Stat/For/Hort 571 — Final Exam, Fall 99 — Partial Solutions

1. (a) The ANOVA table looks like:

Source	df	SS	MS
Trt	4	683.3	178.83
Error	50	2656.4	53.13
Total	54	3339.7	

Thus $F = 3.215$ and $.01 < \text{p-value} < .05$.

Also, LSD $= T_{50,.025} \times s_p\sqrt{2/11} = 2.009 \times 3.108 = 6.244$. This leads to the table:

C A D E B

We conclude that B is significantly different from C, A, and D at $\alpha = .05$ and there are no other significant differences.

(b) Here we want to test the null hypothesis $H_0 : p_{ND} = p_{NE} = p_{OK}$ where the ps are the probabilities of contamination for the states North Dakota, Nebraska, and Oklahoma.

The observed data are:

4	16	10
38	41	50

The pooled estimate of the probability of contamination is $30/159 = .1887$. Hence, the expected data are:

7.92	10.75	11.32
34.08	46.25	48.68

Thus, $\mathcal{X}^2 = 1.94 + 2.56 + .15 + .45 + .60 + .04 = 5.74$ on 2 df. We have $.05 < \text{p-value} < .10$ and thus there is weak evidence against H_0. (The expected values are all greater than 5, so the chi-squared approximation is okay.)

2. (a) The ANOVA table looks like:

Source	df	SS	MS
Trt	2	580238.0	290119
Error	13	626804.3	48215.7
Total	15	1207042.3	

whence $F = 6.02$ on 2 and 13 df, $.01 < \text{p-value} < .05$ and there is good evidence against H_0.

(b) Let $C_i = (\mu_s + \mu_p)/2 - \mu_r$ and $C_{ii} = \mu_s - \mu_p$. For the standard error of the first contrast, we have $s_{C_i} = s_p\sqrt{\frac{1/4}{5} + \frac{1/4}{4} + \frac{1}{7}} = 110.96$. Thus $T = -250.9/110.96 = -2.26$ on 13 df. The comparison-wise p-value for this test is therefore between .02 and .05. For C_{ii}, we have $s_{C_{ii}} = s_p\sqrt{\frac{1}{5} + \frac{1}{4}} = 147.30$. Hence $T = 411/147.30 = 2.79$. For this contrast, the comparison-wise p-value is between .01 and .02. For C_i the Bonferroni p-value is between .04 and .10, and there is weak evidence against H_0 for the first hypothesis. For C_{ii}, the Bonferroni p-value is between .02 and .04, and there is moderate evidence against the corresponding null hypothesis.

3. (a) $\hat{b}_1 = \frac{\sum x_i y_i - \frac{1}{n}(\sum x_i)(\sum y_i)}{\sum x_i^2 - \frac{1}{n}(\sum x_i)^2} = 36660/908950 = .0403$ and $\hat{b}_0 = 44.5 - .0403 \times 857.5 = 9.94$.

(b) The ANOVA table looks like:

Source	df	SS	MS
Regr	1	1478.6	1478.6
Error	6	471.6	78.567
Total	7	1950	

So, we have $T = \frac{.0403 - .075}{\sqrt{78.567/908950}} = -0.0347/.00930 = -3.73$ on 6 df. Thus $.002 < \text{p-value} < .01$ and there is strong evidence against H_0.

(c) We have $\hat{Y}_{pred} = 9.94 + .0403 \times 1100 = 54.27$ with $\text{se}(\hat{Y}_{pred}) = \sqrt{78.567(1 + \frac{1}{8} + \frac{58806.25}{908950})} = 9.668$. The prediction interval is therefore: $54.27 \pm 9.668 \times T_{6,.025}$ or 54.27 ± 23.66.

4. (a) For the first group we have $\bar{y}_1 = 4.4$, $s_1^2 = .876$. for the second group, $\bar{y}_2 = 3.3$ and $s_2^2 = .84$. Hence $s_p^2 = .866$ and $T = \frac{1.1}{\sqrt{.866(1/6+1/3)}} = 1.67$ on 7 df. The p-value is between .1 and .2, and thus there is no evidence of a difference in the two groups.

(b) i. If we could know the values of those leaves, then using those values would make $\bar{y}_2$ smaller, and thus there would be more of a difference between the two group means. Although we do not know what would happen to the estimated variance, it is possible that the two groups would become significantly different.

ii. We could try plugging in some small number for the missing values, like 2.0, or 1.5 or 1, and see what effect that has on the results. If the conclusions do not vary much, then it follows that the choice of value we plug in is not too important. We might also consider a nonparametric analysis to pursue this.

5. Using the binomial formula with $p = .8$ for the red coin and $p = .9$ for the green coin, we can make a table like this:

Number of Heads	Probability of Number of Heads: Red Coin	Green Coin
5	.3277	—
4	.4096	.6561
3	.2048	.2916
2	.0512	.0486
1	.0065	.0036
0	.0003	.0001

Then: $P(reject) = P(5\text{ H on red \& 4 H on green}) + P(4\text{ H on red \& 4 H on green}) + P(5\text{ H on red \& 3 H on green}) = (.3277 \times .6561) + (.4096 \times .6561) + (.3277 \times .2916) = .580$.