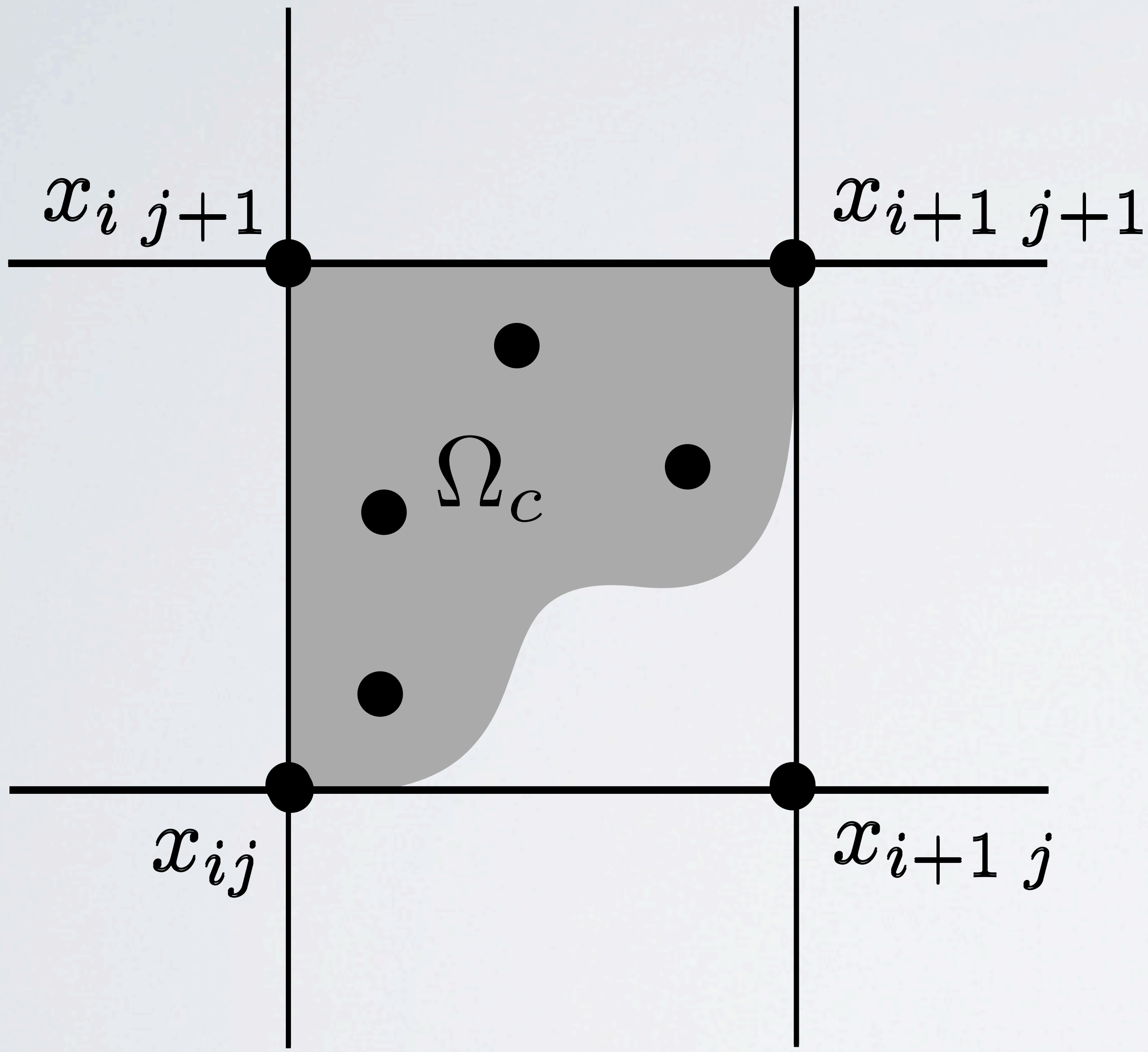
$$\approx \sum_{i=1}^{k} c_i \Psi(X_i)$$

$$\sum c_i = \mathrm{vol}\ (\Omega_c)$$

Note: k = 4 suffices for second order accuracy!

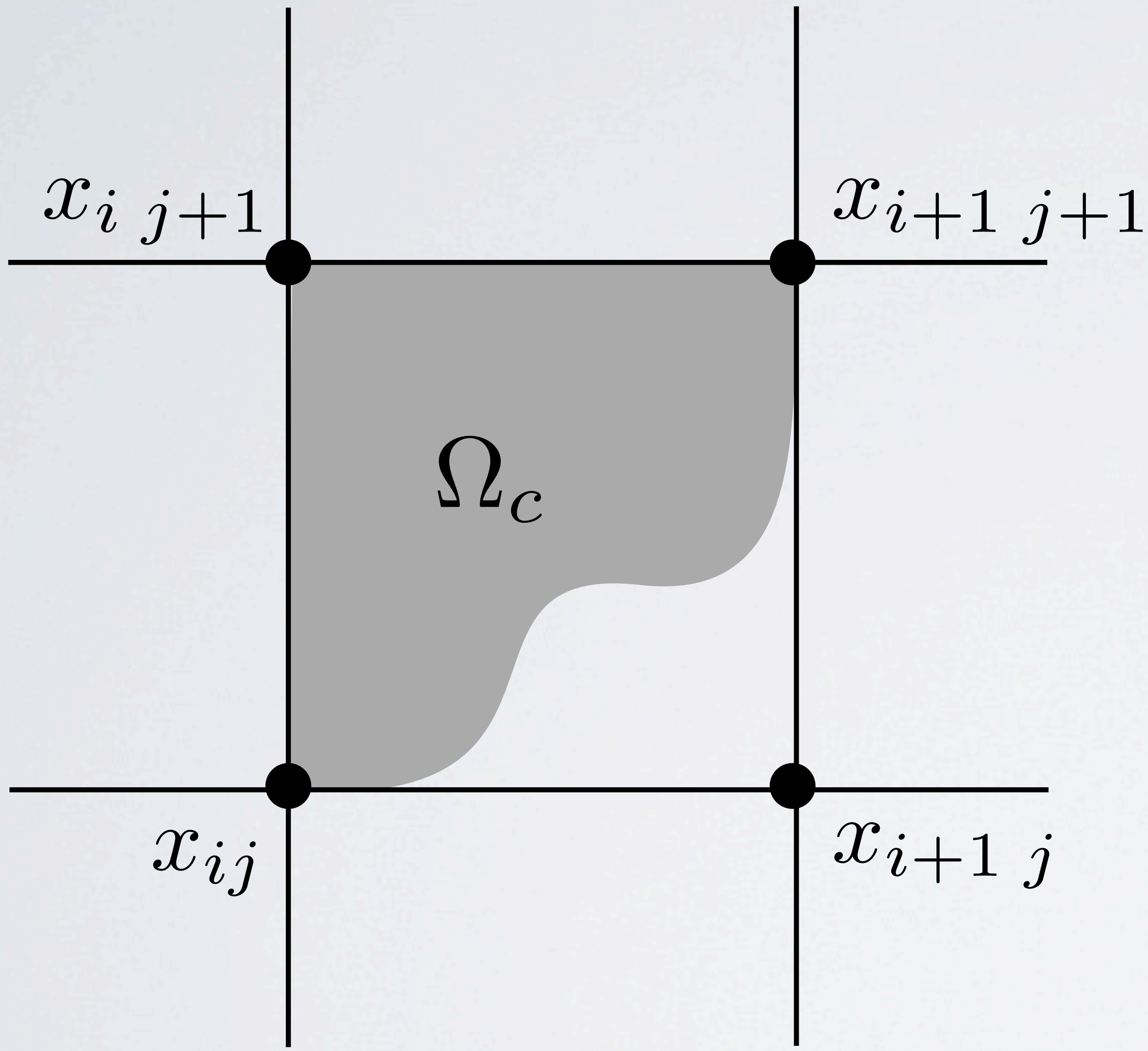
BOUNDARY CELL TREATMENT

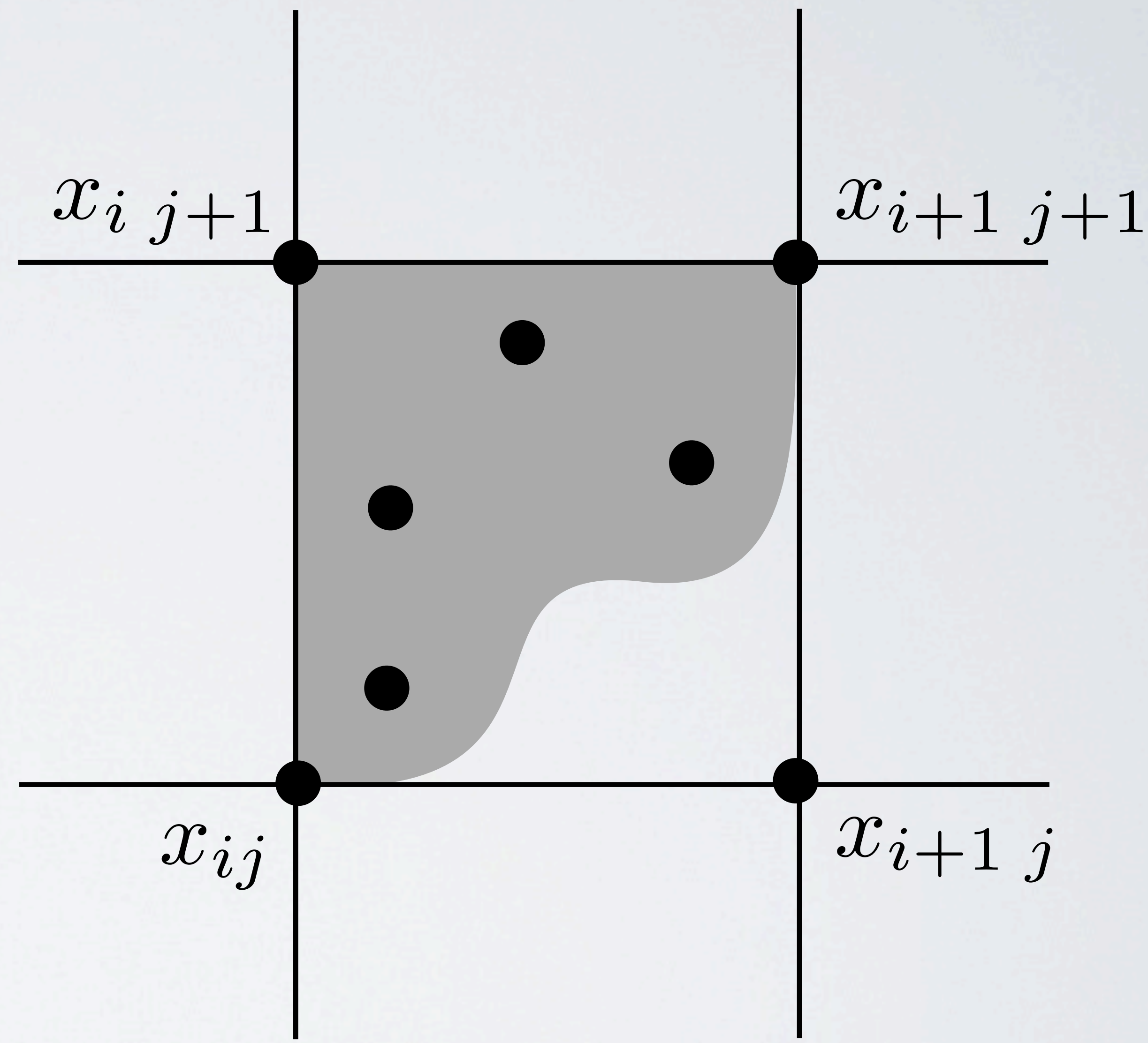
X_i, c_i



BOUNDARY CELL TREATMENT

Choose X_i, c_i such that





BOUNDARY CELL TREATMENT

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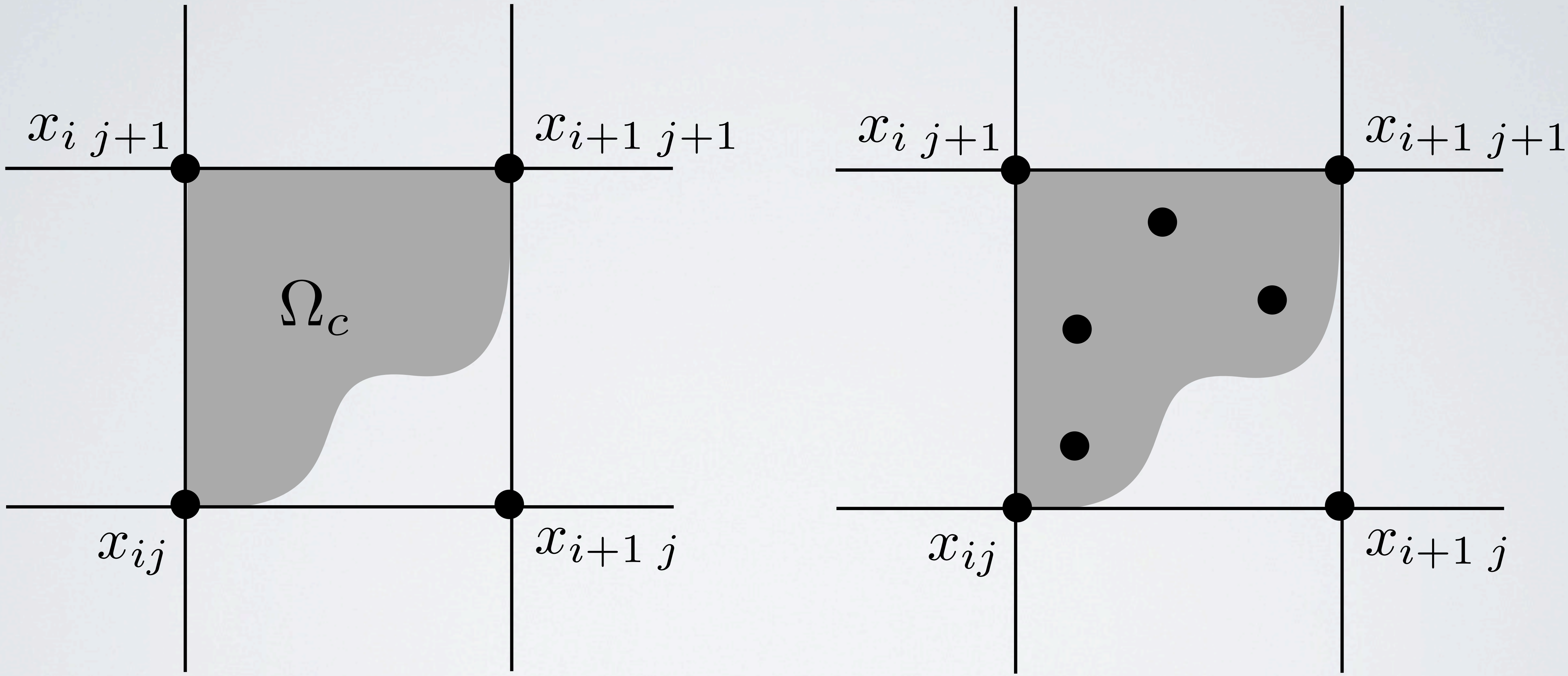
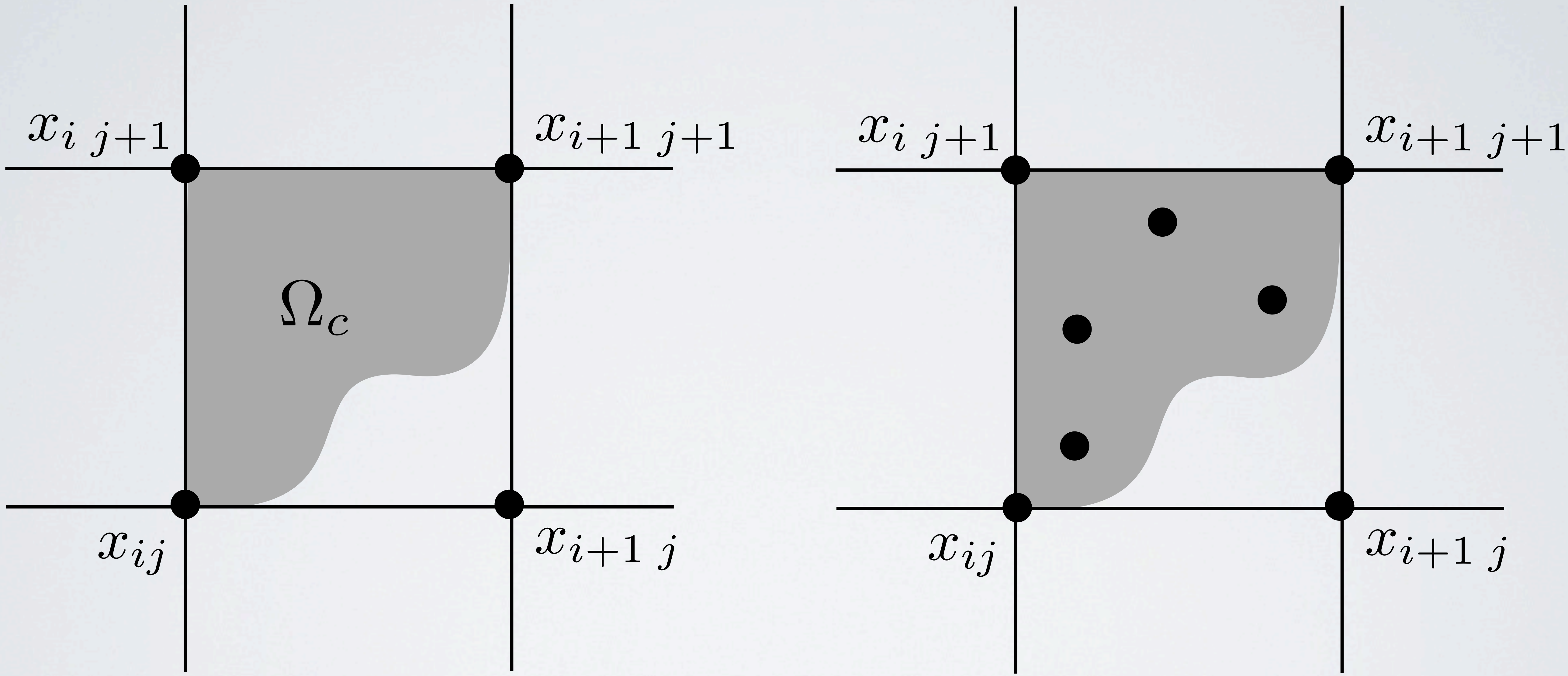


Continuous Distribution Average = Discrete Distribution Average

BOUNDARY CELL TREATMENT

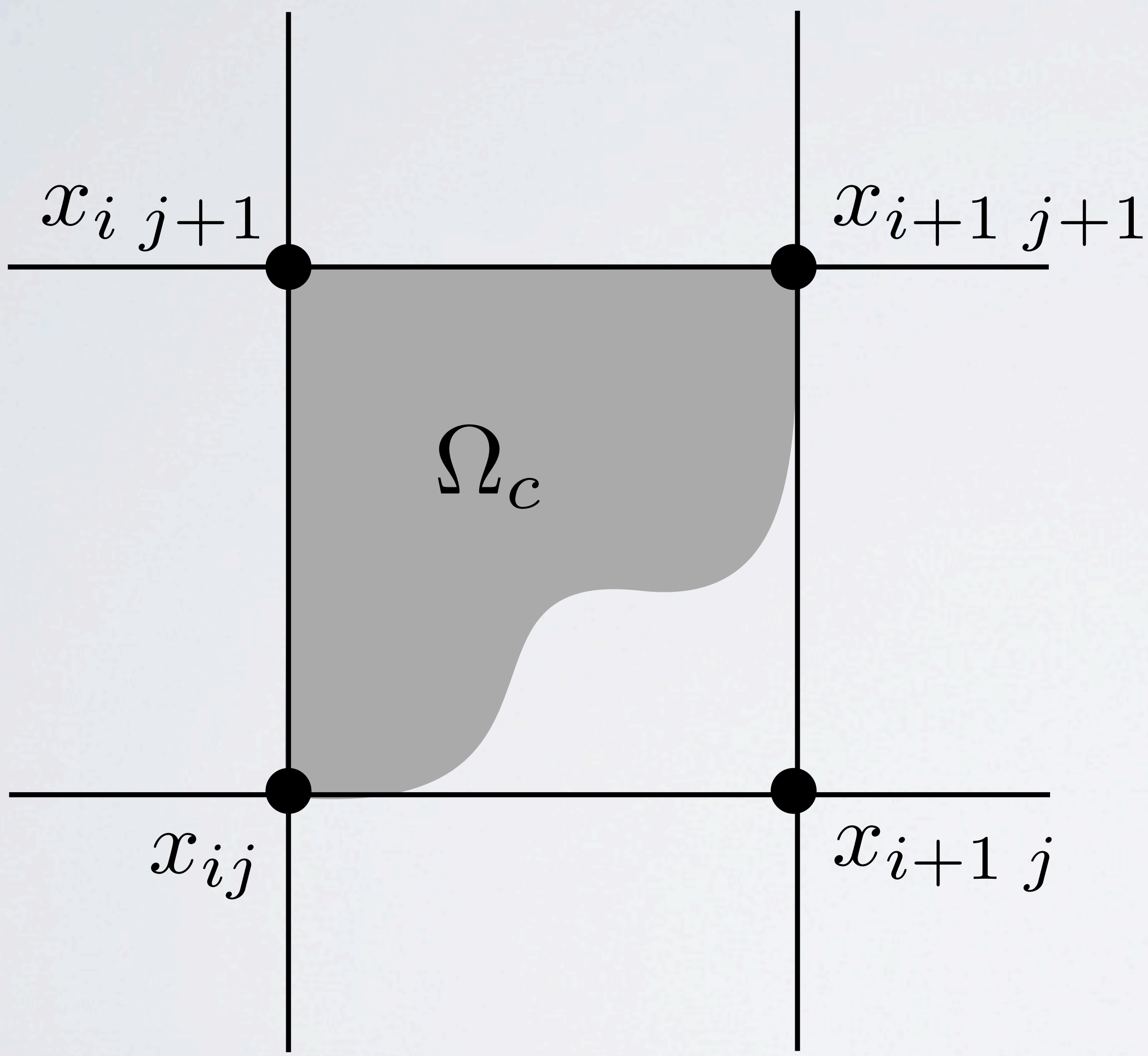
Choose X_i, c_i such that



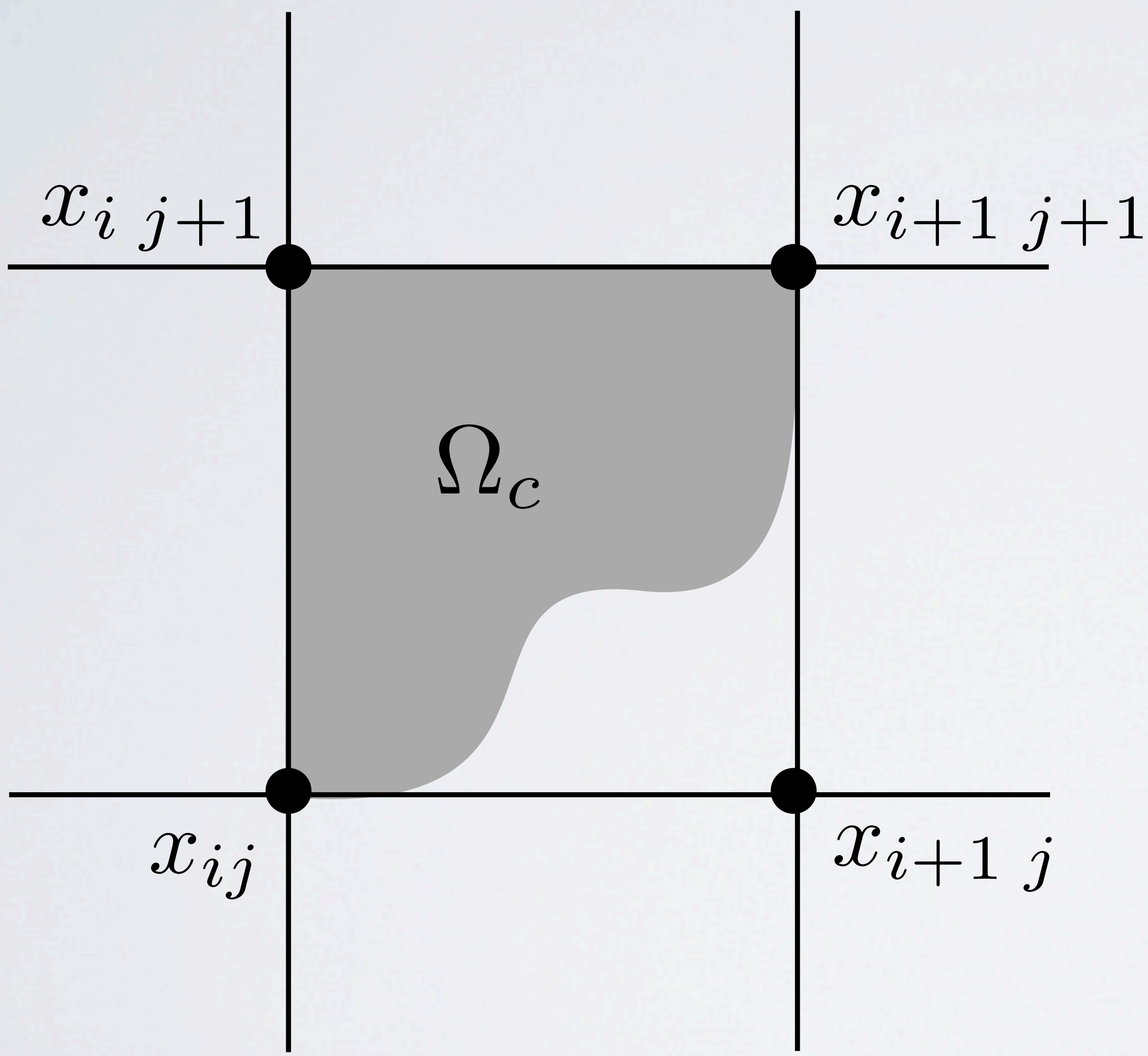
Continuous Distribution Average $=$ Discrete Distribution Average

Continuous Distribution Variance $=$ Discrete Distribution Variance

BOUNDARY CELL TREATMENT

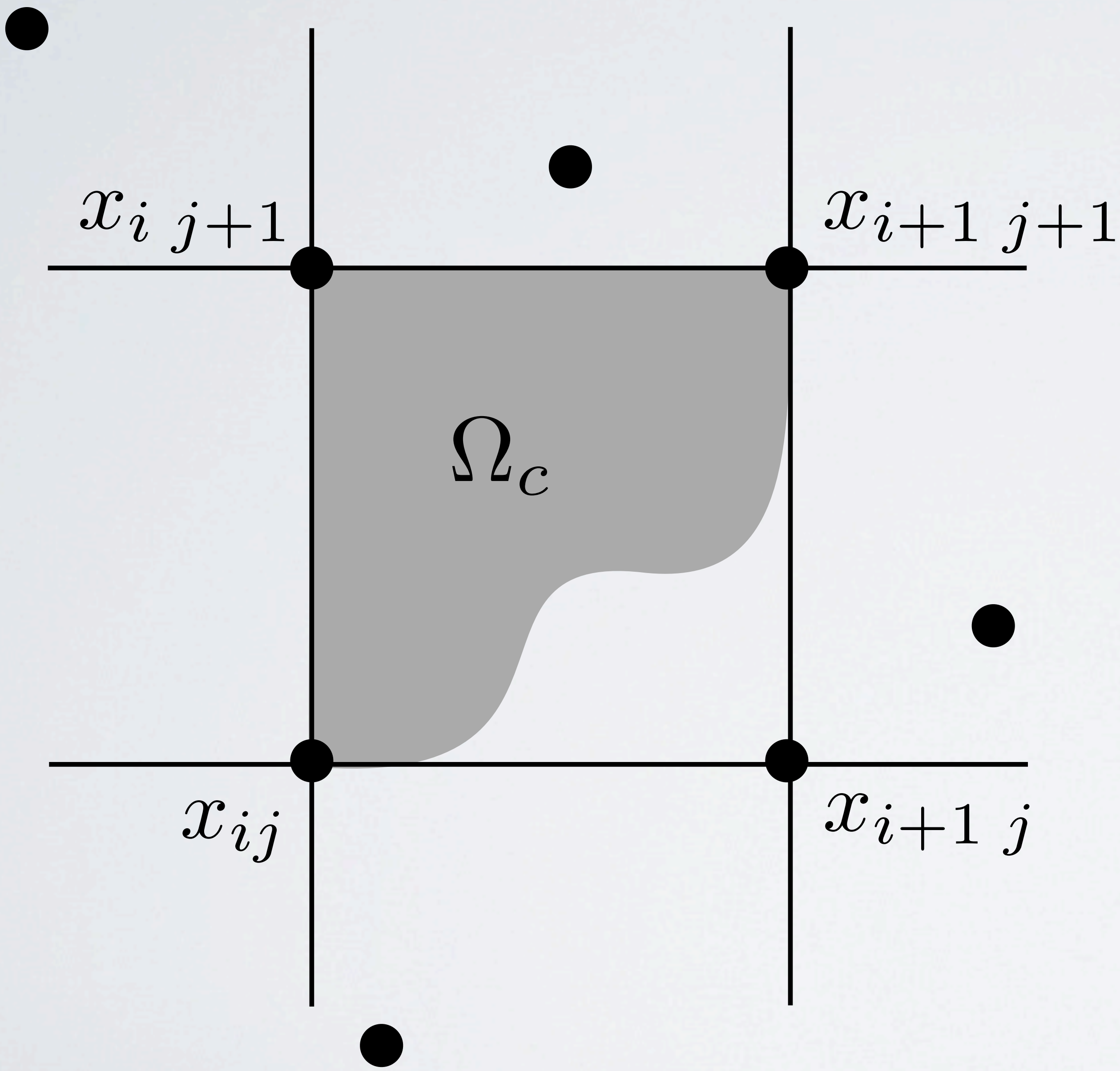


BOUNDARY CELL TREATMENT



Monte Carlo to determine Ω_c

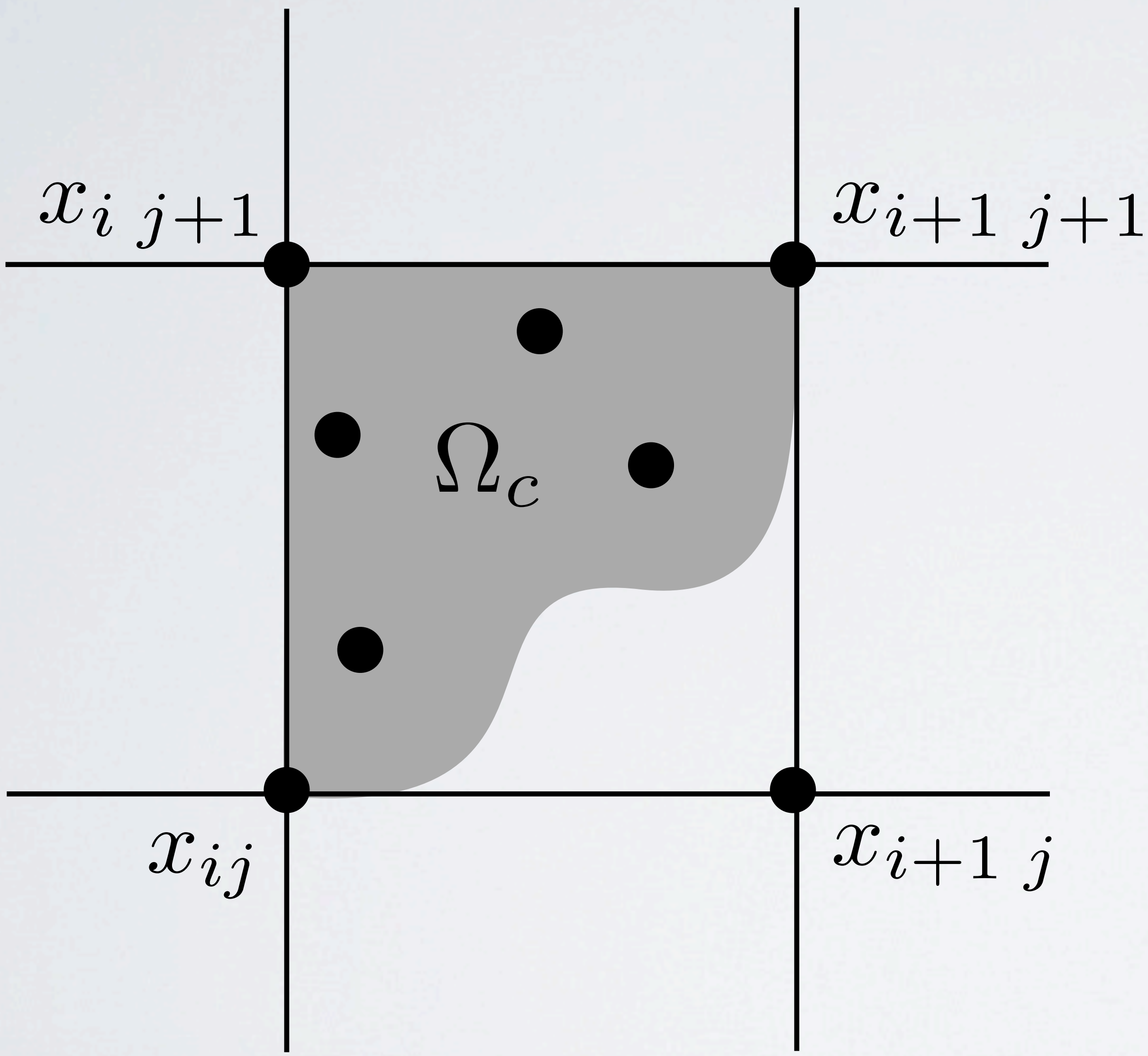
BOUNDARY CELL TREATMENT



Monte Carlo to determine Ω_c

Compute 4 representative points

BOUNDARY CELL TREATMENT

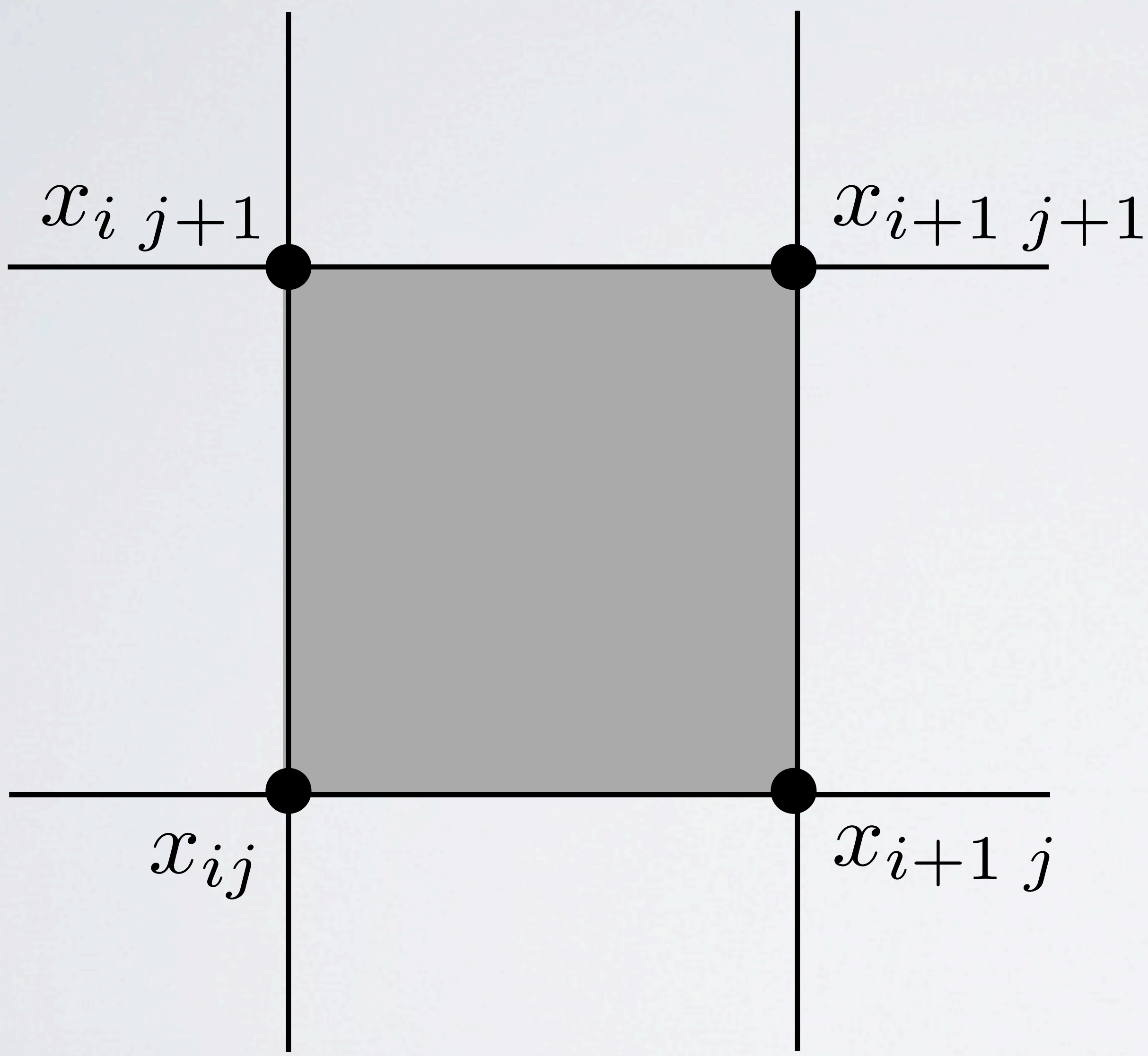


Monte Carlo to determine Ω_c

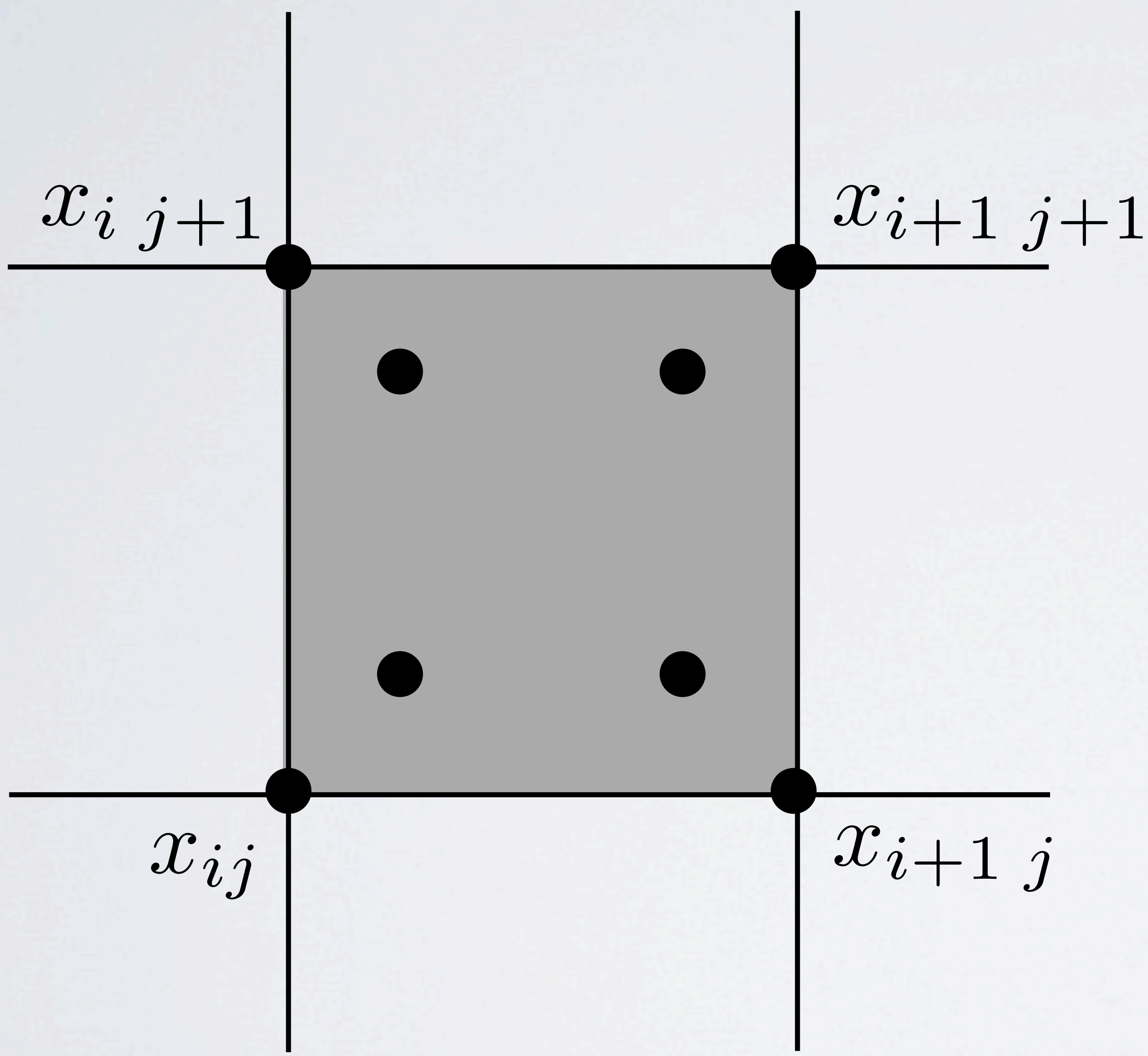
Compute 4 representative points

Ensure interior to the material

BOUNDARY CELL TREATMENT



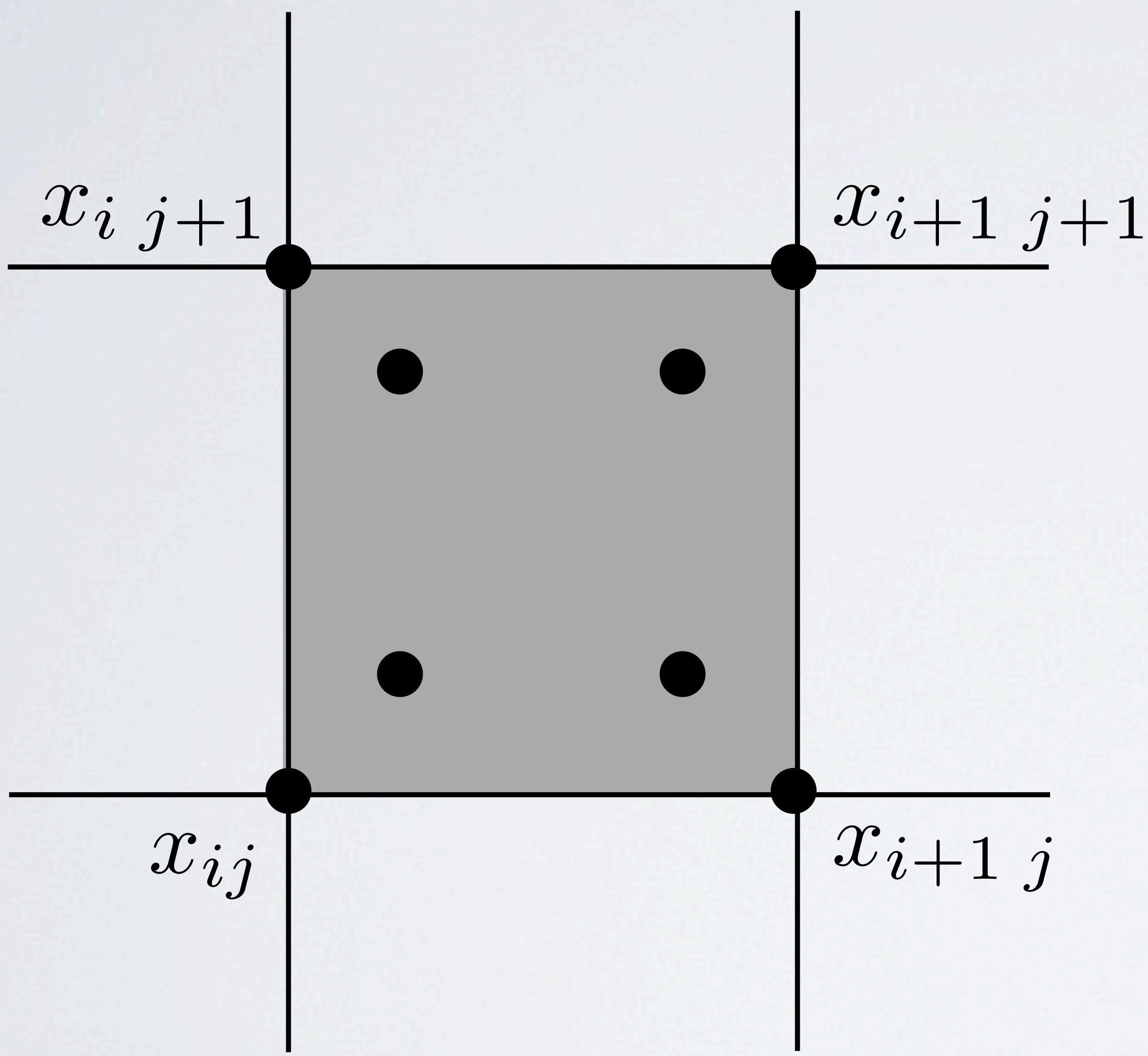
BOUNDARY CELL TREATMENT



Identical positions

Identical weights

BOUNDARY CELL TREATMENT

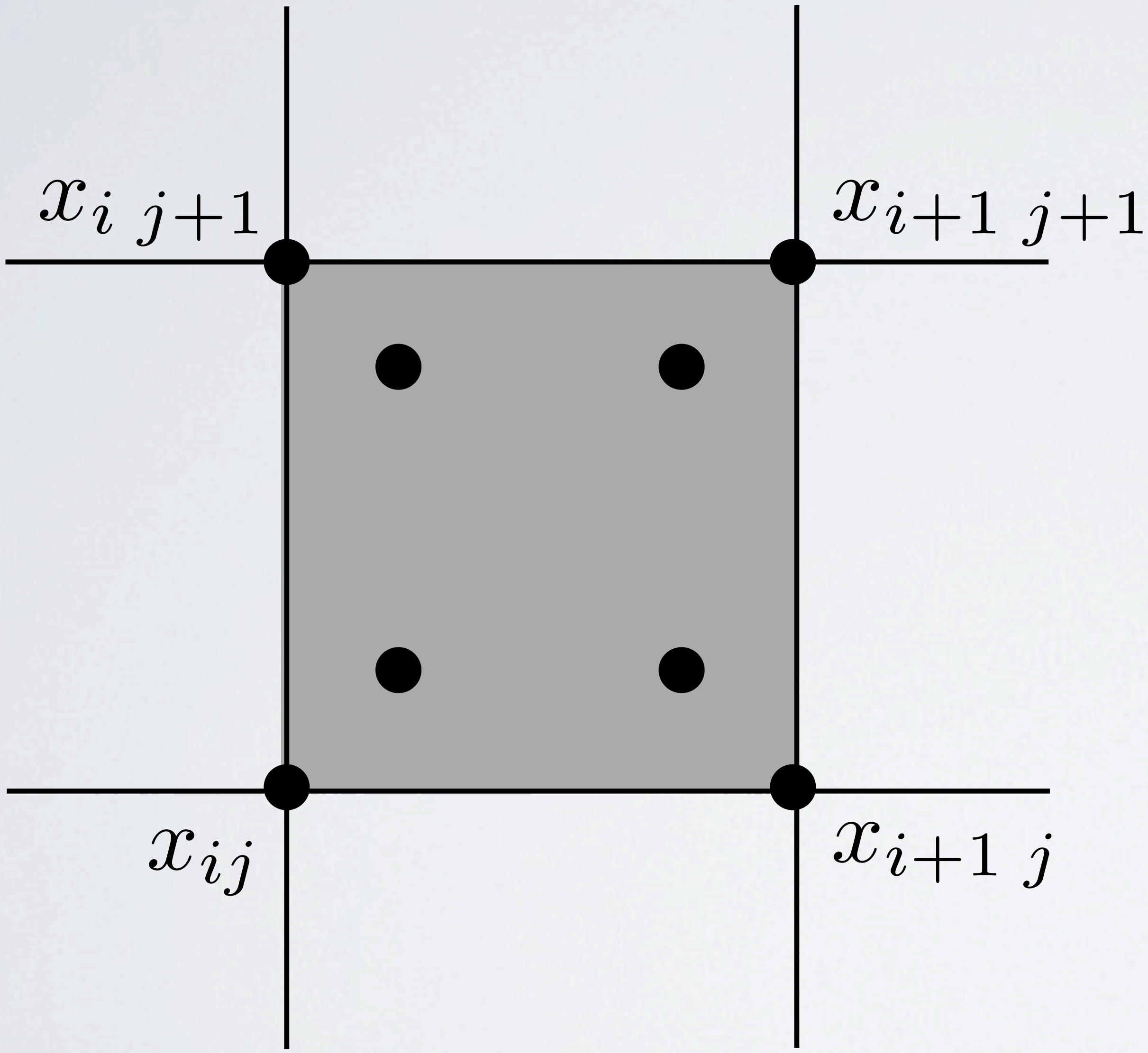


Identical positions

Identical weights

Preprocessing procedure done once for all cells

BOUNDARY CELL TREATMENT



Identical positions

Identical weights

Preprocessing procedure done once for all cells

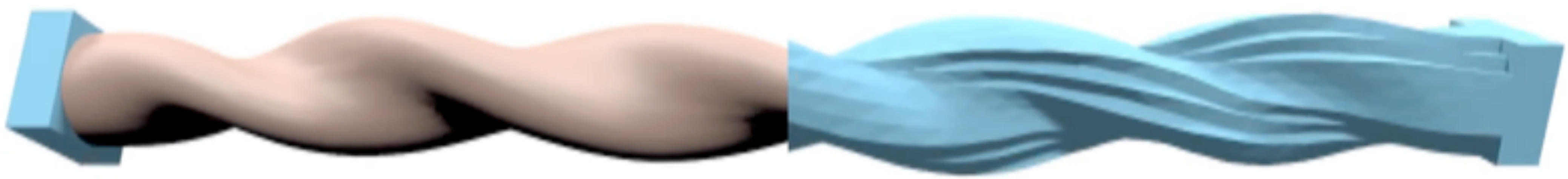
Same data for all cells

Reference Simulation (16x16)

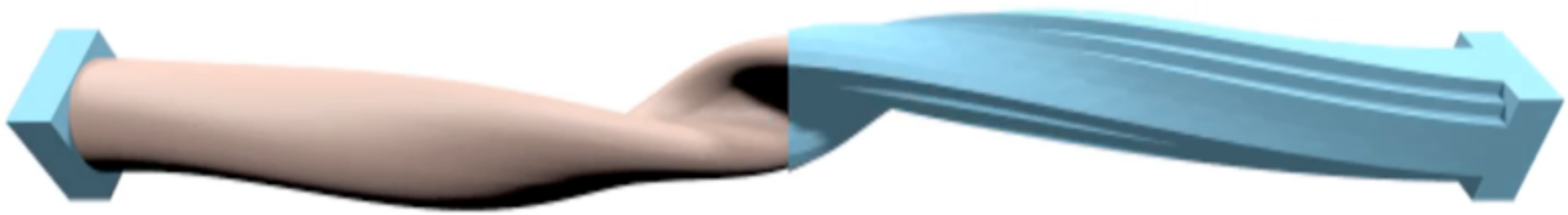
Boundary Cells Treated with Our Method (8x8)

Boundary Cells as Full Cells (8x8)

Reference Simulation (16x16)

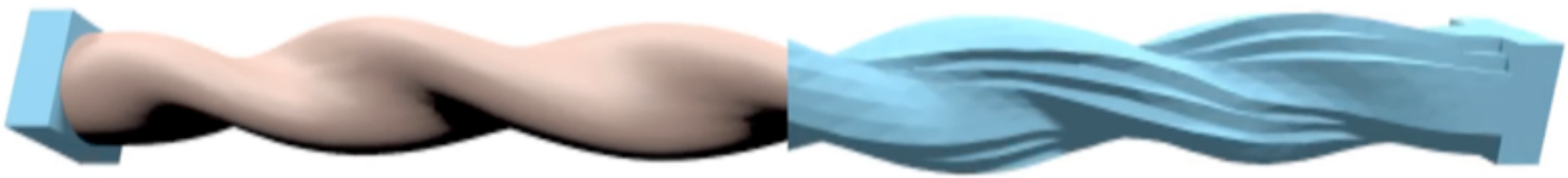


Boundary Cells Treated with Our Method (8x8)



Boundary Cells as Full Cells (8x8)

Reference Simulation (16x16)



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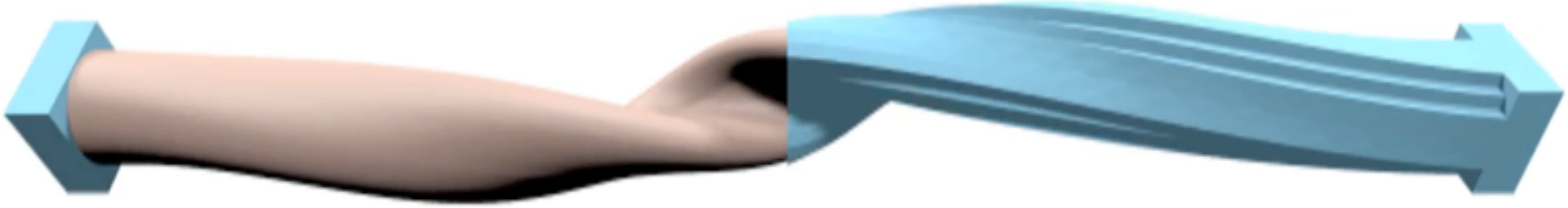

Boundary Cells as Full Cells (8x8)

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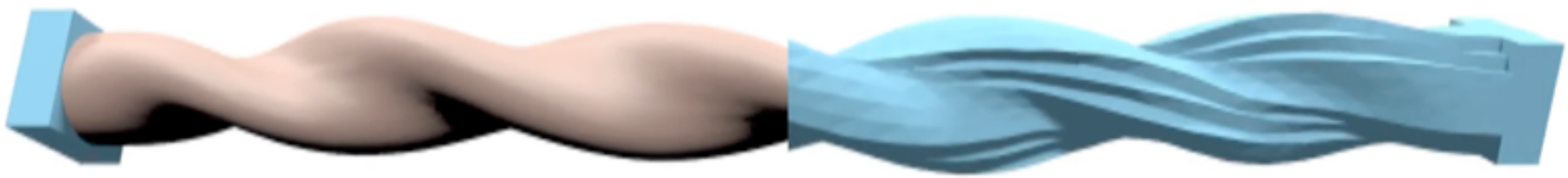

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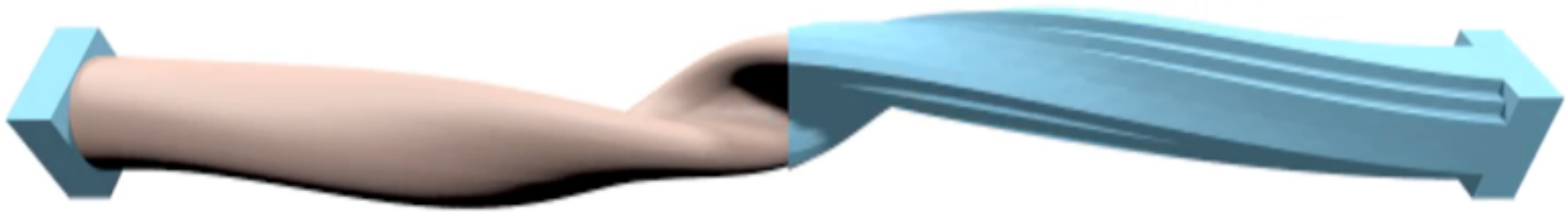
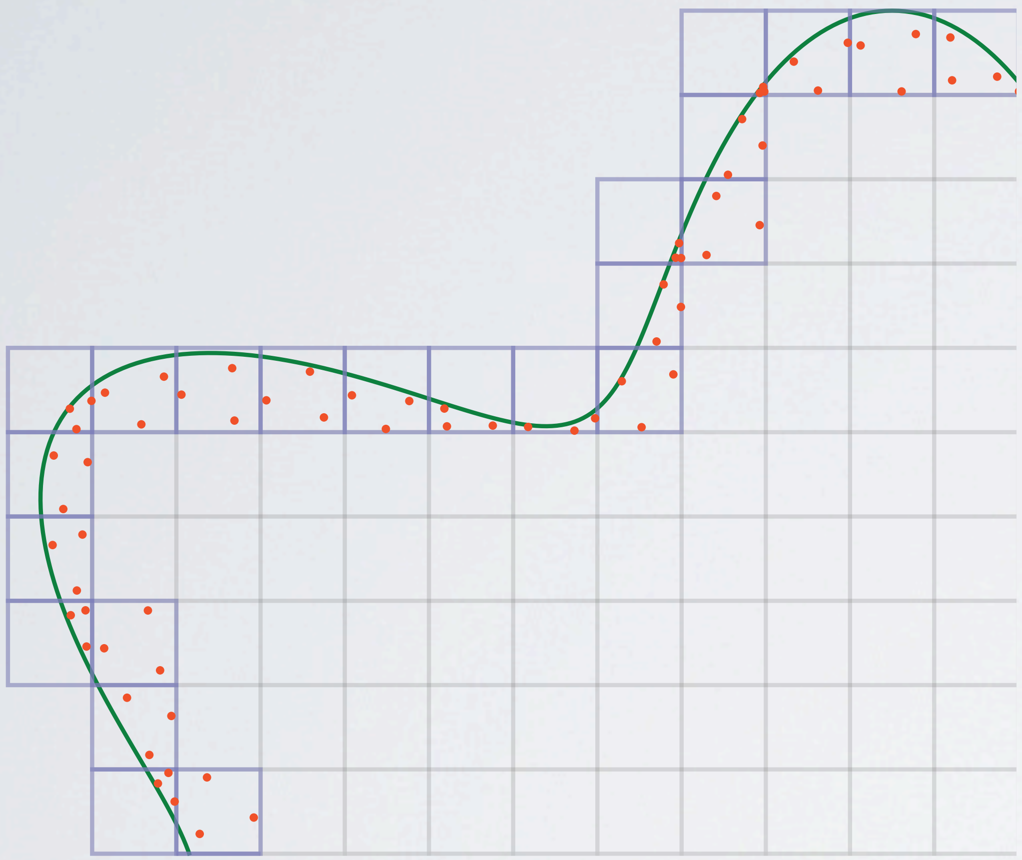
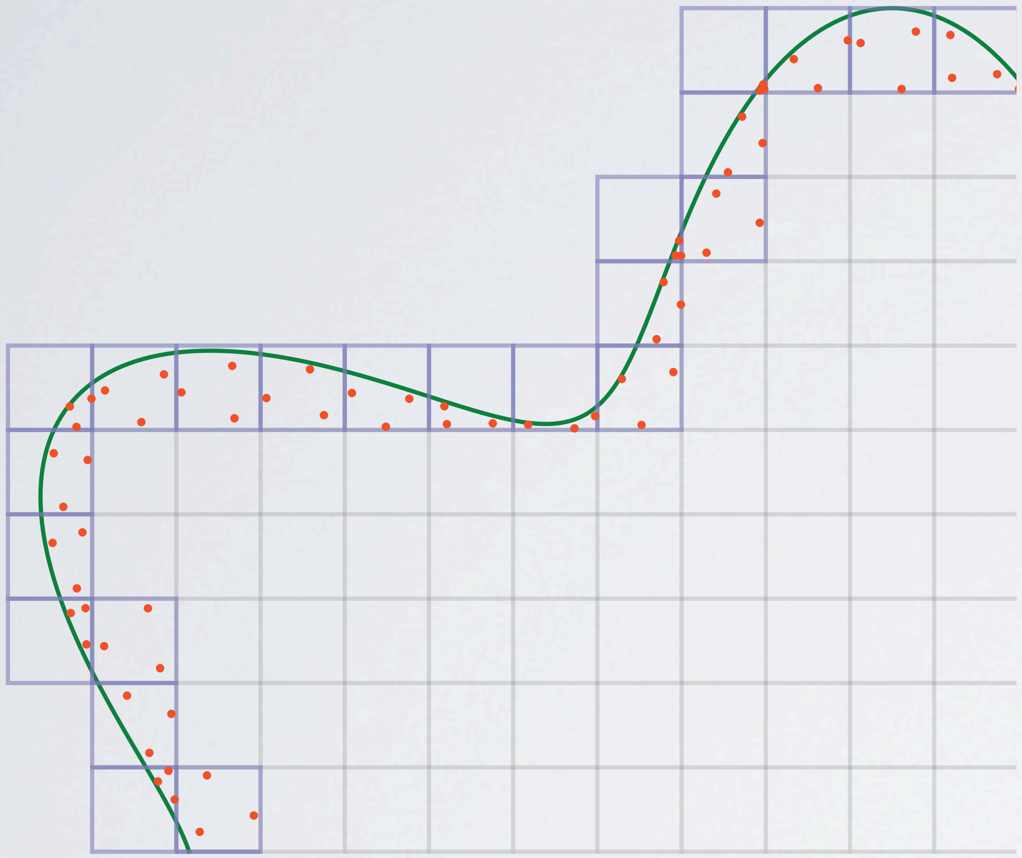


Boundary Cells as Full Cells (8x8)

Boundary Cell Treatment: Recap

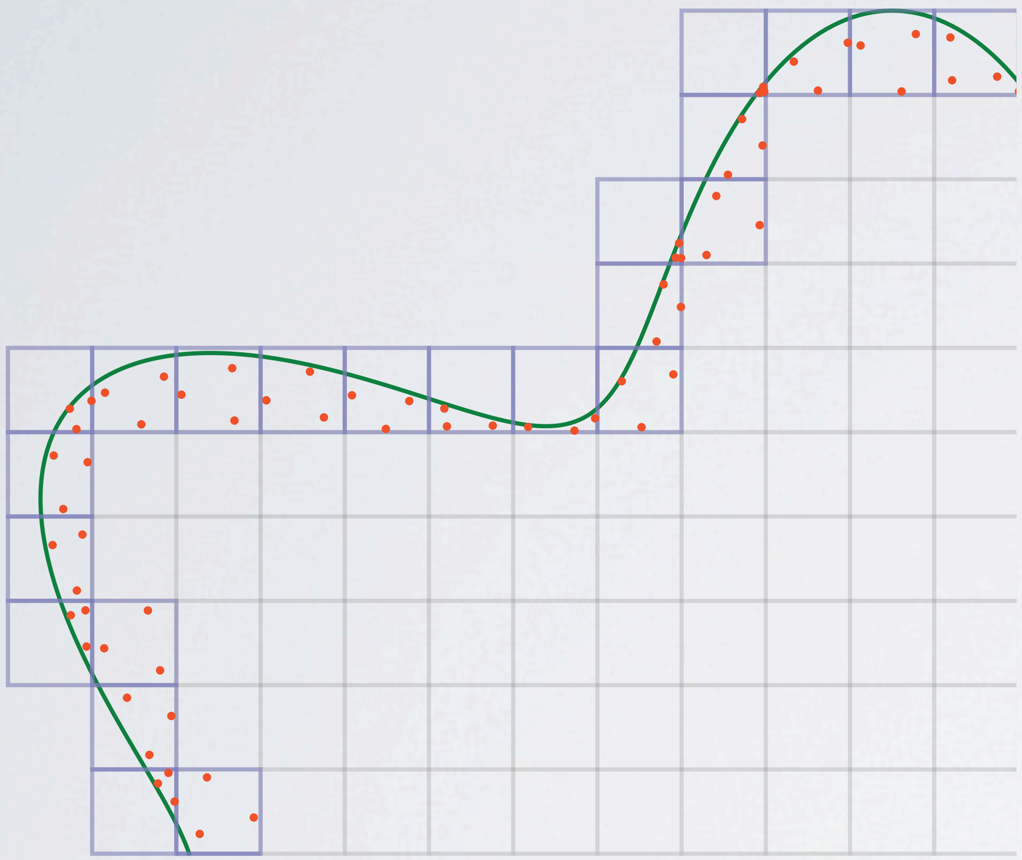


Boundary Cell Treatment: Recap



✓ Produces 2nd order accuracy

Boundary Cell Treatment: Recap



✓ Produces 2nd order accuracy

✓ Fixed work per cell (4 pts max)

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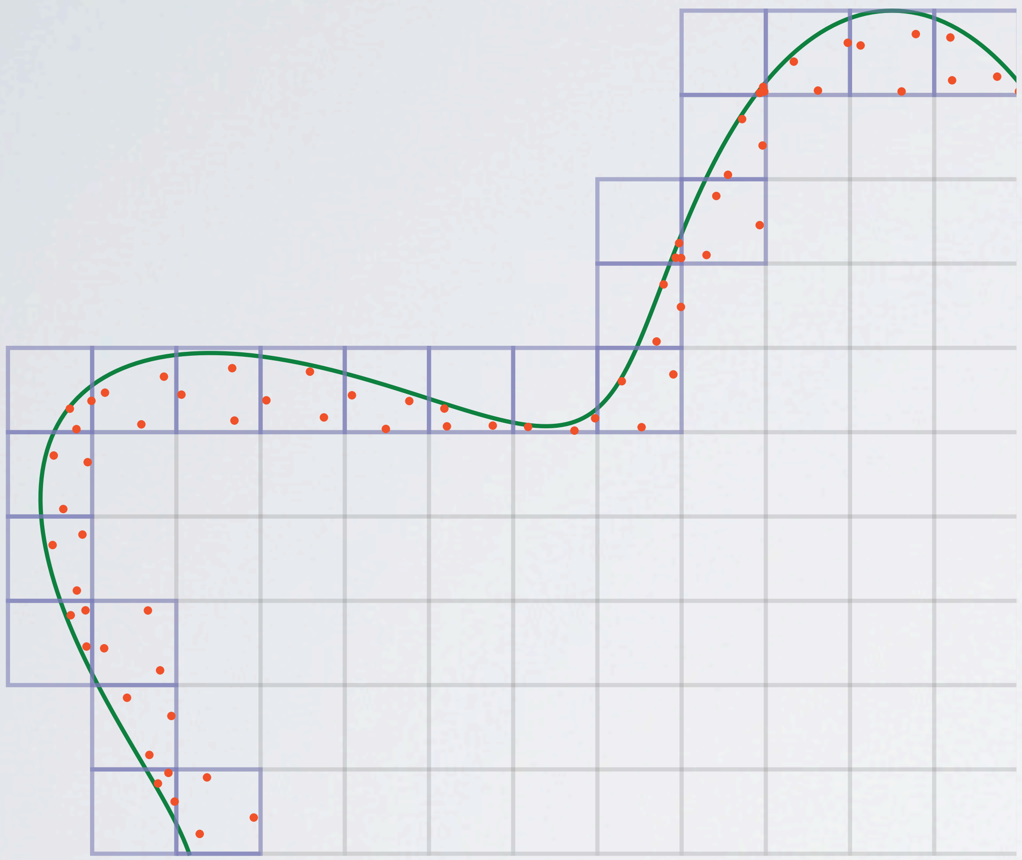
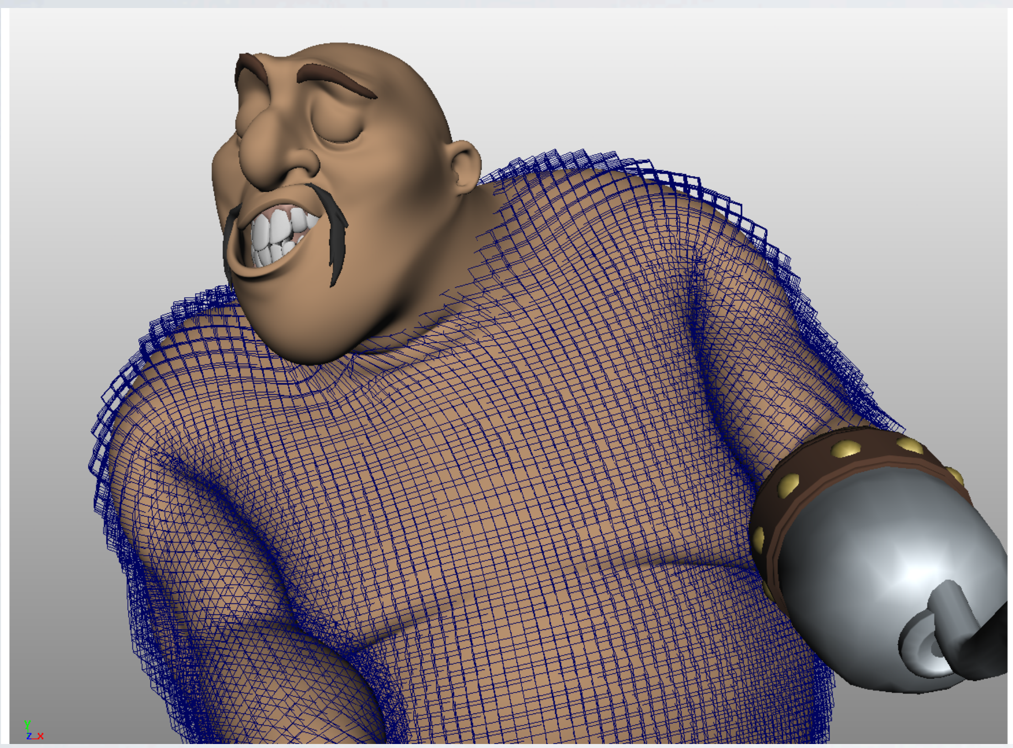


✓ Produces 2nd order accuracy

✓ Fixed work per cell (4 pts max)

✗ Need to compute 4 sets of forces

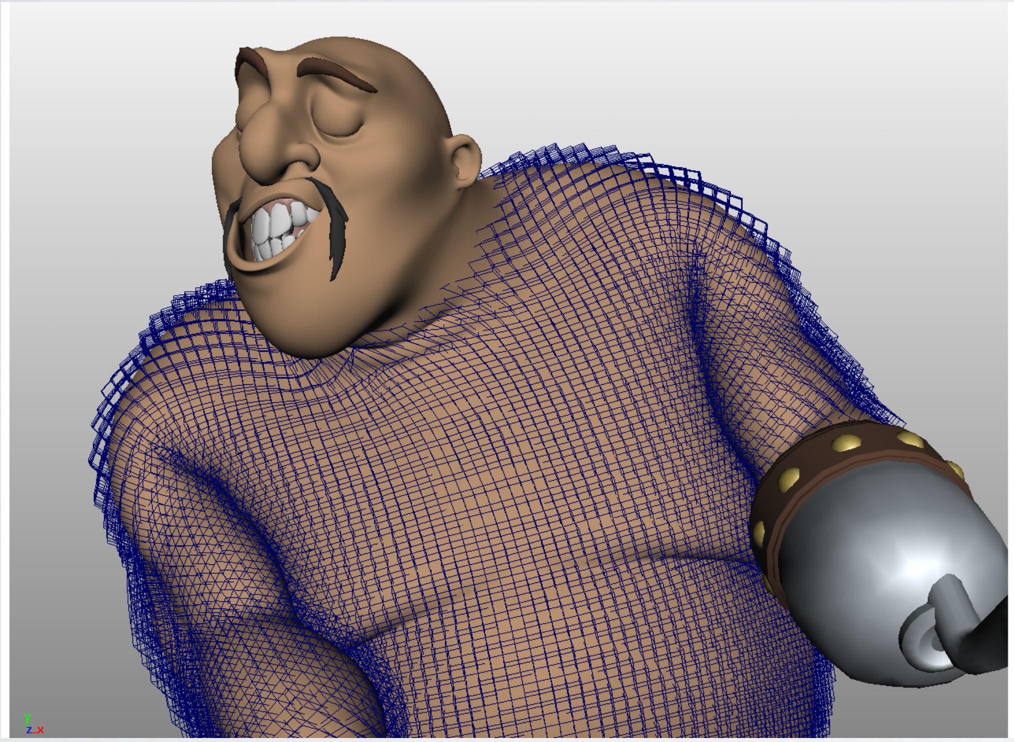
Alternative



[McAdams, et al. 2011]

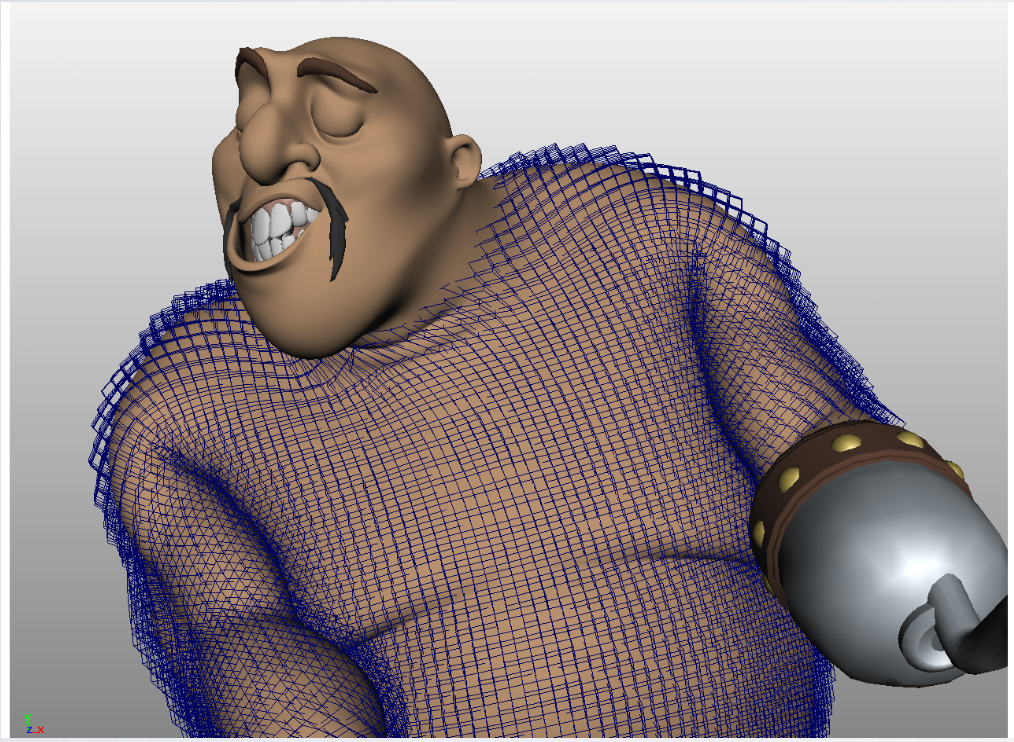
Alternative

✓ Uses whole cells



[McAdams, et al. 2011]

Alternative



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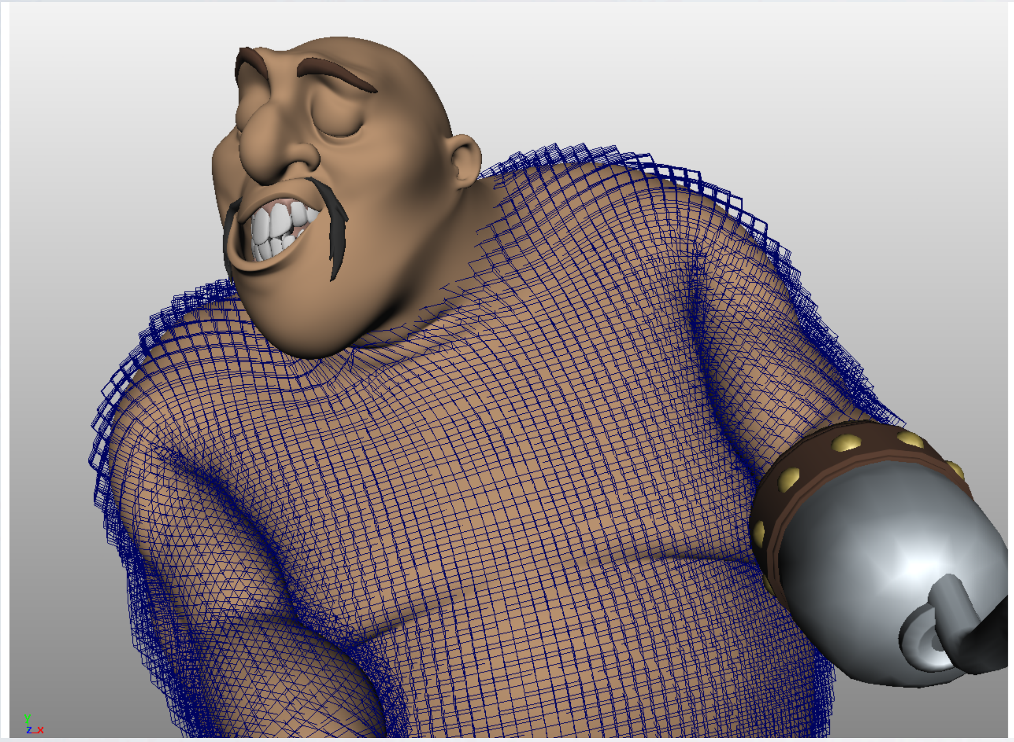
✓ Uses whole cells

✗ Less accurate

Alternative



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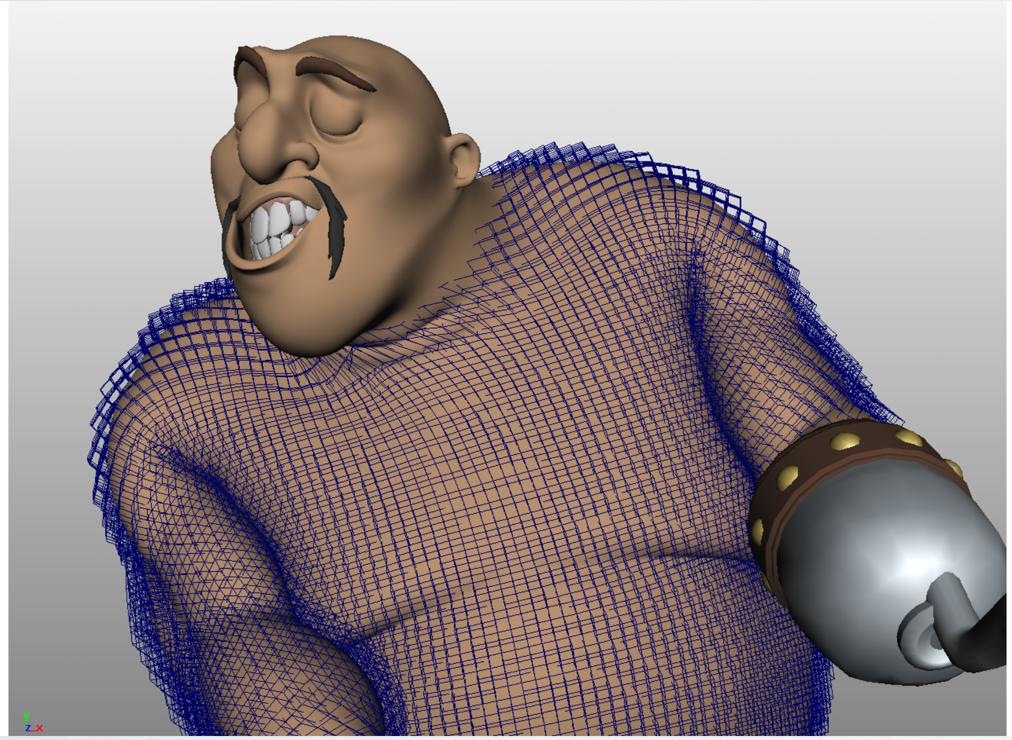
- ✓ Uses whole cells
- ✗ Less accurate
- ✓ Stabilization allows 1-pt evaluation

Alternative



[McAdams, et al. 2011]

✓ Uses whole cells

✗ Less accurate

✓ Stabilization allows 1-pt evaluation

We extended this to arbitrary materials

SOLVING THE SYSTEM

Start with Energy Density

$\downarrow$

Apply Incompressibility Adjustment

$\downarrow$

Compute Discrete Energy $\hat{E}$

$\downarrow$

Derive Nodal Forces $f(x)$

$\downarrow$

Find Equilibrium $f(x) = 0 \implies \min \hat{E}$

SOLVING THE SYSTEM

Find Equilibrium $f(x) = 0 \implies \min \hat{E}$

OBJECTIVE: SOLVE FOR F(X) = 0

Solve $f(x) = 0 \overset{\text{Newton}}{\Rightarrow} x_0 \longleftarrow$ Initial guess

for $k = 0, 1, \ldots$

Solve $\left\{ -\left.\frac{\delta f}{\delta x}\right|_{x_k} \right\} \delta x = f(x_k)$

Update $x_{k+1} = x_k + \delta x$

$\Rightarrow \lim x_k =$ Sub-voxel accurate solution of

$f(x) = 0$

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for $k = 0, 1, \ldots$

Solve $\left\{ -\left.\frac{\delta f}{\delta x}\right|_{x_k} \right\} \delta x = f(x_k)$

Compute with four point method

Update $x_{k+1} = x_k + \delta x$

$\Rightarrow \lim x_k =$ Sub-voxel accurate solution of $f(x) = 0$

FEATURES

✓ Arbitrary Materials

✓ Incompressibility

✓ Sub-voxel Precision

Robust

Parallelism

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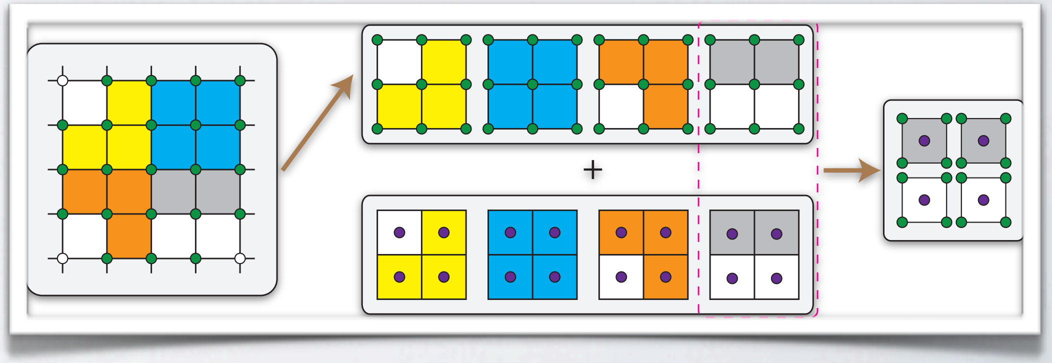

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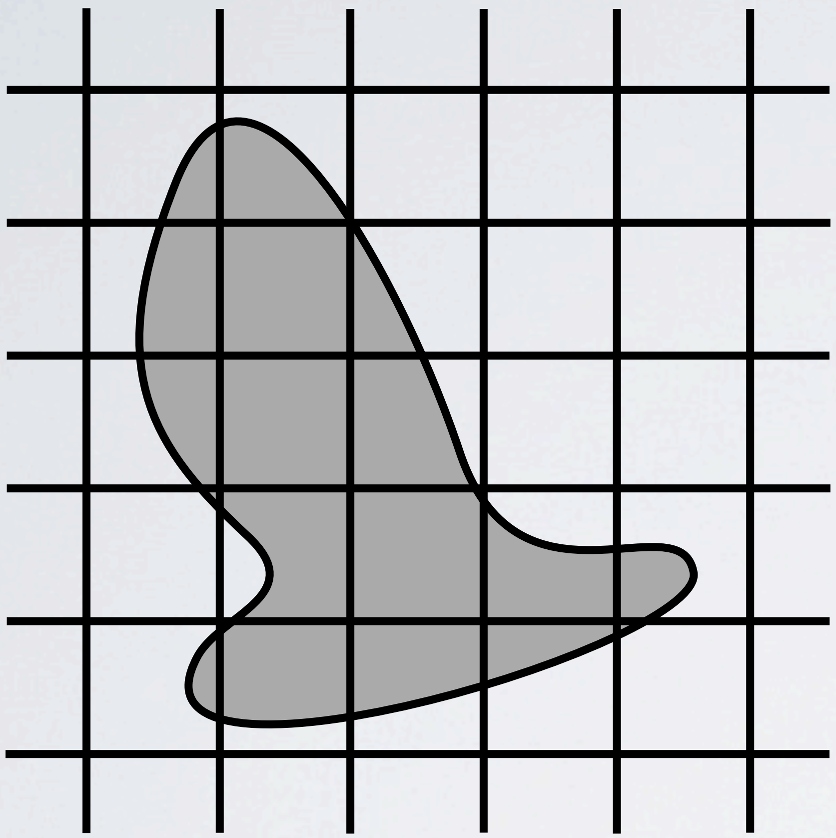

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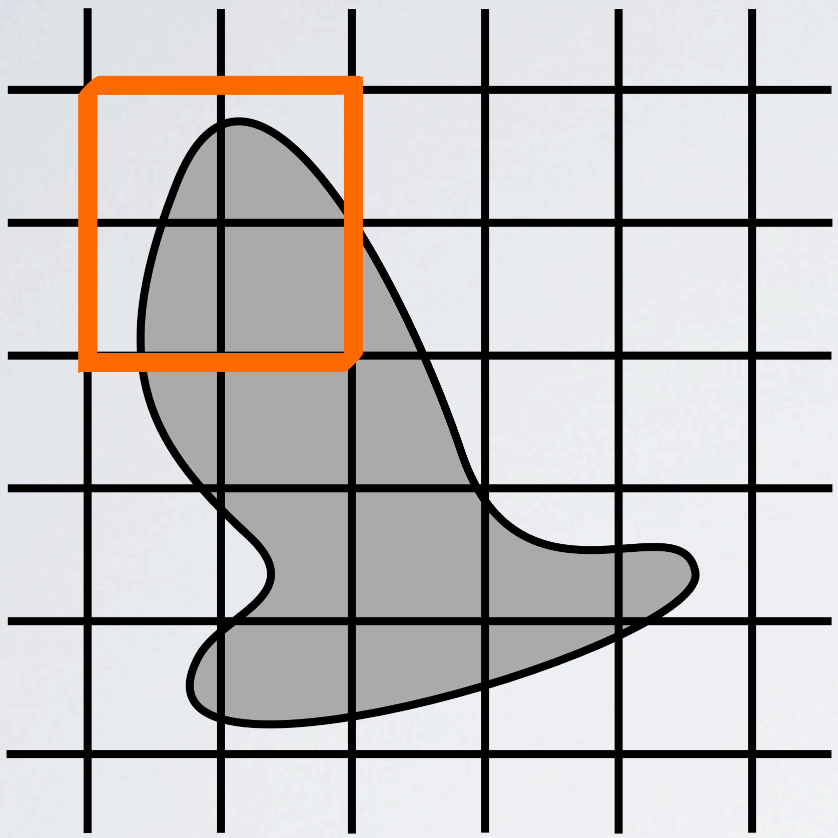

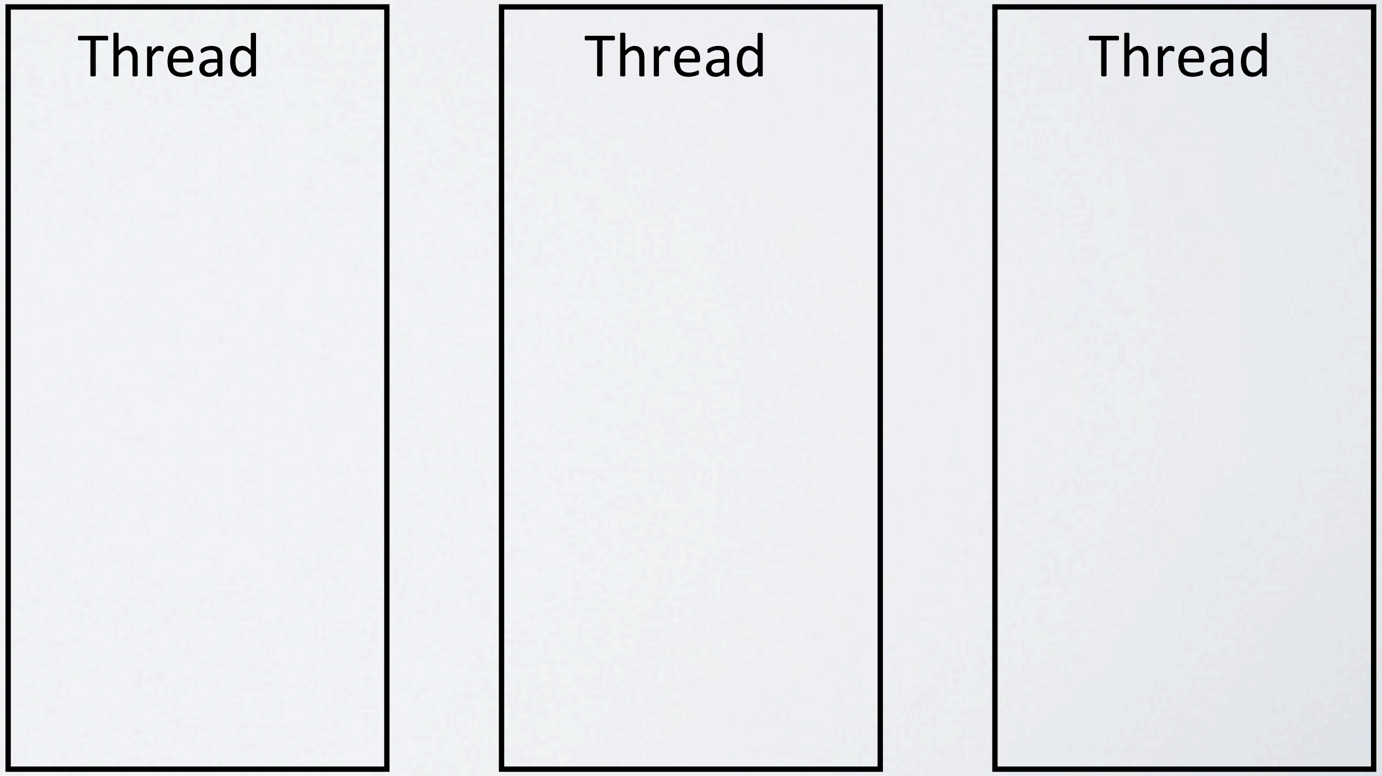

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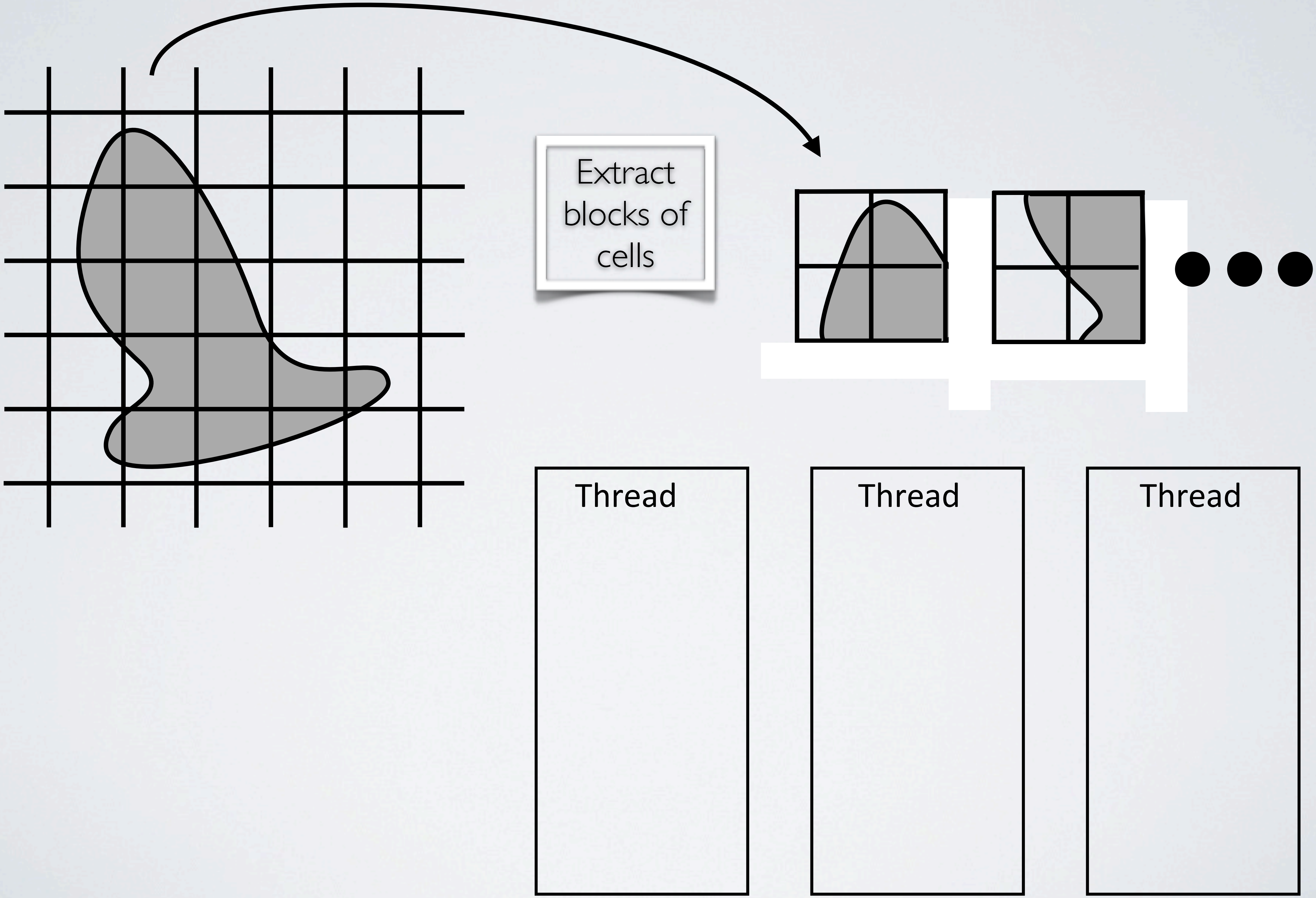
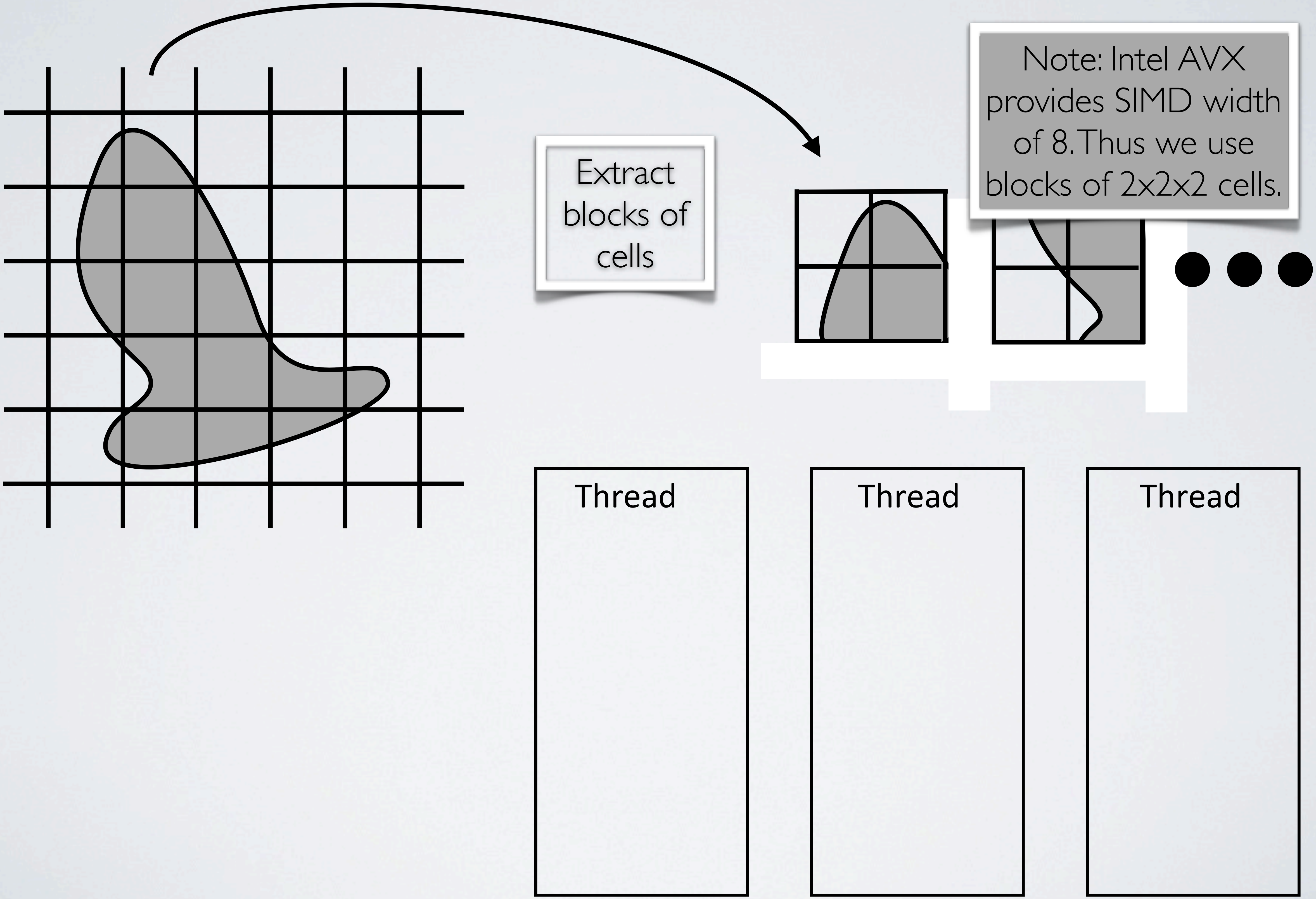


Parallelism

Extract blocks of cells

Note: Intel AVX provides SIMD width of 8. Thus we use blocks of 2x2x2 cells.



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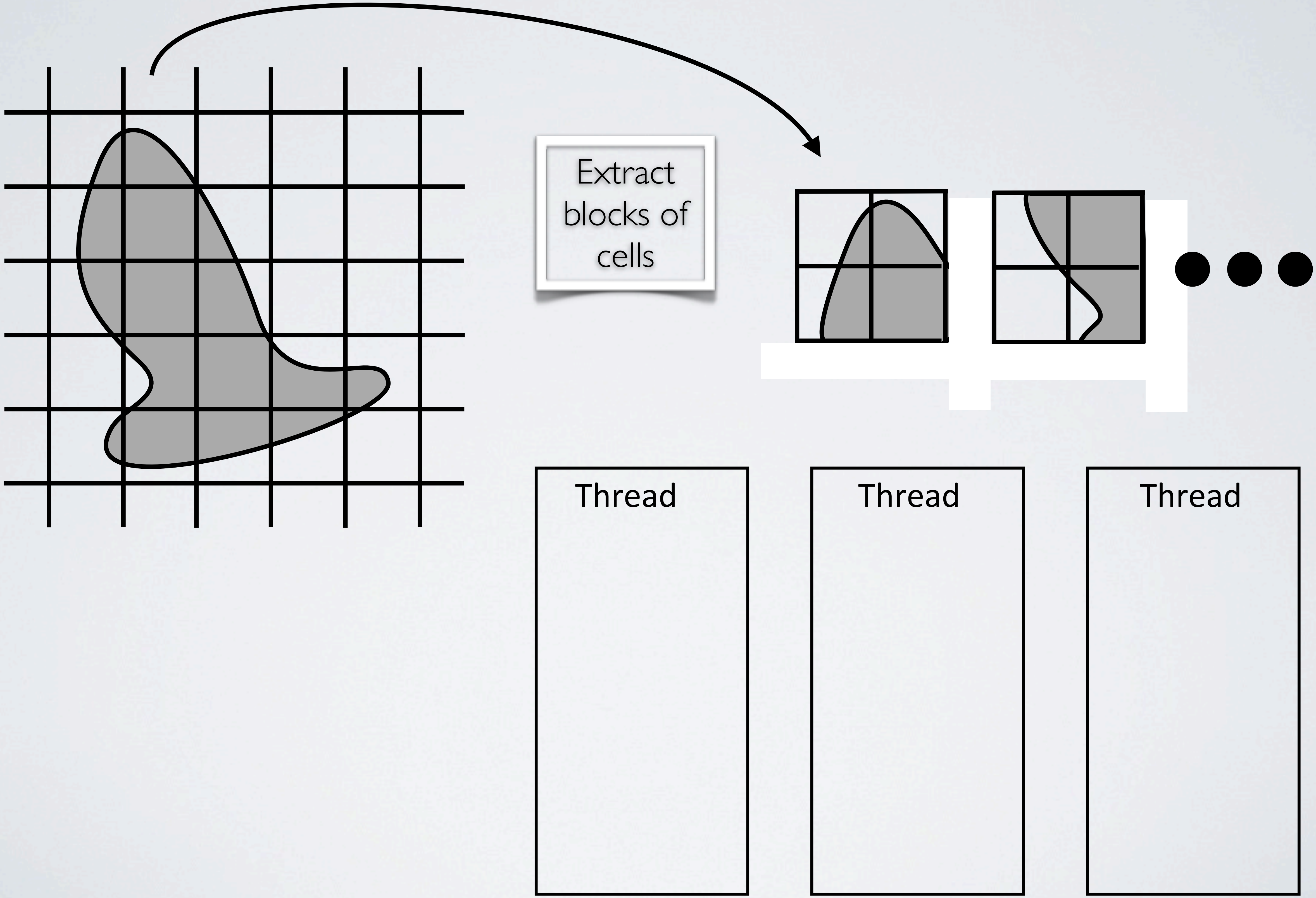

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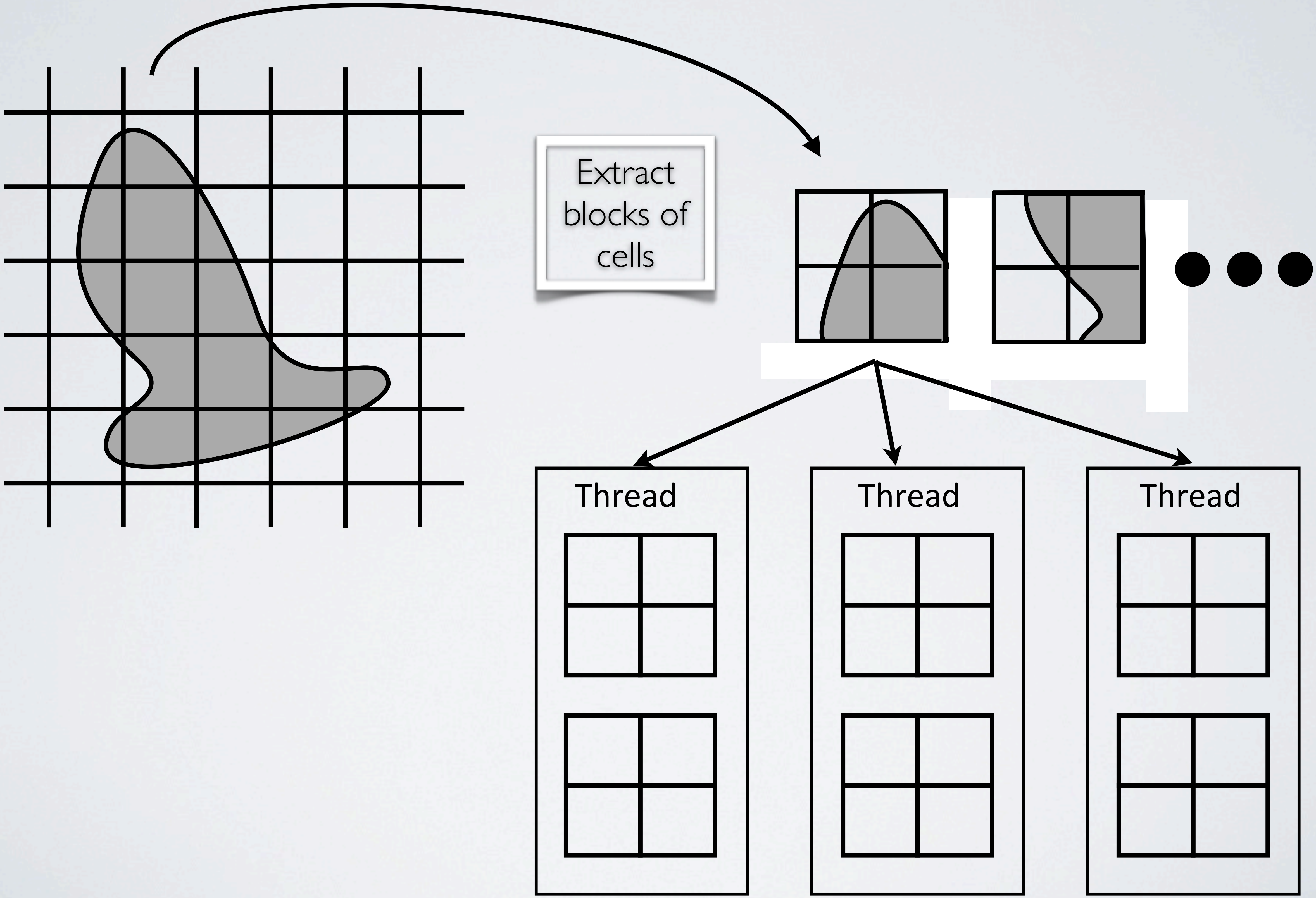

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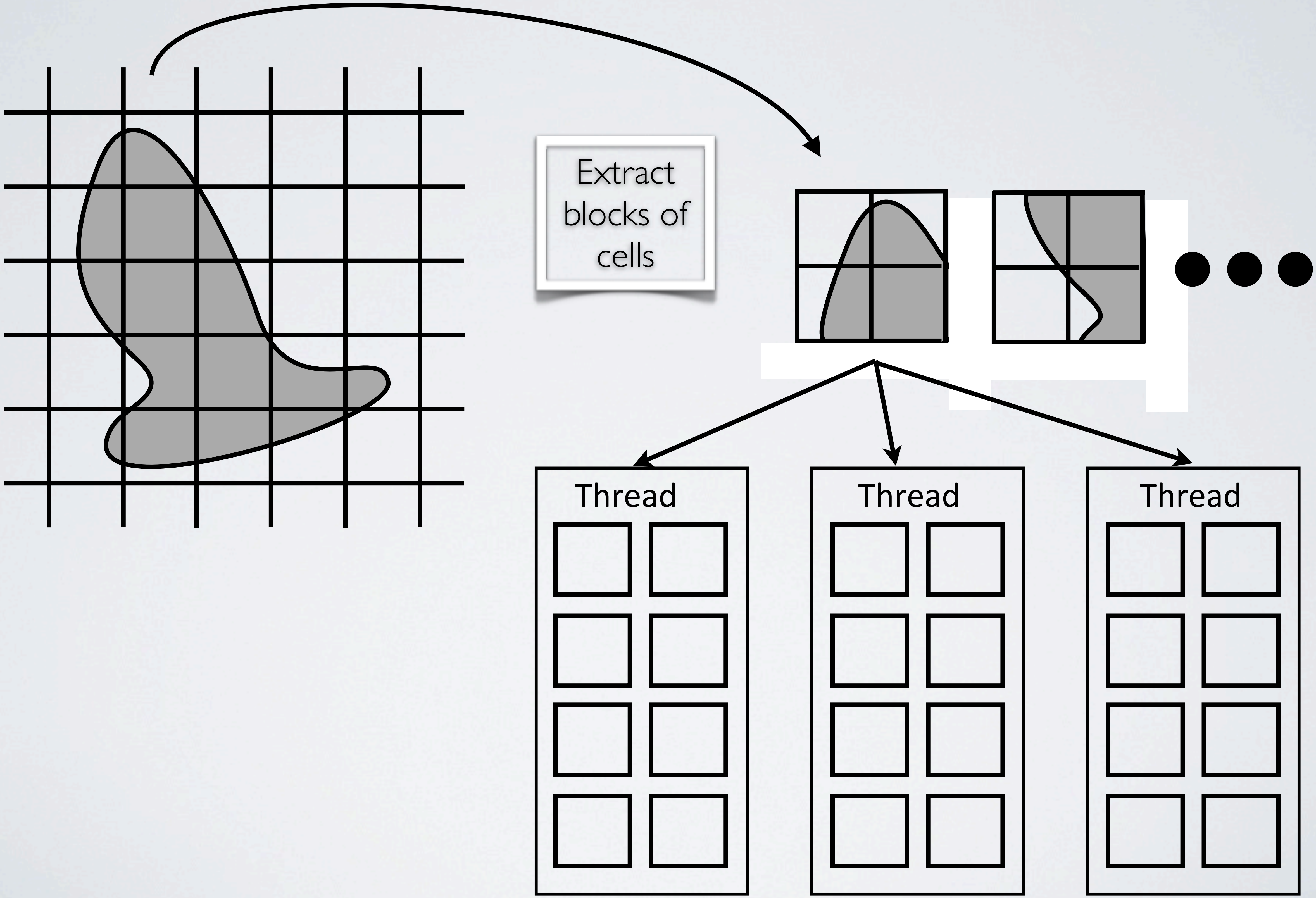

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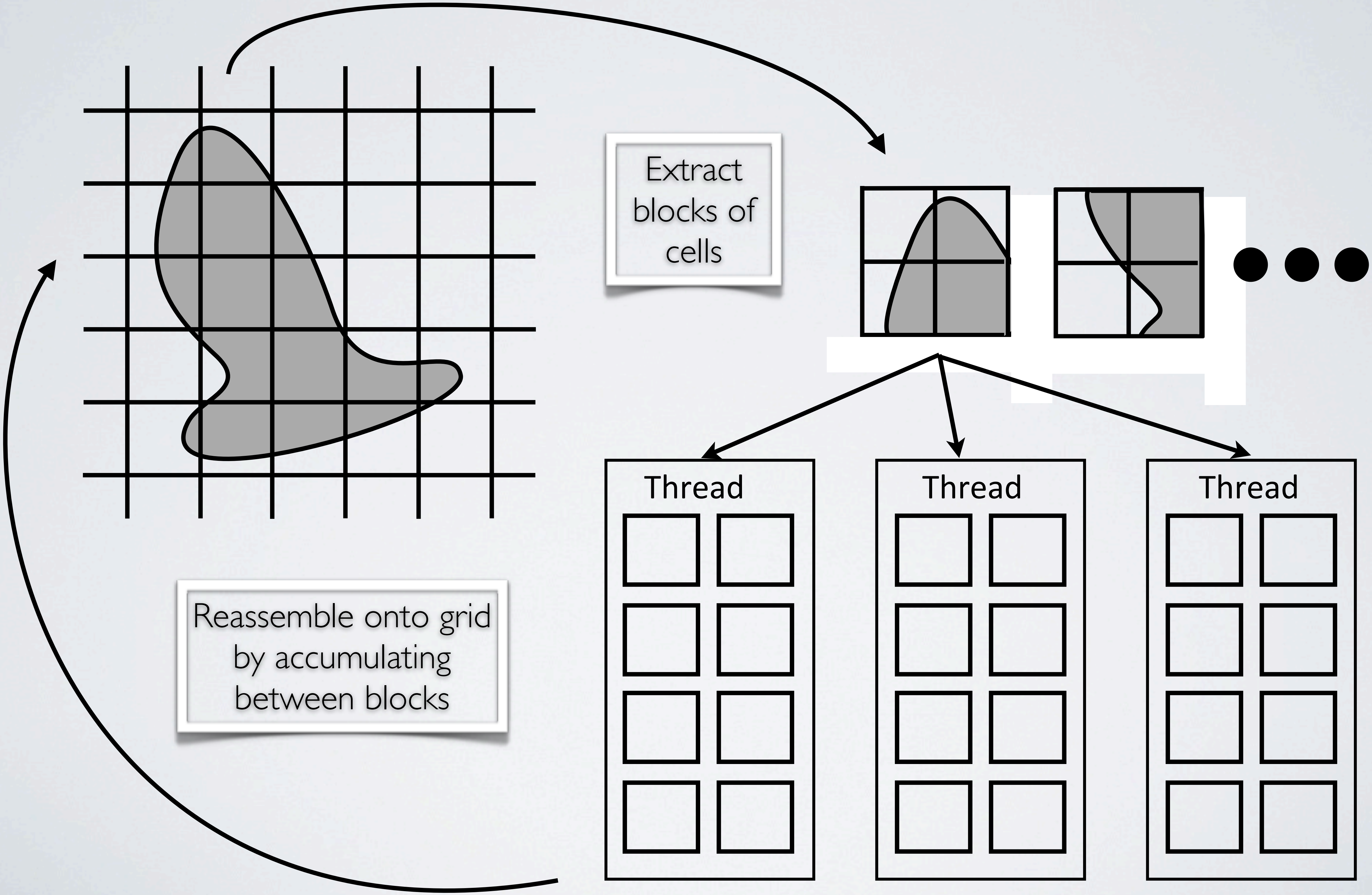

PERFORMANCE

		1 core (AVX) 1 thread	2 cores (AVX) 2 threads	4 cores (AVX) 4 threads	4 cores (AVX) 8 threads
Human model (101K cells)	Force Differentials	0.041	0.023	0.015	0.015
	One QMR Iteration	0.067	0.037	0.025	0.024
	Newton Iteration (including 100 QMR iterations)	7.035	4.066	2.727	2.645
	Typical frame (4-6 Newton iterations)	31.825	18.186	12.193	11.919
Human model (13.5K cells)	Force Differentials	0.005	0.003	0.002	0.002
	One QMR Iteration	0.007	0.005	0.003	0.003
	Newton Iteration (including 50 QMR iterations)	0.422	0.244	0.184	0.177
	Typical frame (3-5 Newton iterations)	1.466	0.842	0.640	0.626

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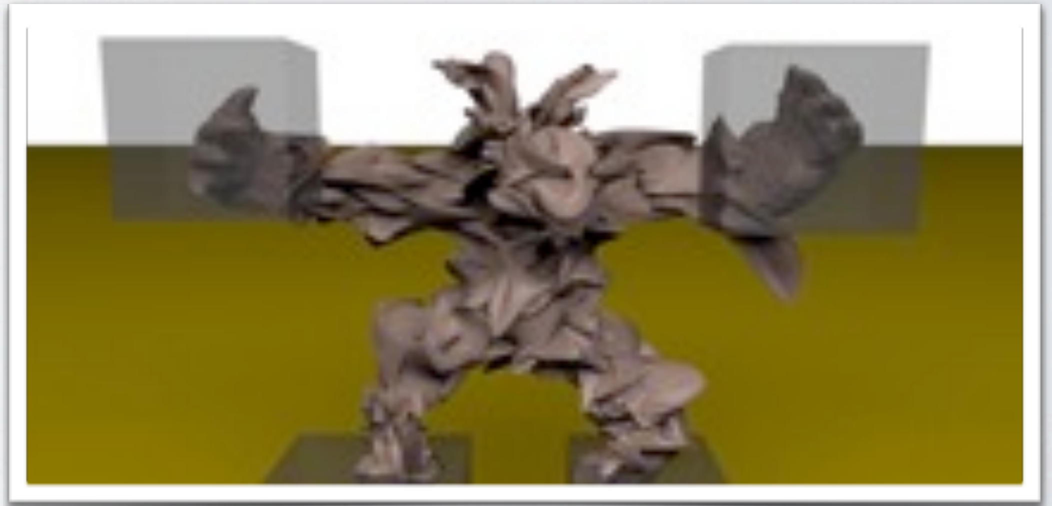
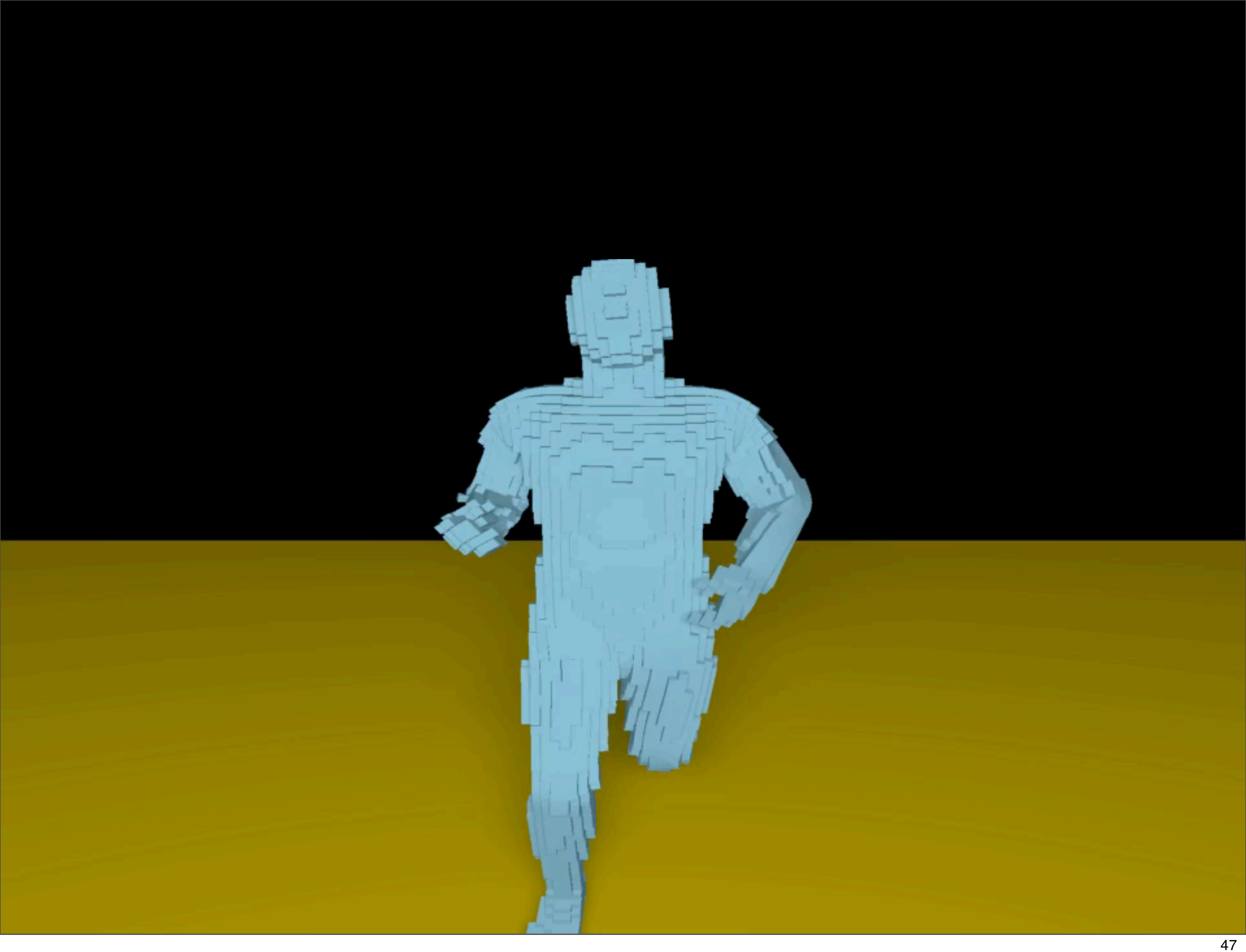
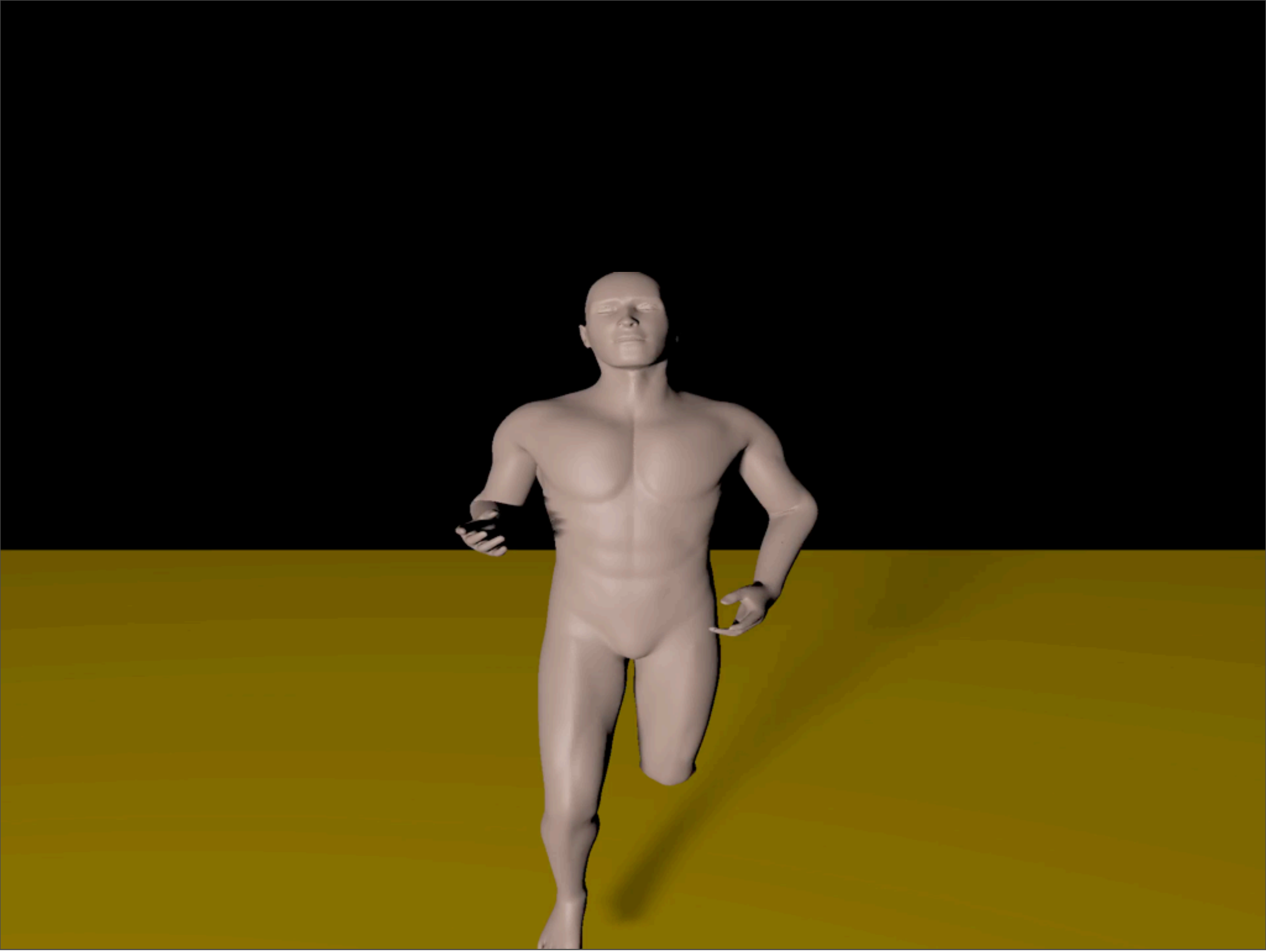


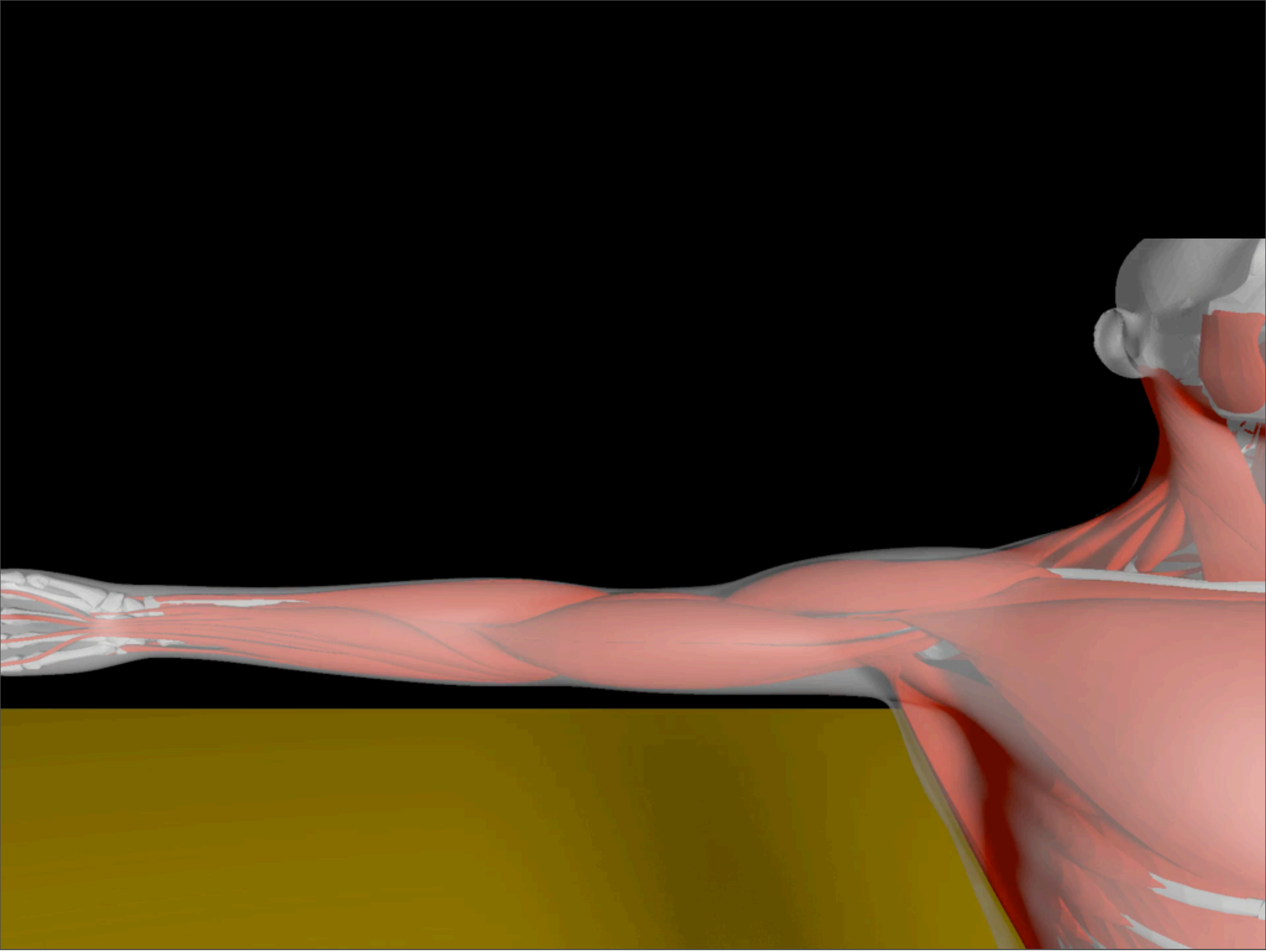
ROBUSTNESS

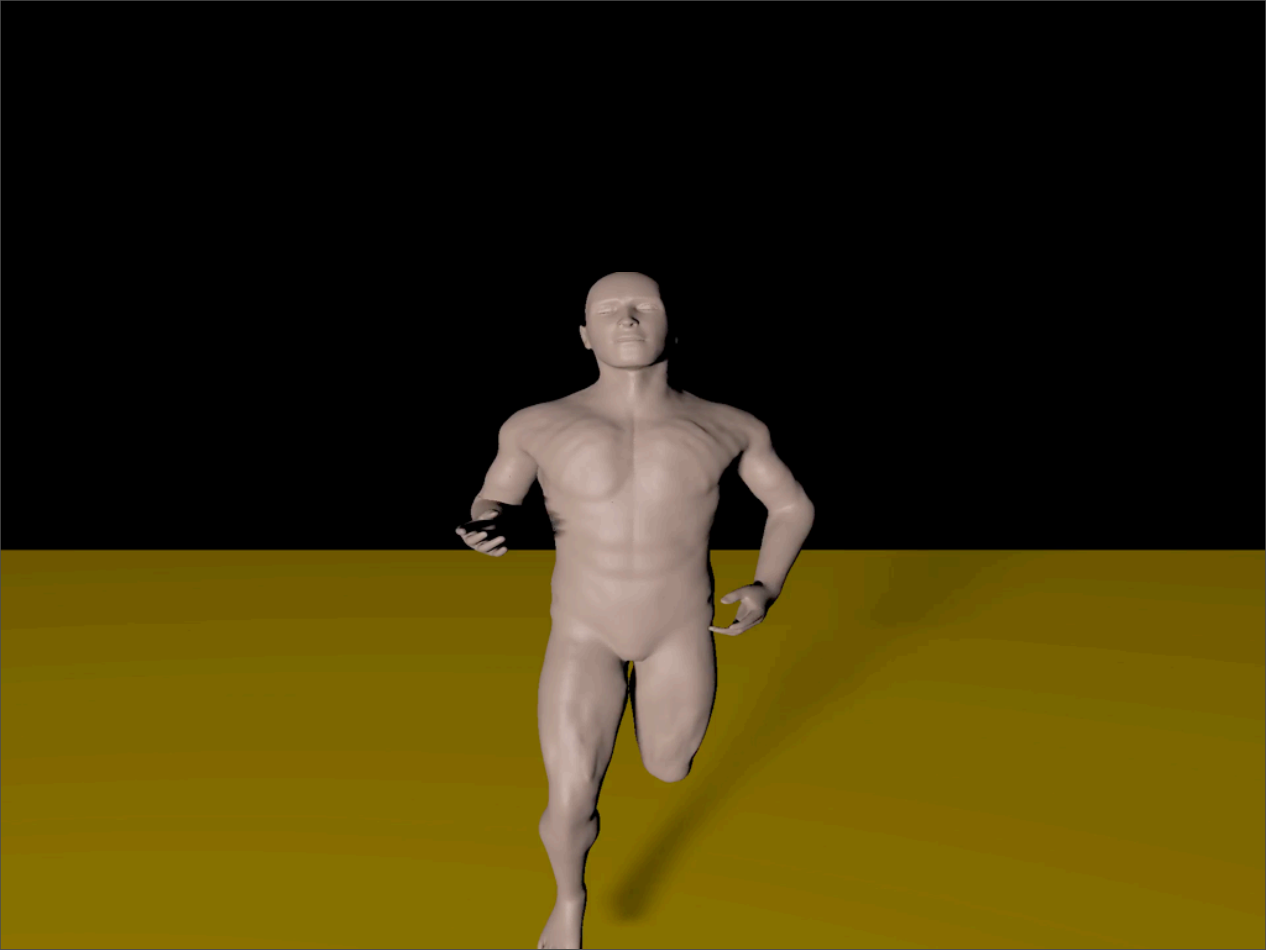
ROBUSTNESS

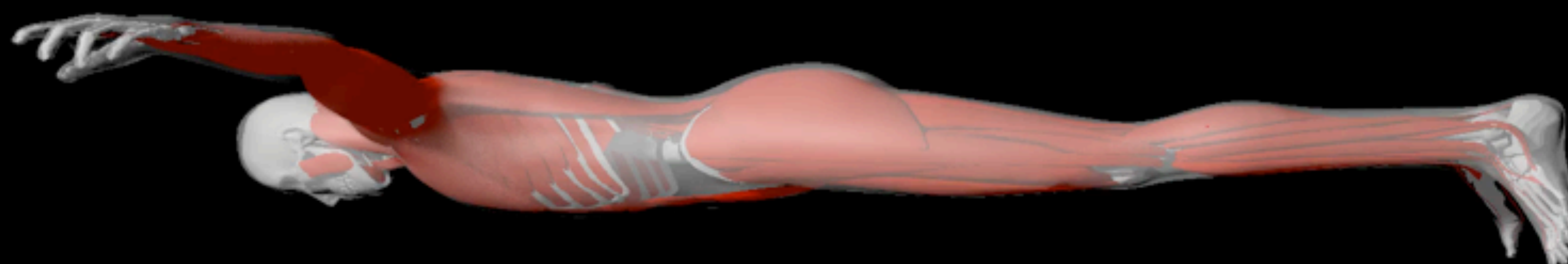
[redacted]











CONCLUSION OF COMPLETED WORK

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Sub-voxel Precision

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Parallelism

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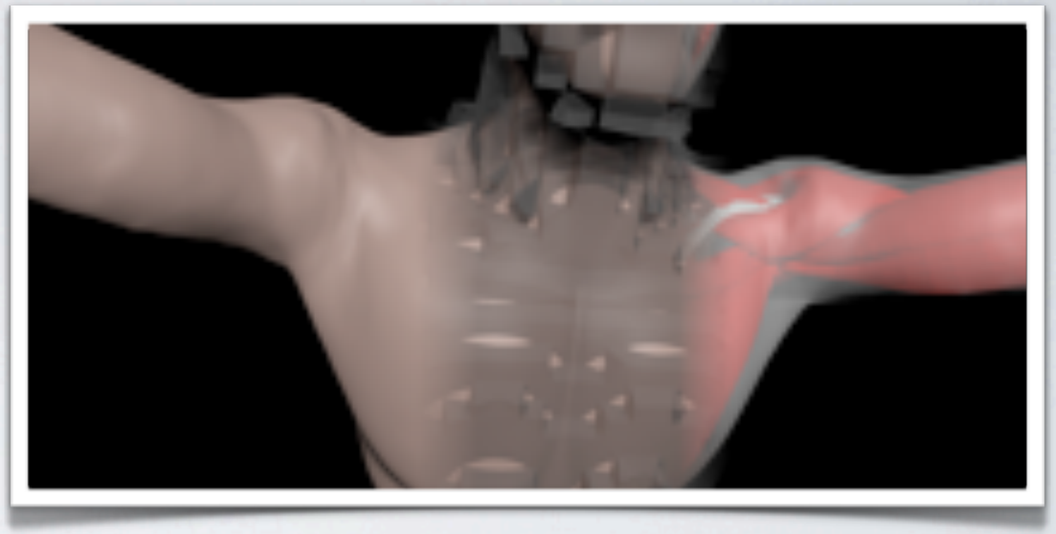

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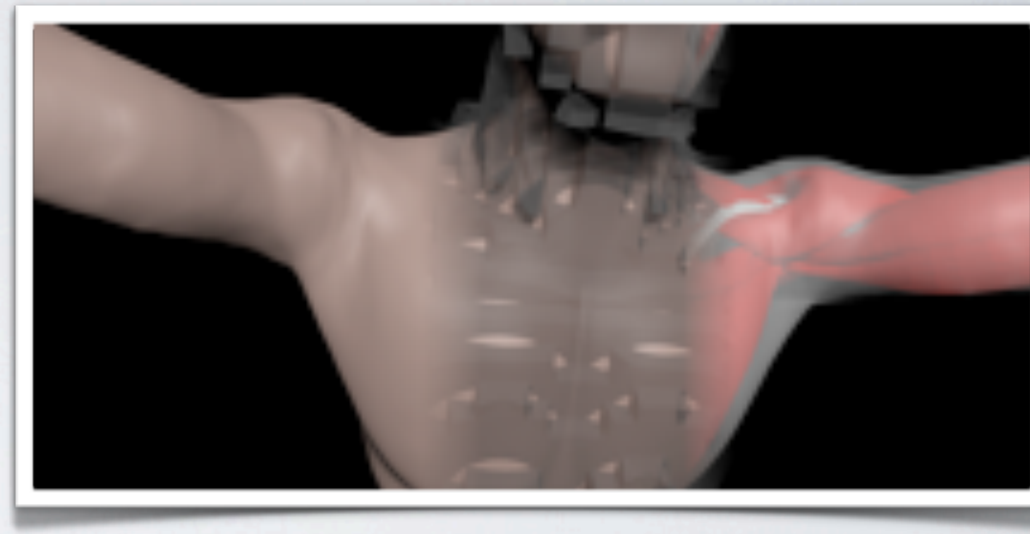

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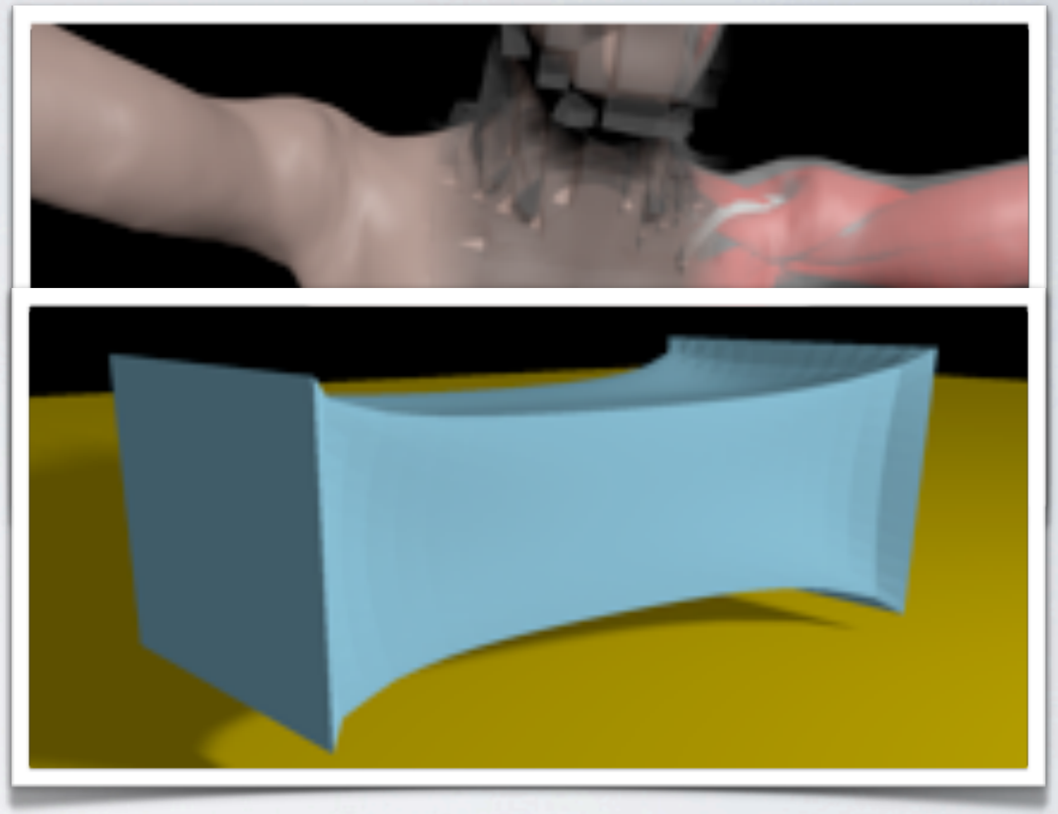

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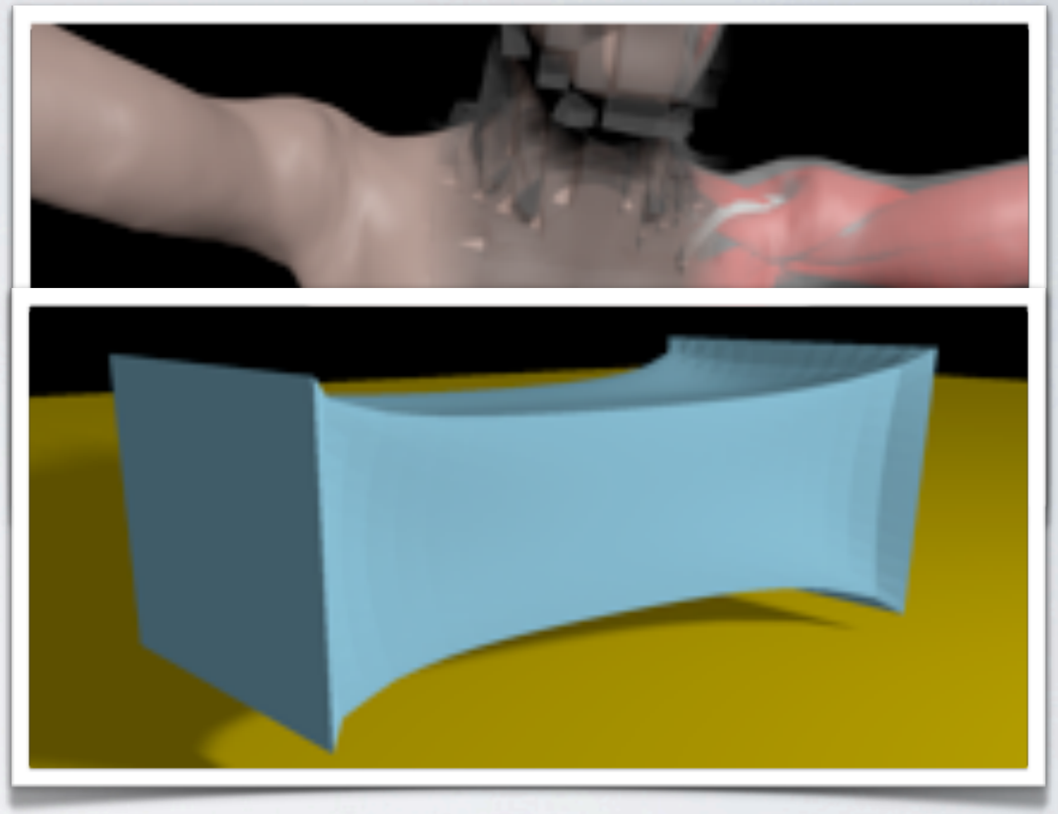

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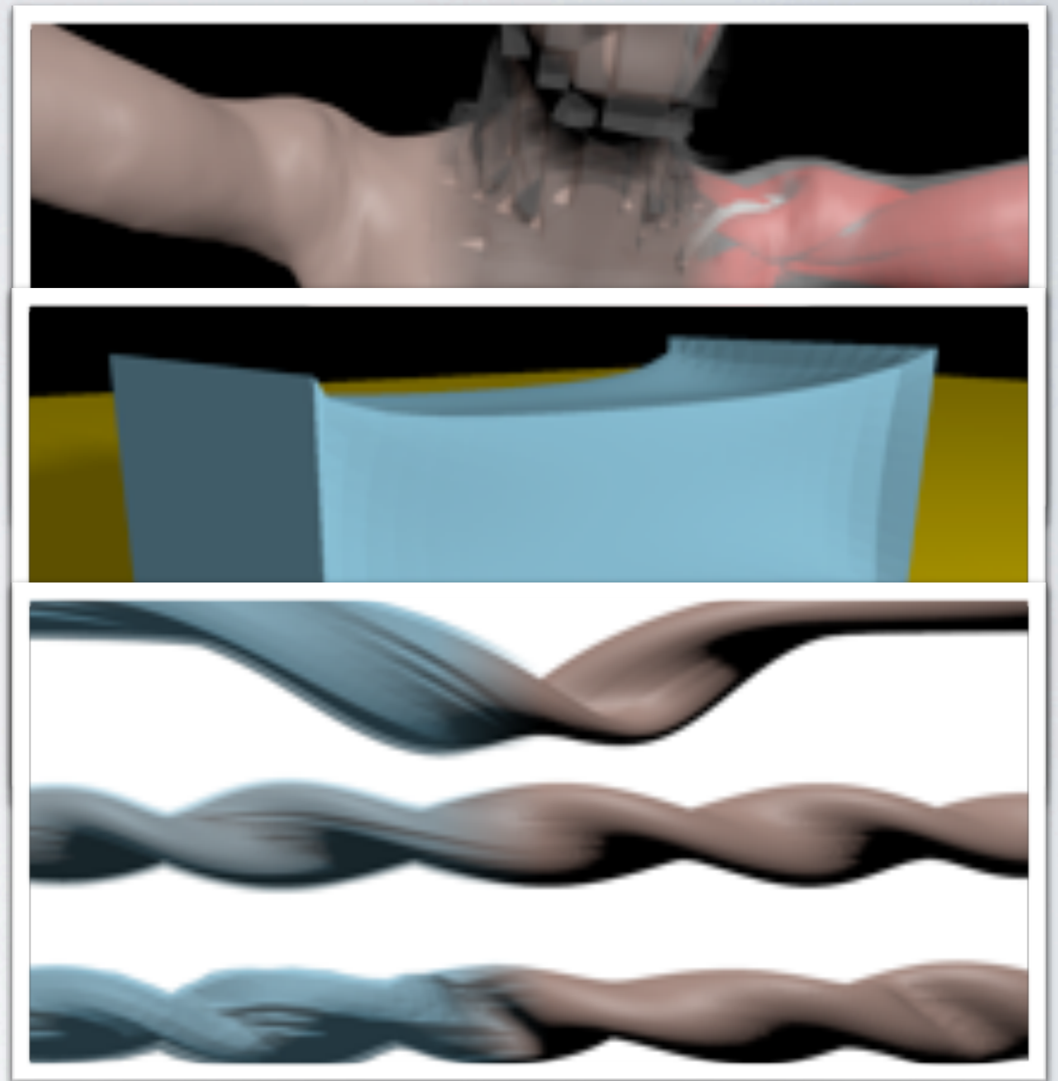

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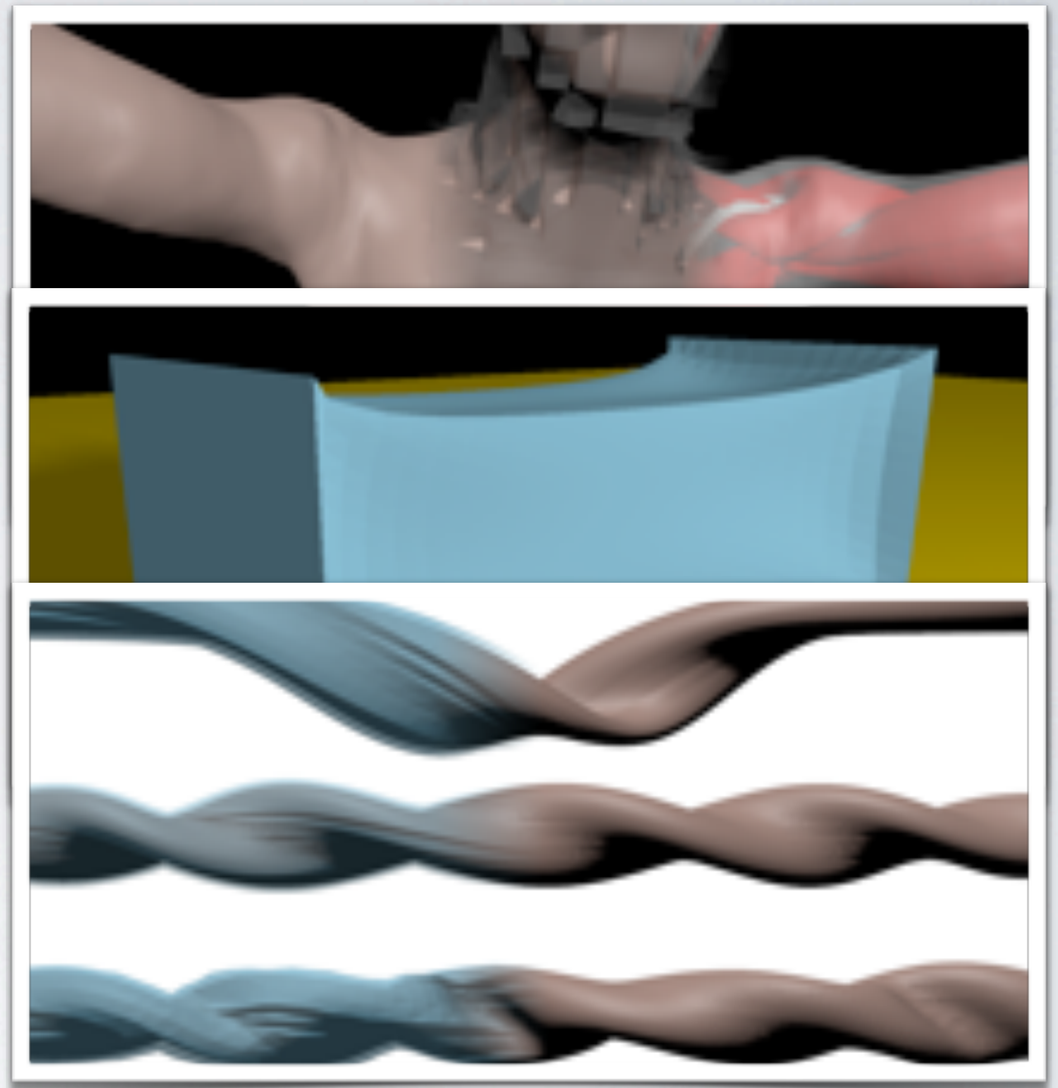

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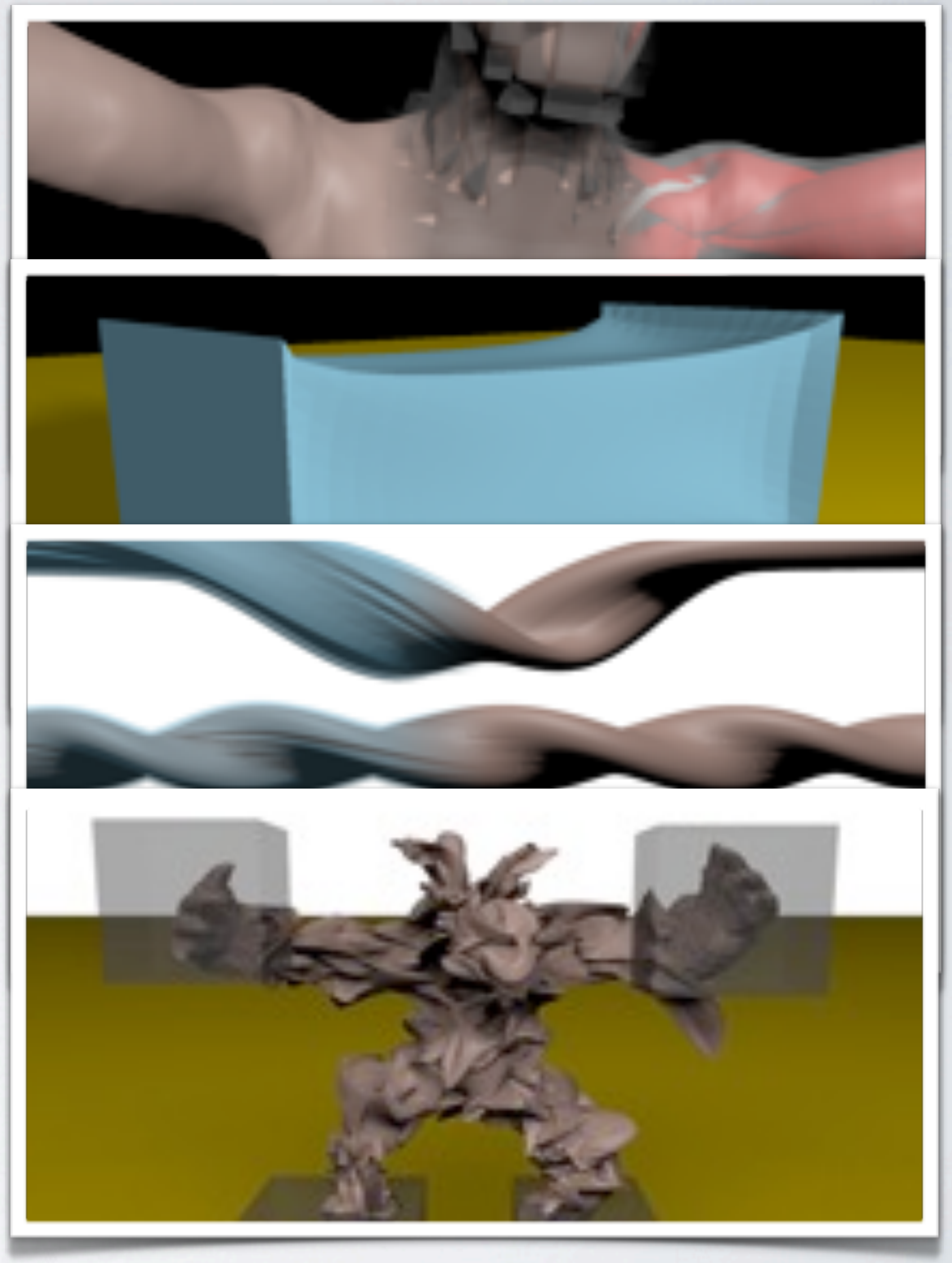

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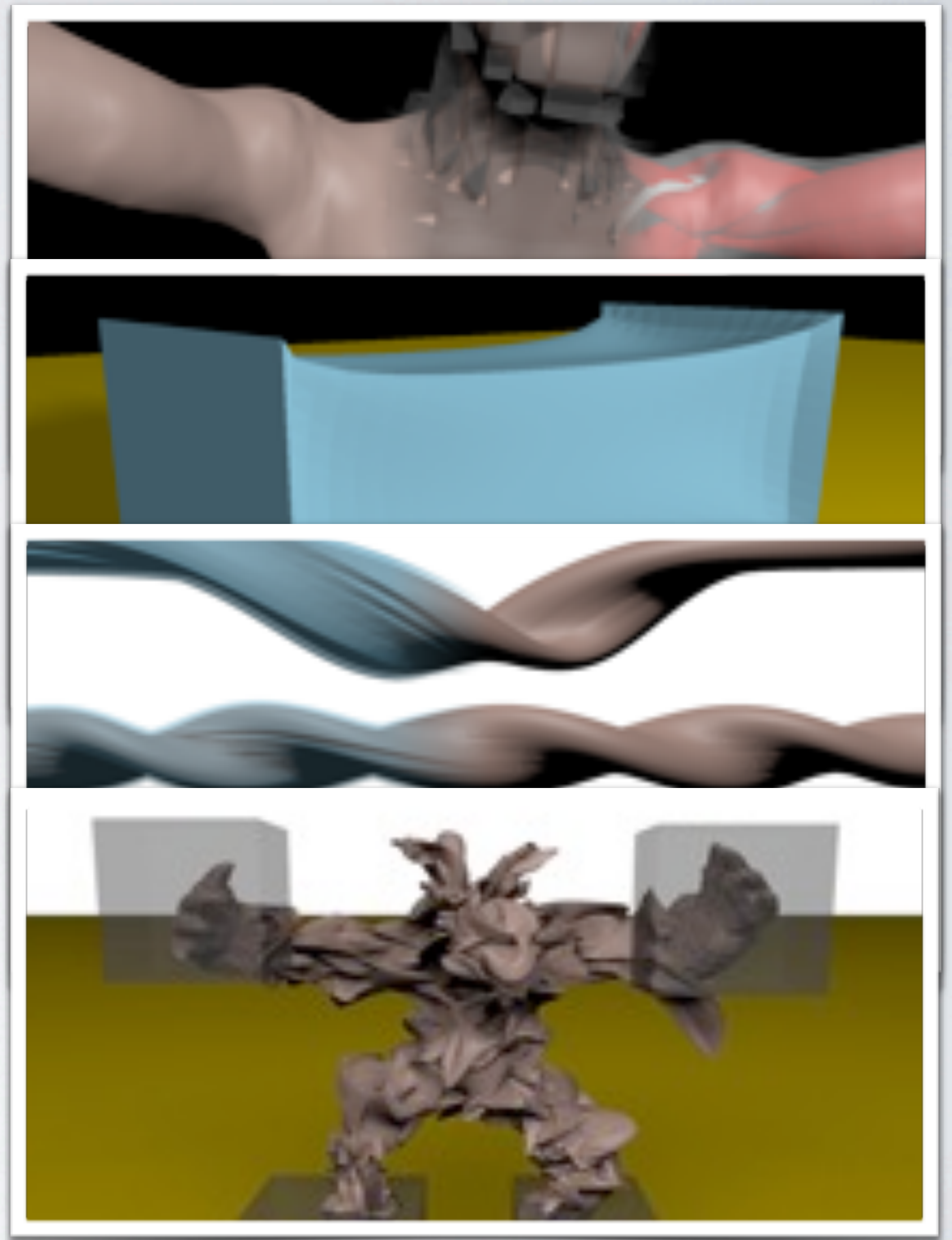

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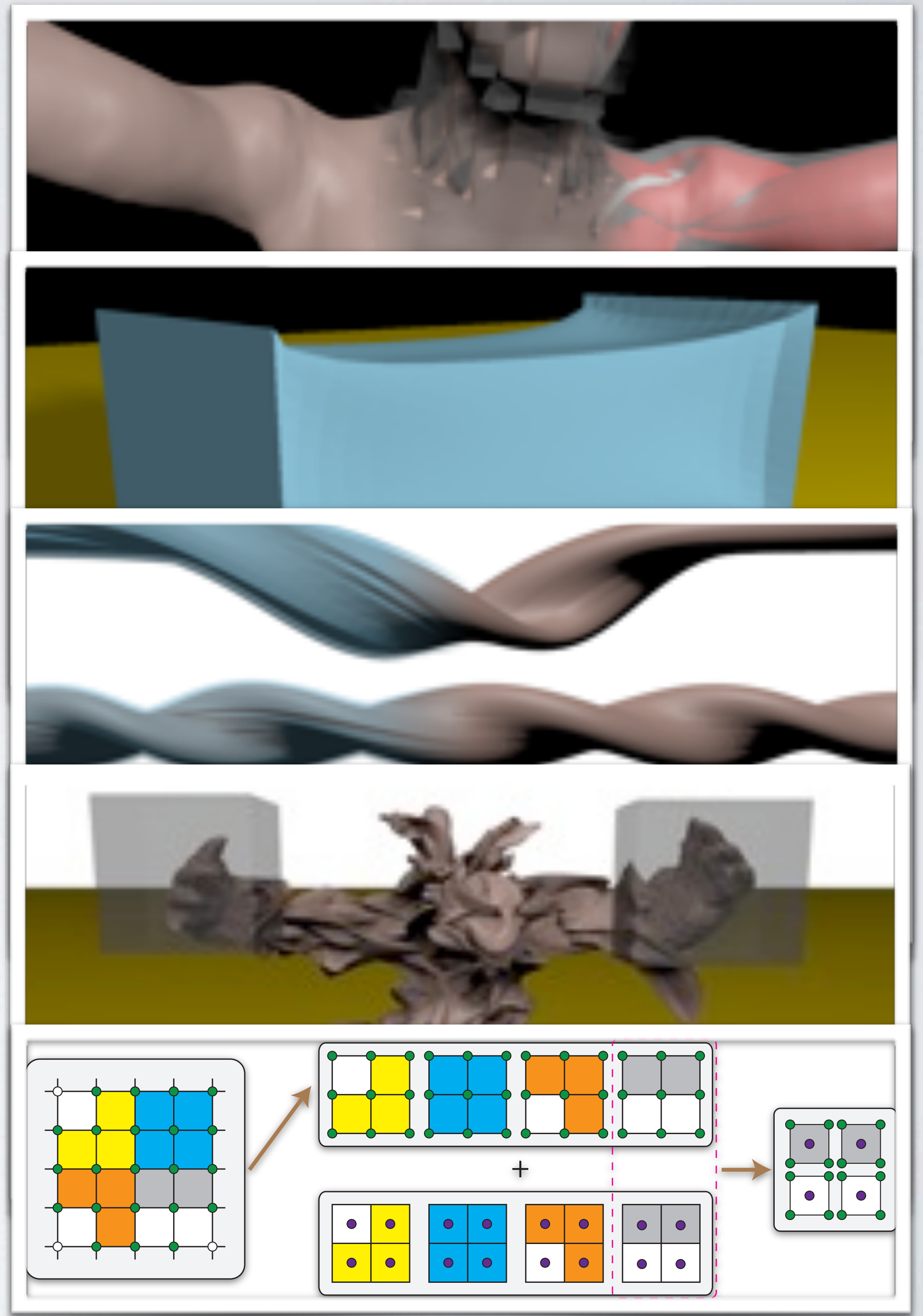

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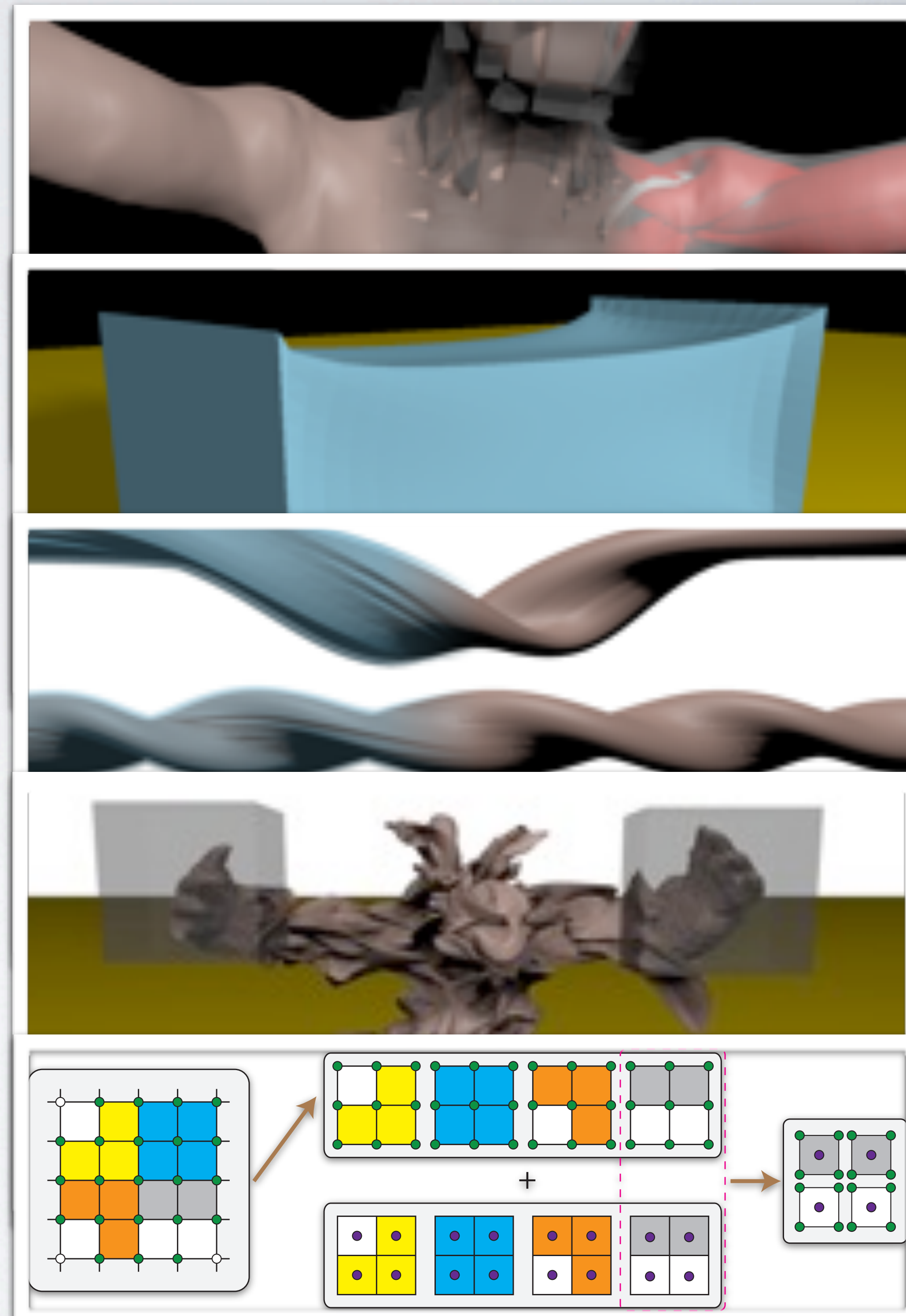

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$\Longrightarrow$ No collisions yet

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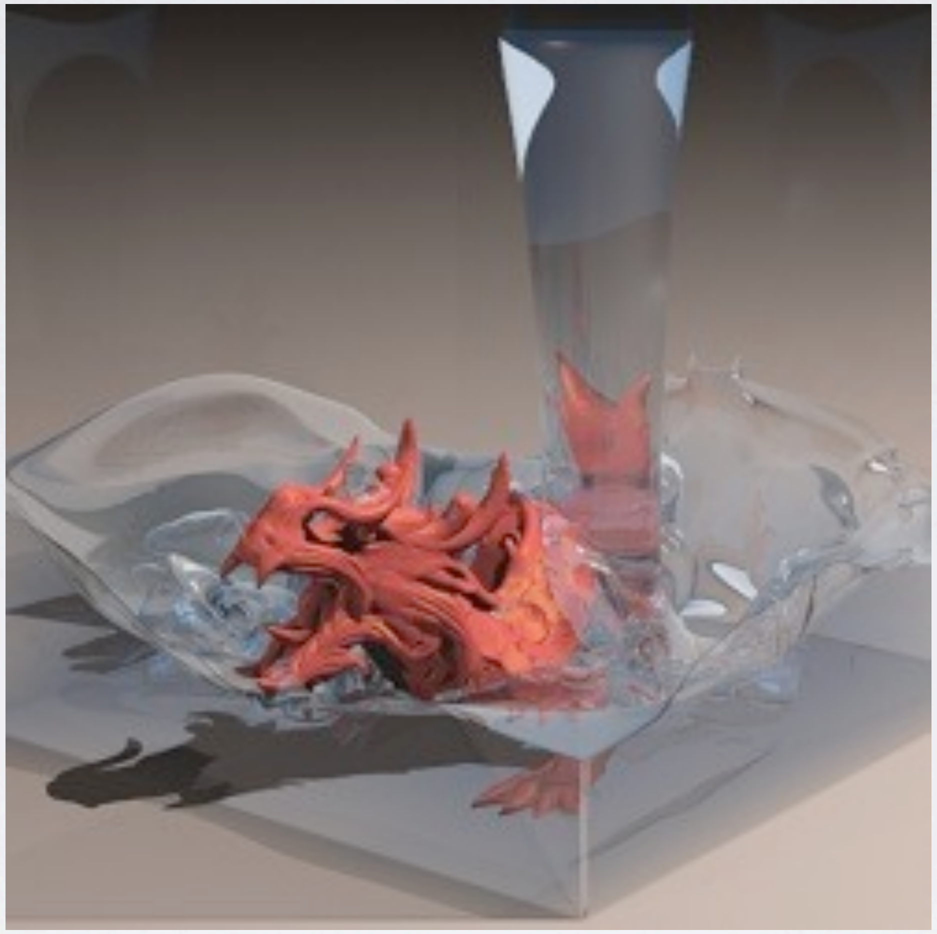

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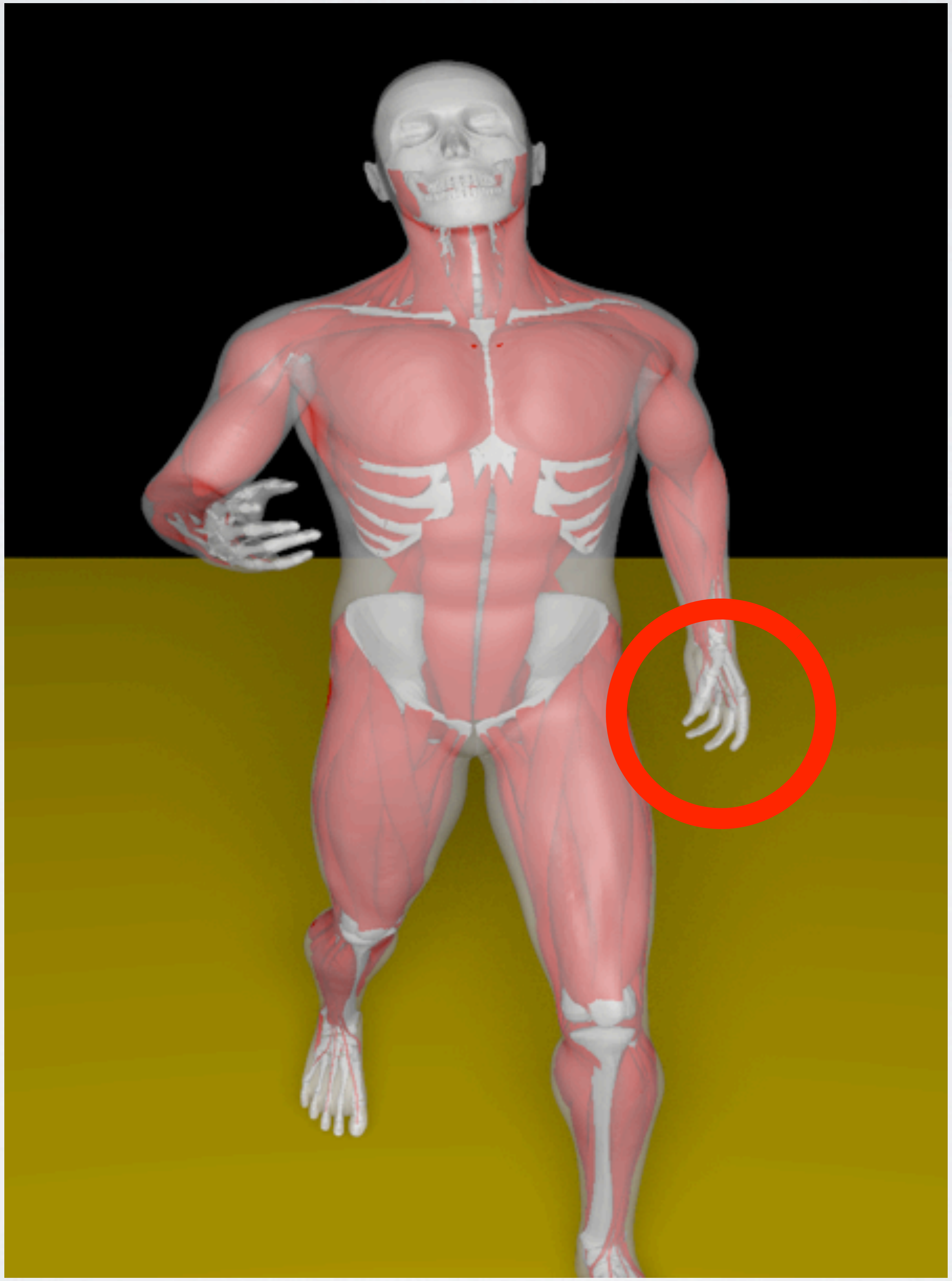

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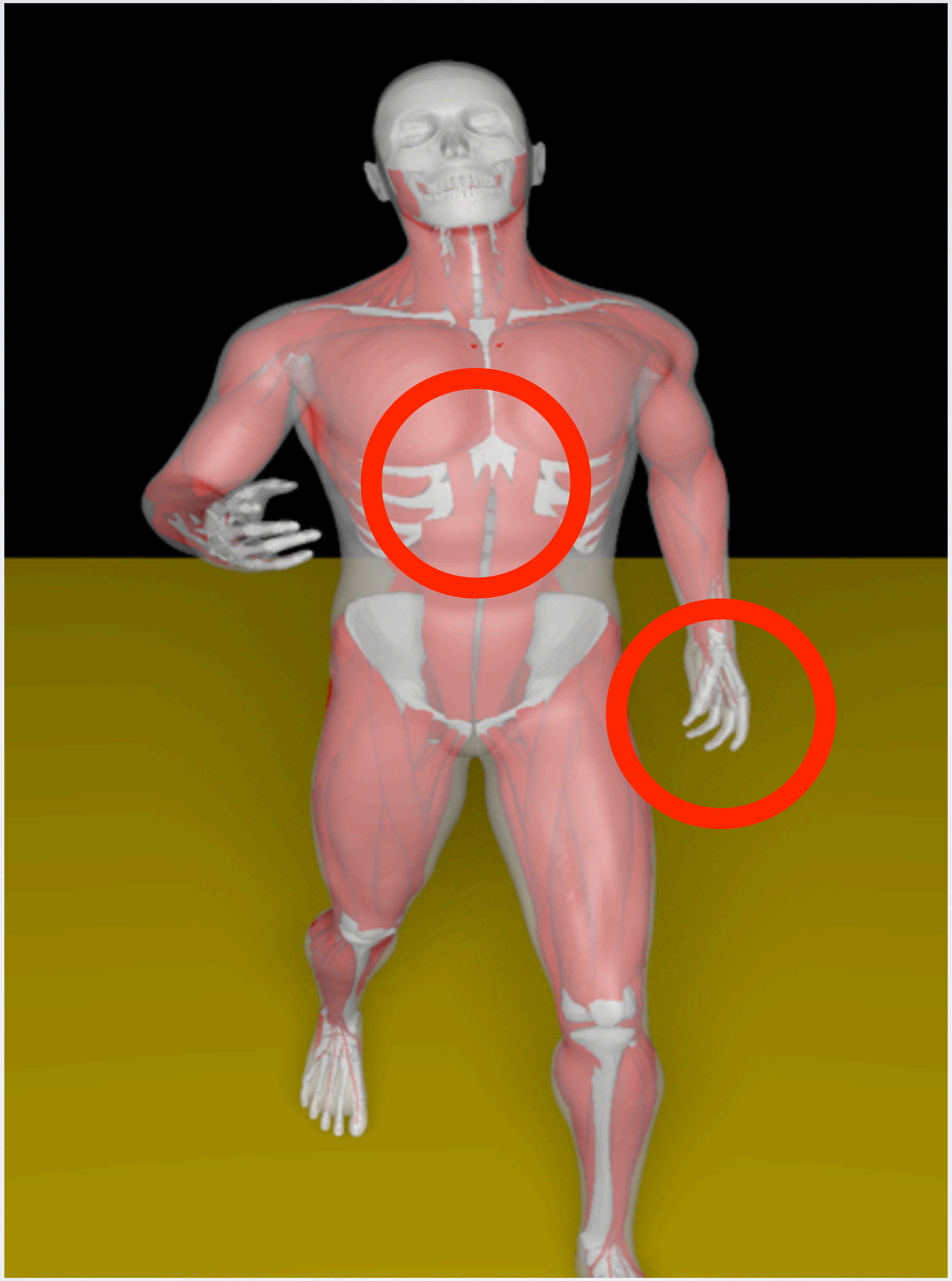

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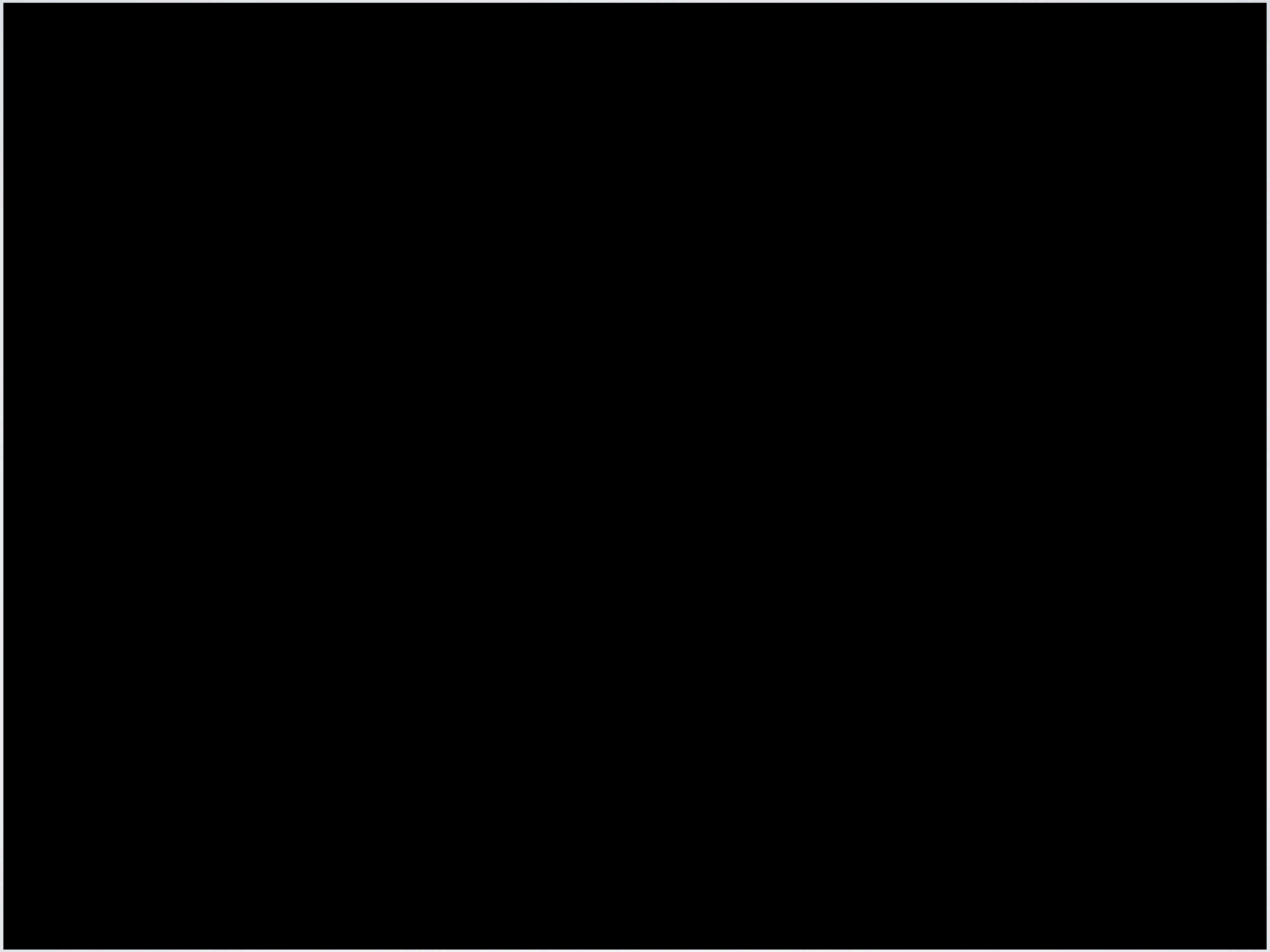
TOPOLOGICAL CHANGE

[Sifakis et al 2009]

TOPOLOGICAL CHANGE



[Sifakis et al 2009]

ACKNOWLEDGEMENTS

University of Wisconsin-Madison

Eftychios Sifakis
Nathan Mitchell
Raj Setaluri

Thank you

Questions?