Consistent Query Answering for Primary Keys on Path Queries

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Data model

- Primary key constraint as the only integrity constraint
- Inconsistent relational databases violating the primary constraint
- A repair is an inclusion-maximal consistent subinstance

Univ

Acronym	City
UW	Madison
UW	Seattle
UMONS	Mons

Weather

City	Weather
Madison	Snow
Seattle	Rain
Mons	Sunny

Q: Is there a university snowing today?

$$Q() : -\text{Univ}(\underline{x}, y), \text{Weather}(\underline{y}, \texttt{`Snow'})$$

The problem statement

Consistent query answering – CERTAINTY(q)

INPUT: an inconsistent database db
QUESTION: Is the **fixed** Boolean query q true in <u>all</u> repairs of db?

$q_2() : -R(\underline{x}, z), S(\underline{y}, z)$

$q_1() : -R(\underline{x}, y), S(\underline{y}, z)$

$$\text{CERTAINTY}(q_1) \iff \exists x(\exists y R(\underline{x}, y) \land \forall y R(\underline{x}, y) \rightarrow \exists z S(\underline{y}, z))$$

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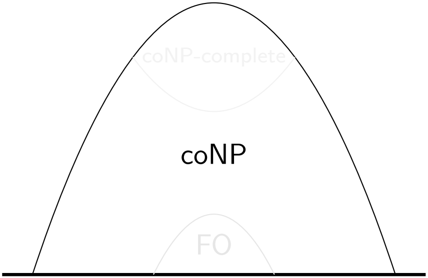

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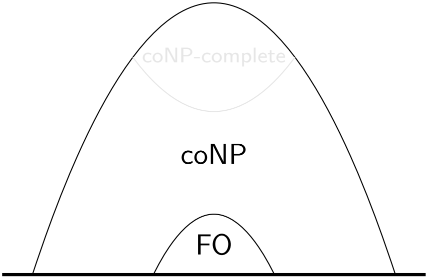

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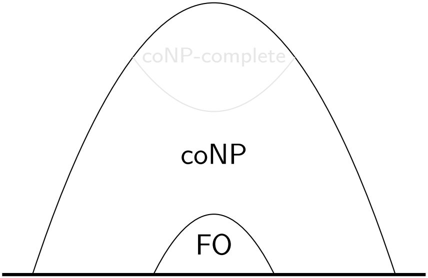

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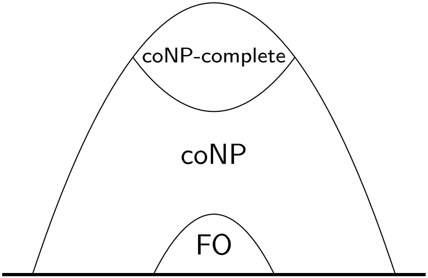

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Complexity classification

Conjecture

For any BCQ q, $\mathsf{CERTAINTY}(q)$ *is either in* PTIME *or* coNP*-complete.*

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Classification	BCQ Class	Result
FO, non-**FO**	$\mathcal{C}_{forest}$ (SJF$_{\text{(self-join-free)}}$)	[Fuxman and Miller, ICDT'05]
FO, non-**FO**	SJF α-acyclic queries	[Wijsen, PODS'10]
P, **coNP**-comp.	SJF two atoms	[Kolaitis and Pema, 12]
P, **coNP**-comp.	SJF simple keys	[Koutris and Suciu, ICDT'14]
FO, **P** $\setminus$ **FO**, **coNP**-comp.	SJF	[Koutris and Wijsen, PODS'15]
FO, **L**-comp., **coNP**-comp. $\star$	SJF	[Koutris and Wijsen, ICDT'19]
FO	SJF path	(implied by [KW, PODS'15])

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$$q() : -R_1(\underline{x_1}, x_2), R_2(\underline{x_2}, x_3), \cdots, R_n(\underline{x_n}, x_{n+1}) \quad \Longrightarrow \quad R_1 R_2 \cdots R_n$$

distinct variables x_i, relation names R_i

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FO, **NL**-comp., **P**-comp., **coNP**-comp.	SJF **path**	**(Our result)**

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$$q() :- R(\underline{x}, y), R(\underline{y}, z), X(\underline{z}, w) \quad \Longrightarrow \quad RRX$$

The curse of self-joins

Theorem (Deletion Propagation, TODS 2012)

For every CQ without self-joins, deletion propagation is either APX-hard or solvable (in polynomial time) by the unidimensional algorithm.

Theorem (Pricing, JACM 2015)

Let Q be a CQ without self-joins. The data complexity for PRICE(Q) *is either in* PTIME *or* NP*-complete.*

Theorem (Query resilience, PODS 2020)

Let q be a single-self-join-CQ with at most two occurrences of the self-join relation. The problem RES(q) *is either in* PTIME *or* NP*-complete.*

Theorem (Our result)

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Outline

1. Handling self-joins
2. Classification result
3. Proof sketch

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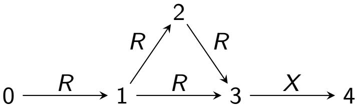
The notion of "rewinding"

INPUT to CERTAINTY(RRX):

R	$\underline{A_1}$	A_2
	0	1
	1	2
	1	3
	2	3

X	$\underline{B_1}$	B_2
	3	4



QUESTION: Do all repairs contain an RRX path?

r_1

RRX *RRX* *RRX*

r_2

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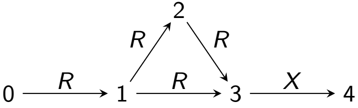

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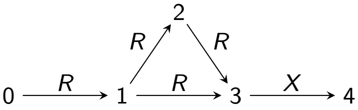

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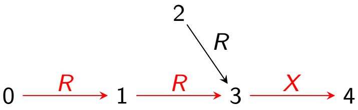

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r_2

RRX RRX

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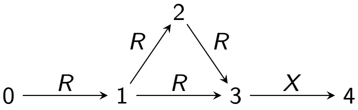

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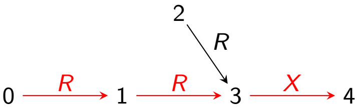

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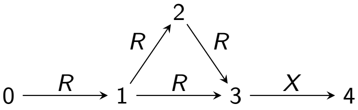

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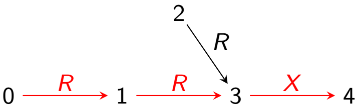

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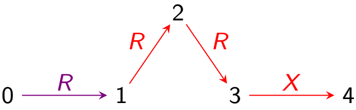

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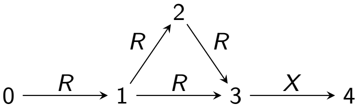

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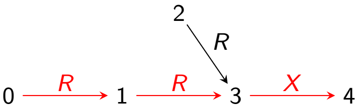

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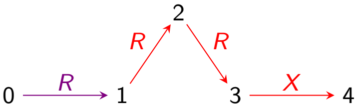

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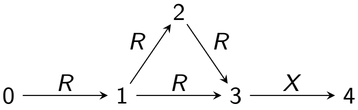

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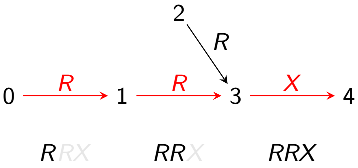

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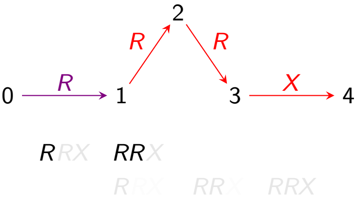

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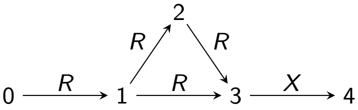

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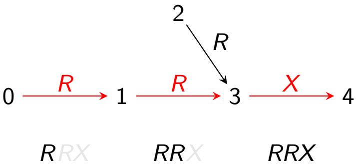

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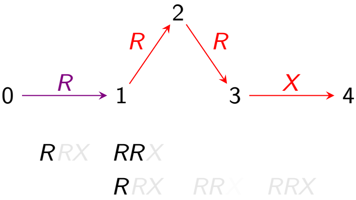

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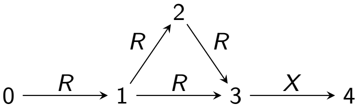

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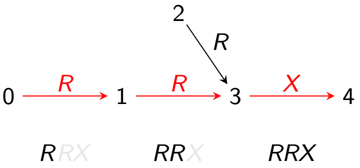

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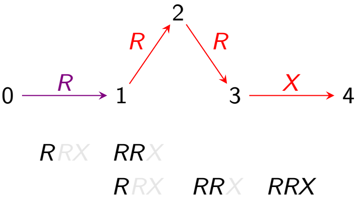

The notion of “rewinding” (cont.)

Proposition

The following statements are equivalent:

1. db *is a “yes”-instance for* CERTAINTY(RRX)*; and*
2. $\exists c$ *such that in all repairs, there exists a path of* $\underline{RR \cdot R^* \cdot X}$ *starting at* c.

“Reachability”, “**NL**-complete”

How to find the regular expression?

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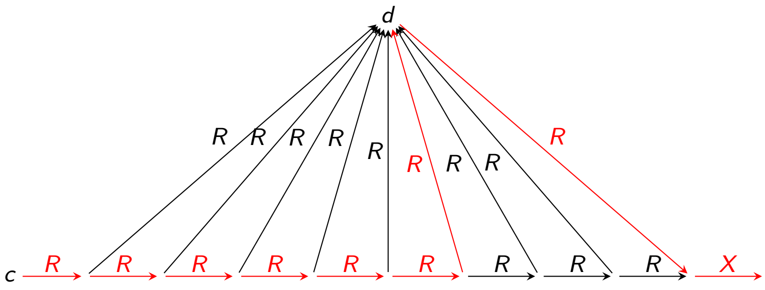

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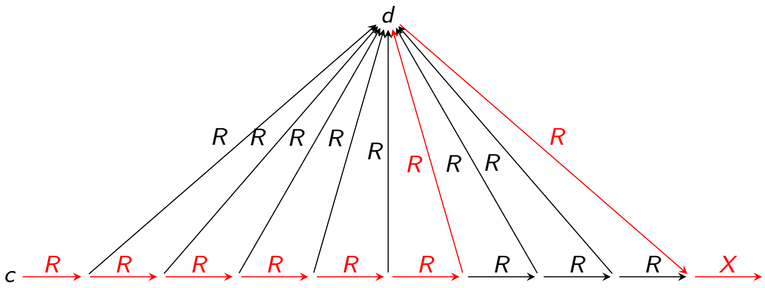

“Reachability”, “**NL**-complete”

How to find the regular expression?

From path query to NFA

NFA(RRX)

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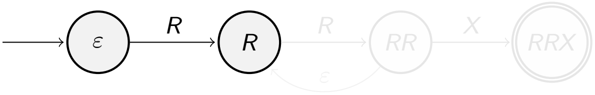

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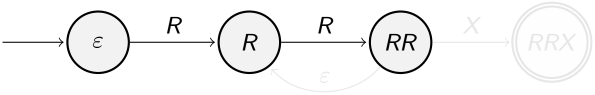

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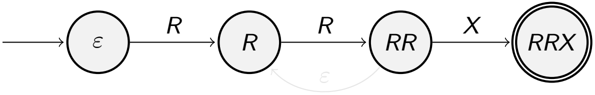

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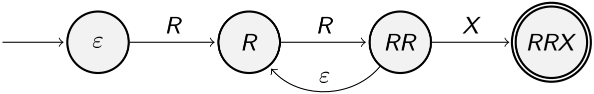

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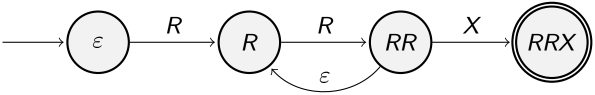
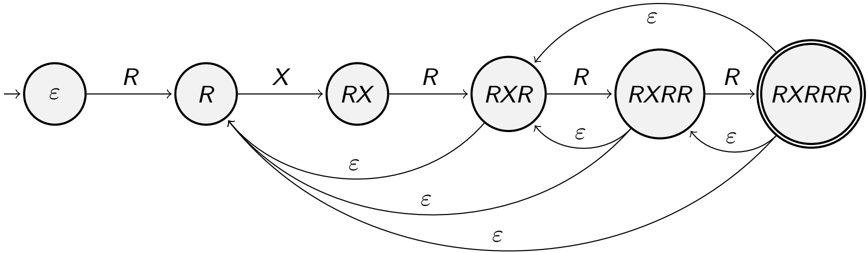


NFA(RRX)

From path query to NFA (cont.)



NFA(*RXRRR*)

Outline

Our result

NL-hard

$q_2 = RXRY$ $\qquad$ $RXRXRY \in \mathrm{NFA}(q_2)$

C$_1$: q is a prefix of every word in NFA(q)

$q_1 = RXRX$ $\qquad$ $RXRXRX \in \mathrm{NFA}(q_1)$

FO-rewritable

Our result

coNP-complete $q_4 = RXRXRYRY$ $RXRXRYRXRYRY \in \text{NFA}(q_4)$

C_3: q is a factor of every word in $\text{NFA}(q)$

PTIME

NL-hard

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Our result

coNP-complete

$q_4 = RXRXRYRY \qquad RXRXRYRXRYRY \in \mathsf{NFA}(q_4)$

C_3: q is a factor of every word in $\mathsf{NFA}(q)$

PTIME

$q_3 = RXRYRY$

C_2: Whenever $q = uRvRw$, q is a factor of $uRvRvRw$; and whenever $q = uRv_1Rv_2Rw$ for consecutive occurrences of R, $v_1 = v_2$ or Rw is a prefix of Rv_1.

NL-hard

$q_2 = RXRY \qquad RXRXRY \in \mathsf{NFA}(q_2)$

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PTIME-complete

$q_3 = RXRYRY$

C$_2$: Whenever $q = uRvRw$, q is a factor of $uRvRvRw$; and whenever $q = uRv_1Rv_2Rw$ for consecutive occurrences of R, $v_1 = v_2$ or Rw is a prefix of Rv_1.

NL-complete

$q_2 = RXRY \qquad RXRXRY \in \text{NFA}(q_2)$

C$_1$: q is a prefix of every word in NFA(q)

$q_1 = RXRX \qquad RXRXRX \in \text{NFA}(q_1)$

FO-rewritable

C_1, C_2 and C_3 are decidable

C_1 : q is a prefix of every word in NFA(q)

$\Longleftrightarrow$ Whenever $q = u \cdot Rv \cdot Rw$, q is a prefix of $u \cdot Rv \cdot Rv \cdot Rw$.

C_3 : q is a factor of every word in NFA(q)

$\Longleftrightarrow$ Whenever $q = u \cdot Rv \cdot Rw$, q is a factor of $u \cdot Rv \cdot Rv \cdot Rw$.

Outline

Lemma (**PTIME**)

Let q be a path query satisfying C_3. The following statements are equivalent:

1. db is a "yes"-instance for CERTAINTY(q); and
2. $\exists c$ such that in all repairs, there exists a path accepted by NFA(q) starting at c.

Moreover, item 2 can be decided in **PTIME** using dynamic programming/least fixedpoint logic.

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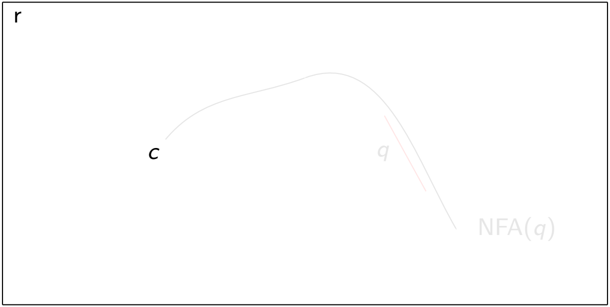

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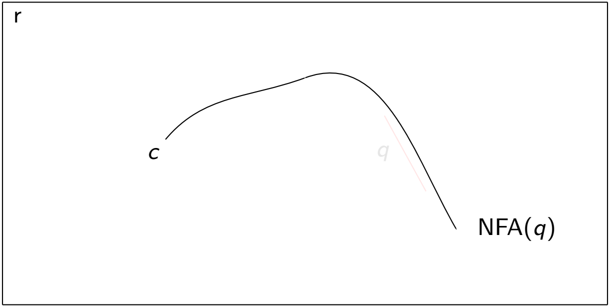

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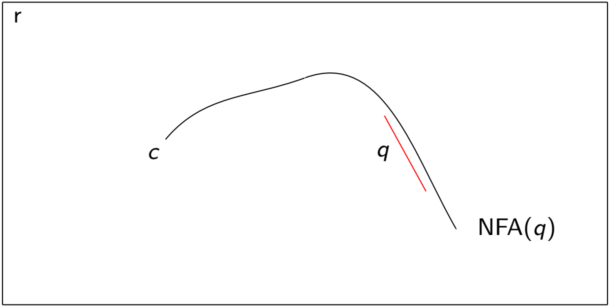
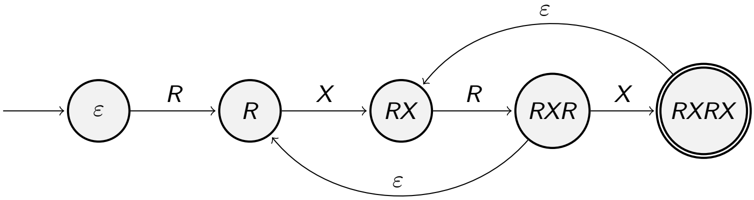


FO-rewritability

C$_1$: *q is a prefix of every word in* NFA(q)



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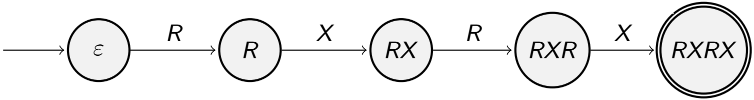

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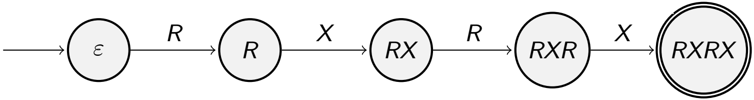


Lemma (**FO**)

Let q be a path query satisfying C_1. The following statements are equivalent:

1. db is a "yes"-instance for CERTAINTY(q); and
2. $\exists c$ such that in all repairs, there exists a path of $\underline{q}$ starting at c.

Moreover, item 2 can be decided in **FO**.

Hardness results

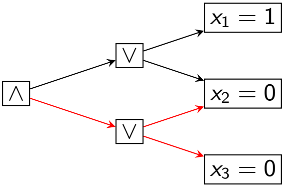
Lemma

Let q be a path query. Then we have

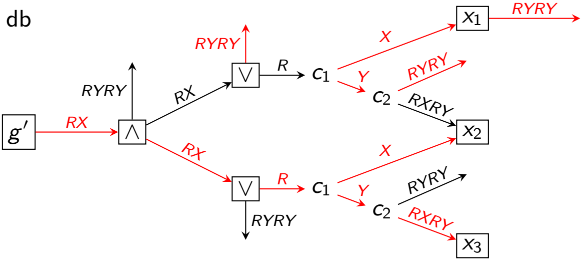
- if q does not satisfy C_1, then CERTAINTY(q) is **NL**-hard (via Reachability);
- if q does not satisfy C_2, then CERTAINTY(q) is **PTIME**-hard (via Monotone Circuit Value); and
- if q does not satisfy C_3, then CERTAINTY(q) is **coNP**-hard (via Satisfiability).

$$q = \underbrace{\quad}_{u}\underbrace{RX}_{Rv_1}\underbrace{RY}_{Rv_2}\underbrace{RY}_{Rw}$$

$\mathcal{C}$



db



Future works

Conjecture

Let q be a CQ. Then we have

- CERTAINTY(q) *is either in* PTIME *or* coNP*-complete, and it is decidable which of the two cases applies; and*
- *it is decidable whether or not* CERTAINTY(q) *is in* FO*.*

- Acyclic queries with self-joins
- Multiple key constraints, negated atom, aggregation etc.

Conclusion

coNP-complete	$q_4 = RXRXRYRY$	$RXRXRYRXRYRY \in \mathrm{NFA}(q_4)$

C$_3$: q is a factor of every word in NFA(q)

PTIME-complete	$q_3 = RXRYRY$	

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